Reservation-Based Distributed Medium Access in Wireless Collision Channels

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ABSTRACT

We consider an uplink wireless collision channel, shared by multiple mobile users. As part of the medium access protocol, channel reservation is carried out by using request-to-send (RTS) and clear-to-send (CTS) control packets. Consequently, collisions are reduced to the relatively short periods where mobiles request channel use. In our model, users are free to schedule their individual channel requests, while the objective of each user is to minimize its own power investment subject to a minimum-throughput demand. Our analysis reveals that for feasible throughput demands, there exist exactly two Nash equilibrium points in stationary strategies, with one being superior to the other uniformly for all users. We then show how this better equilibrium point can be obtained through a distributed mechanism. Finally, we discuss the optimal design of the reservation periods, while considering capacity, power and delay tradeoffs.

Keywords

Collision Channel, Channel Reservations, Nash Equilibrium

1. INTRODUCTION

1.1 Background and Motivation

Current wireless networks consist of a relatively large number of users with heterogeneous Quality of Service (QoS) requirements (such as bandwidth, delay, and power). To reduce the management complexity, decentralized control of such networks is often to be preferred to centralized one. This requirement leads to distributed (or at least partially distributed) network domains, in which end-users take autonomous decisions regarding their network usage, based on their individual preferences. This framework is naturally formulated as a non-cooperative game, and has recently been an active research area (see e.g. [11] for a recent survey).

In the context of wireless networks, self-interested user behavior can be harmful, as network resources (such as bandwidth) are usually limited, and might be abused by a subset of greedy users. Consequently, a central question that arises in the design and management of networks is the following: What is the *right* degree of freedom that should be given to end-users during network operation? This dilemma is incorporated in mechanism or protocol design, as by restricting users to some protocol rules, an adequate performance level can be preserved, despite user selfishness.

In this paper, we examine a distributed access control mechanism, in which a reservation mechanism is applied to efficiently use the shared medium. Specifically, we adopt 802.11's virtual carrier sense mechanism [1]: A mobile station ready to transmit, first sends a short control packet, called RTS (Request to Send), which includes the source, destination and duration of the transaction (according to data-length). The base station then responds to the RTS request by sending an CTS (Clear to Send) packet with the same information as above.

As part of the reservation protocol, a mobile station that hears a CTS message addressed to another station will not use the channel until the end of the current transaction. This virtual carrier sense mechanism essentially reduces the probability of collision to the short duration of the RTS transmission. We note that non-interference with on-going transmissions may be seen to be in the self-interest of any legitimate user of the channel, and strictly so when the protocol mandates priority to interrupted transmissions. We do not consider here malicious users (jammers), whose sole interest is to interfere with the performance of other users.

Several recent papers have analyzed non-cooperative user-behavior over wireless collision channels (see [2, 5, 3, 8, 10, 9]), yet none seems to have incorporated reservation mechanisms as part of the model. In particular, the basic framework of this paper is similar to [8, 10, 9], where the first two papers consider rate-based equilibria in collision channels with fading, and the third considers the same model with power-level control. A more general capture channel model has been considered in [7].

1.2 Contribution and Paper organization

In this paper we study the RTS/CTS reservation mechanism under the assumption that mobiles are free to schedule their individual channel requests. The objective of each user is to minimize its average power investment subject to a minimum-throughput demand. We focus on the sim-

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plest scheduling strategy, in which each user sets a stationary probability (or rate) for its requests[1]. This user interaction may be viewed as a noncooperative game [4], the properties of which are the main focus of this work. We start by analyzing the equilibrium points of this game, focusing on their number and efficiency properties. Our analysis largely relies on the results of [8, 10], after showing that the present model (with channel reservation) may be reduced to that of [8, 10] through a simple transformation. Based on our analysis, we focus on system *design and control* issues in two significant directions:

- Leading the network to a power-efficient equilibrium in a distributed manner.

- Tuning the reservation and data transmission periods based on capacity and power versus delay tradeoff.

The paper is organized as follows. The networking model, along with basic properties thereof, is presented in Section 2. In Section 3 we analyze the basic equilibrium properties of the underlying noncooperative game. Section 4 focuses on distributed mechanisms for leading the network to an efficient equilibrium point. In Section 5 we address the issue of setting the length of the data transmission period. Conclusions and future research directions are outlined in Section 6.

2. THE MODEL

Our model consists of a finite set of mobile users $\mathcal{I} = \{1, \ldots, n\}$ who connect to a common base station over a shared channel. We focus on the uplink direction, where users transmit their data to the base station.

2.1 The Network Model

Medium Access Protocol. Time is slotted, and the access to the channel is obtained as follows. All mobiles wishing to send data ask for transmission permission from the base station by sending short RTS packets. The base station in turn sends a CTS acknowledgement to at most a single station (see below). The total duration of this RTS-CTS phase is a constant of T_1 slots duration (whether the base station sends a CTS or not). In case that the channel has not been assigned to any of the stations, the above step is repeated for the same duration of T_1 slots. Otherwise, the user that obtained the CTS occupies the channel for a fixed number of T_2 slots, called the data transmission period, in which its data is transmitted without interruption. See Figure 1 for a graphic illustration. Although the 802.11 standard allows for reserving the channel for a variable duration, we assume that all mobiles use a fixed-length data trasmission period. This assumption is commensurate with the worst-case scenario where all users always have packets to send, and therefore reserve that channel for the largest allowed duration at each transmission.

Reception Model. We assume the following.

[1]It may easily be verified that a best-response strategy for each user to a stationary multi-strategy of the others is stationary as well. Thus, an equilibrium point in stationary strategies remains an equilibrium point within the general set of non-stationary strategies, see [10].

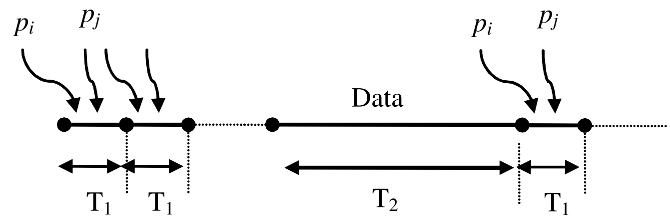

Figure 1: Illustration of the reservation-based medium access control. Each user requests the channel with some probability. In case of a single request, the channel is granted to the requesting user for a transmission period of T_2 slots.

- Simultaneous RTS transmissions of two or more users result in a collision. That is, in case of multiple requests, the base station is unable to recognize which of the users have requested the channel.

- During the data transmission period, the average (effective) data rate of user i is a constant, say C_i, determined by the station characteristics (transmission power, distance from the base station, channel gain, etc.).

2.2 User Model

We associate with each user i a fixed throughput requirement ρ_i, which stands for the (minimal) data rate that this user wishes to maintain[2]. To simplify notations, we normalize the throughput requirement for each user by C_i, its effective data rate during the data transmission period (T_2-slots). Thus, $0 < \rho_i \leq 1$.

We assume that a user always has packets to send, yet it may postpone its transmission requests in order to accommodate its required rate ρ_i (as we formally define below). Hence, each user i chooses a request probability p_i, which is the probability for sending an RTS frame in the designated time-period. We further assume that users transmit at a fixed power level (regardless of whether the transmission is an RTS packet or a data packet). Consequently, the joint strategy vector $\mathbf{p} = (p_1, \ldots, p_n)$ determines the average power investment of each user. To simplify notations, we shall normalize the per-slot power investment of each user's transmission to 1.

The underlying assumption of our model is that users are selfish and do not cooperate or coordinate in any manner in order to satisfy their throughput demands. Obviously, the transmission schedule of each user affects the throughput of all others. This state of affairs gives rise to a conflict situation, which we formalize as a non-cooperative game between the users. We are interested in the Nash equilibrium point of that game. To define the Nash equilibrium, we require some basic quantities, which are derived in the next subsection.

2.3 Basic Performance Measures

[2]We view here the throughput requirement as a lower bound mandated by the user application. Alternatively, this rate may be assigned (and policed) as an upper bound by the system as a control and management tool, leading to a similar model. In either case, the rate assignment procedure is exogenous to our model.

In this subsection we obtain explicit expressions for the rate and power-investment averages, as a function of the stationary strategy vector $\mathbf{p} = (p_1, \dots p_n)$.

Define

$$q_i \equiv q_i(\mathbf{p}) \stackrel{\triangle}{=} p_i \prod_{j \neq i} (1 - p_j)\,, \tag{1}$$

which is probability that user i alone transmits at a given RTS slot. Let $r_i(\mathbf{p})$ be user i's throughput (in slot percentage) as determined by the request probabilities of all other users.

LEMMA 1. *The long-term average throughput of user i is given by*

$$r_i(\mathbf{p}) = \frac{q_i T_2}{T_1 + \sum_j q_j T_2}. \tag{2}$$

PROOF. Let X_k, $k = 1, 2, \dots$ be the renewal process whose renewal points are the starting times of the user channel requests (i.e., transmission of an RTS packet by any user). Then $E(X_k) = T_1 + \sum_j q_j T_2$. The expected throughput of user i over the k-th renewal interval may be regarded as a reward R_k earned at the k-th renewal, so that (X_k, R_k) is a reward-renewal process [12]. The expected reward is given by $E(R_k) = q_i T_2$. Then, by Proposition 7.3 in [12],

$$r_i(\mathbf{p}) = \lim_{k \to \infty} \frac{\sum_{m=1}^{k} R(m)}{k} = \frac{E(R_k)}{E(X_k)} = \frac{q_i T_2}{T_1 + \sum_j q_j T_2}. \tag{3}$$

□

Denote by $S_i(\mathbf{p})$ the (long-term) average transmission power investment of user i. For convenience, we normalize the average power and express it as a fraction of the instantaneous power of user i during transmission. Thus, S_i is identical to the fraction of time in which user i is transmitting. Proceeding as in Lemma 1, we obtain the following expression.

LEMMA 2. *The average power investment of user i is given by*

$$S_i(\mathbf{p}) = \frac{p_i (T_1 + \frac{q_i}{p_i} T_2)}{T_1 + \sum_j q_j T_2}. \tag{4}$$

2.4 Game Formulation

A Nash equilibrium point (NEP) for our model is a vector of request probabilities $\mathbf{p} = (p_1, \dots, p_n)$, which is self-sustaining in the sense that all throughput constraints are met, and neither user can lower its average energy investment by unilaterally modifying its transmission request probability. Formally,

DEFINITION 2.1 (EQUILIBRIUM POINTS). *A stationary multi-strategy $\mathbf{p} \stackrel{\triangle}{=} (p_1, \dots, p_n)$ is a Nash equilibrium point if*

$$p_i \in \operatorname*{argmin}_{0 \le \tilde{p}_i \le 1} \{ S_i(\tilde{p}_i, \mathbf{p}_{-i}) : r_i(\tilde{p}_i, \mathbf{p}_{-i}) \ge \rho_i \} \tag{5}$$

for each $i \in \mathcal{I}$, where S_i and r_i are defined in (4) and (2), respectively, and $\mathbf{p}_{-i}$ stands for the vector $\mathbf{p}$ excluding its i-the element p_i.

Noting that both functions S_i and r_i are strictly increasing in p_i, it follows that the above minimization over $\tilde{p}_i$ is equivalent to satisfying the second inequality with equality, namely $r_i(\mathbf{p}) = \rho_i$. We thus obtain the following equivalent characterization of a Nash equilibrium point $\mathbf{p}$ in our model:

$$r_i(\mathbf{p}) \equiv \frac{q_i(\mathbf{p}) T_2}{T_1 + \sum_j q_j(\mathbf{p}) T_2} = \rho_i \tag{6}$$

for every user i. We shall refer to (6) as the *equilibrium equations*.

3. EQUILIBRIUM ANALYSIS

We start our analysis by reformulating the equilibrium equations in an equivalent form that is more convenient for analysis. To that end, denote

$$\rho = \sum_{i=1}^{n} \rho_i$$

(the total throughput demand of all users), and let $\alpha = \frac{\rho}{1-\rho}$.

PROPOSITION 1. *The equilibrium equations (6) are equivalent to the following set of equations:*

$$q_i(\mathbf{p}) = \tilde{\rho}_i\,, \quad i \in \mathcal{I} \tag{7}$$

where

$$\tilde{\rho}_i = \frac{\rho_i (1 + \alpha) T_1}{T_2} = \frac{\rho_i T_1}{(1 - \rho) T_2}\,. \tag{8}$$

PROOF. We sum the equilibrium equations (6) over all users i and obtain the following equation

$$\frac{\sum_i q_i T_2}{T_1 + \sum_j q_j T_2} = \rho. \tag{9}$$

Therefore, $\frac{T_1}{T_2 \sum_i q_i} + 1 = \frac{1}{\rho}$, or $\frac{T_1}{T_2 \sum_i q_i} = \frac{1-\rho}{\rho}$. Thus,

$$\sum_i q_i = \frac{\alpha T_1}{T_2}. \tag{10}$$

By substituting (10) into (6) we obtain that $\frac{q_i T_2}{T_1 + \alpha T_1} = \rho_i$, which leads to (7). □

The significance of the last proposition is that our equilibrium equations reduce to those of the model without reservations studied in [8], with the throughput requirements of the users set to the modified rates $\tilde{\rho}_i$. Consequently, the following results from [8] carry over to the current model. By *feasible throughput region* we refer to the set of throughput requirement vectors $(\rho_1, \dots, \rho_n)$ for which there exists at least one equilibrium point.

THEOREM 2 ([8, 10]). *(i) The feasible throughput region is a non-empty and closed set.*
(ii) In every interior point of the feasible region, there exist exactly two Nash equilibria, denoted $\mathbf{p}^a$ and $\mathbf{p}^b$, and one of those (say $\mathbf{p}^a$) is uniformly better than the other in the sense that $p_i^a < p_i^b$ for every user i.
(iii) For the (better) equilibrium point $\mathbf{p}^a$, it holds that

$$\sum_i p_i^a < 1\,. \tag{11}$$

We note that any feasible throughput vector in the modified model must satisfy $\sum_i \tilde{\rho}_i \le 1$ (fully-utilized channel), which is equivalent here to $\rho = \sum_i \rho_i \le \frac{T_2}{T_1+T_2}$. Clearly, except for the case of a single user, the total throughput ρ will be smaller due to possible collisions. In fact, it was shown in [8, 10] that for many small users, the maximal channel throughput approaches e^{-1}, which translates to $\rho \approx \frac{T_2}{eT_1+T_2}$ in the present model. Some further properties of the throughput region will be discussed in Section 5.1 below.

We next provide a simple expression for the average power investment at equilibrium.

THEOREM 3. *The average power investment in any equilibrium* $\mathbf{p}$ *is linear in* p_i *and given by*

$$S_i(\mathbf{p}) = \rho_i + (1-\rho)p_i. \tag{12}$$

PROOF. Observe that the shared channel is either in reservation mode (where users send channel requests) or in data mode (where users send data packets). In every equilibrium point $\mathbf{p}$, each users consumes a fraction of ρ_i in data mode, which is the first summand in (12). Overall, ρ is the fraction of time in which the channel is in data mode, while a fraction of $1-\rho$ is spent for reservations. Since user i is active in reservation mode with probability p_i, its corresponding power investment in that mode is given by $(1-\rho)p_i$, which is the second summand in (12). □

Note that the expression (12) does not include the terms T_1 and T_2. However, the power investment is a function of p_i, which is a function of these parameters through the equilibrium equations (7). Observe further that the power investment in equilibrium is linear in p_i. The immediate conclusions from both Theorem 2 and Theorem 3 are summarized below.

COROLLARY 1. *Let* $\mathbf{p}^a$ *and* $\mathbf{p}^b$ *be the two equilibrium points, such that* $p_i^a < p_i^b$ *for every user* i*. Then,*
(i) The power investment at $\mathbf{p}^a$ *is strictly lower for each user, namely* $S_i(\mathbf{p}^a) < S_i(\mathbf{p^b})$ *holds for every* i*.*
(ii) $\sum_i S_i(\mathbf{p}^a) \leq 1$*: the total power investment at the better equilibrium* $\mathbf{p}^a$ *is bounded by* 1*.*

PROOF. (i) is immediate from (12) and Theorem 2.(ii). Part (ii) follows by summing (12) over all users and by (11). □

It is interesting to note that the total (normalized) power at the better equilibrium is never higher than for the single-user, fully utilized channel case. This bound does not hold in for the other equilibrium point.

The above results clearly motivate the design and study of distributed algorithms that converge to the *better* equilibrium $\mathbf{p}^a$. This is the subject of the next section.

4. CONVERGENCE TO THE BETTER EQUILIBRIUM

A Nash equilibrium point for our system represents a strategically stable working point, from which no user has incentive to deviate unilaterally. Still, the question of if and how the system arrives at an equilibrium needs to be addressed. Furthermore, since our system has two Nash equilibria with one strictly better than the other, it is of major importance (from the system viewpoint, as well as for each individual user) to employ mechanisms that converge to the better equilibrium rather than the other, worse equilibrium point.

We start by considering the natural best-response scheme, where each user reacts to the observed system conditions by adjusting its strategy to achieve its required throughput. We analyze the convergence of this scheme, and discuss the feasibility of its distributed implementation. We then consider a simplified scheme, the naive best response, which is easier to implement and may seem more natural to less sophisticated users. We note that the two schemes coincide when channel reservations are not employed (as in [8, 10]).

4.1 Best Response Dynamics

A best-response strategy for a given user is generally defined as the optimal strategy for that user given the strategies of all other users (see [4]). In our model, the best response for user i is the transmission request probability that equalizes the user's throughput with its throughput demand ρ_i. Observing (6), the best response $\tilde{p}_i$ of user i to a multi-strategy $\mathbf{p} = (p_1, \ldots, p_n)$ is the probability that solves

$$r_i(\tilde{p}_i, \mathbf{p}_{-i}) = \frac{q_i T_2}{T_1 + \sum_j q_j T_2} = \rho_i\,. \tag{13}$$

An explicit expression for $\tilde{p}_i$ is given below in (15).

Our mechanism can now be described as follows: Each user updates its transmission probability from time to time through its best response map, namely to satisfy (13). The update times of each user need not be coordinated with other users.

This mechanism reflects what myopic, self-interested users would naturally do: Repeatedly observe the current network conditions and react to bring their throughput to the required level. For the analysis of the best-response scheme, we assume the following.

ASSUMPTION 1.
(i) Fixed Demands: *The user population and the users' throughput requirements are fixed.*
(ii) Frequent Updates: *Users repeatedly update their transmission probabilities (i.e., an infinite number of updates for each user) using Eq. (13) .*
(iii) Slow Start: *The initial transmission probabilities* $\mathbf{p}^0$ *of the users are set to low values, so that for each user i (a) the individual probabilities are not larger than their better-equilibrium values:* $p_i \leq p_i^a$*, and (b) the resulting throughput is not larger than the demand:* $r_i(\mathbf{p}) \leq \rho_i$*.*

The slow-start requirements may be satisfied individually by each user by selecting any $p_i^0 \leq \rho_i$.

Our convergence result is summarized below.

THEOREM 4. *Assume that* $\rho = (\rho_1, \ldots, \rho_n)$ *is feasible. Then under Assumption 1, the best response dynamics asymptotically converges to the better equilibrium.*

The proof proceeds by showing that the vector of user strategies $\mathbf{p}$ monotonously increases until convergence. The details are provide in the Appendix.

The Slow Start requirement (Assumption 1(iii)) is a key one. This (or similar) requirement is essential to ensure convergence to the better equilibrium. For example, if all users start at the worse equilibrium value, they will evidently stay at that equilibrium forever.

An important question is whether the users would obey this slow-start requirement. The incentive to do so is the possibility that the system may be led to the worse equilibrium otherwise. As this operating point is uniformly worse for all users, it is in the interest of each user to cooperate in a protocol that leads to the better operating point.

We next consider the practical implementation of the proposed mechanism in a distributed environment. Note that r_i can be written as

$$r_i(\mathbf{p}) = \frac{p_i f_i(\mathbf{p}_{-i})}{T_1/T_2 + p_i f_i(\mathbf{p}_{-i}) + (1-p_i) g_i(\mathbf{p}_{-i})} \qquad (14)$$

where

$$f_i(\mathbf{p}_{-i}) = \prod_{j\neq i}(1-p_j)\,, \quad g_i(\mathbf{p}_{-i}) = \sum_{j\neq i} p_j \prod_{k\neq i,j}(1-p_k)\,.$$

Both f_i and g_i now have a clear meaning and can be estimated by the user without detailed information on the other individual users. Indeed, f_i is the fraction of times where user i's transmission is successful, and g_i the fraction of successful transmissions over the channel when user i does not transmit. The latter can be counted by listening to the number of CTS's (or alternatively inferred from centralized channel utilization information provided by the base station). So, if f_i and g_i are known (for the current channel conditions), the best-response (13) is easily computed as

$$\tilde{p}_i = \frac{T_1/T_2 + g_i}{(1-\rho_i) f_i + \rho_i g_i}\,. \qquad (15)$$

We note that this aspect of estimating a global (rather than private) quantity, namely g_i, does not appear in the basic model without reservations; see [8, 10].

4.2 Naive Best-Response

As discussed above, computing the best response of each user requires some active measurement of the shared medium, as well the use of equation (14). In some cases, the users may be limited in their sensing abilities, or alternatively may be more simple-minded in their computational approach. We next provide an alternative and easier to implement mechanism, which still converges to the better equilibrium under plausible conditions.

The update rule we consider here is simple: Update the transmission request probability in proportion to the required increase (or decrease) in throughput. In more detail, let p_i denote users i's current transmission request probability, and $r_i(\mathbf{p})$ its current throughput. The modified transmission request probability is computed as

$$\tilde{p}_i = \frac{\rho_i}{r_i(\mathbf{p})}\, p_i\,. \qquad (16)$$

While the update rule above is simple to implement, we should emphasize that it does *not* lead to the required rate ρ_i in a single step (as does the best-response update), even if the other users freeze their strategies. The reason is that it does not take into account the effect of p_i on the denominator of (14). Such an approach, which neglects the users own effect on some system parameters, corresponds in the literature to *price-takeing* users, as opposed to *price-anticipating* users (see [6] and references therein).

A slightly different definition of the update rule in (16) can be given as follows. From equations (1) and (2), the rate of each user given $\mathbf{p}$ can be written as

$$r_i(\mathbf{p}) = p_i R_i(\mathbf{p})\,, \qquad (17)$$

where

$$R_i(\mathbf{p}) = \frac{T_2 \prod_{j\neq i}(1-p_j)}{T_1 + T_2 \sum_j q_j} \qquad (18)$$

R_i can be interpreted as the average rate obtained per channel request (successful or not). Clearly this quantity can be locally monitored by the user. The modified transmission request probability may now be computed as

$$\tilde{p}_i = \frac{\rho_i}{R_i(\mathbf{p})}, \qquad (19)$$

This rule reflects the (false) assumption that $R_i(\mathbf{p})$ is not affected by p_i. It is easily seen to be equivalent to (16).

The convergence properties of the proposed scheme are summarized in the next theorem.

THEOREM 5. *Consider the asynchronous mechanism defined by the update rule (16) or (19). Then, under Assumption 1, this mechanism converges to the better equilibrium point $\mathbf{p} = \mathbf{p}^a$, provided that this equilibrium satisfies*

$$\sum_i \frac{p_i}{1-p_i} < 1. \qquad (20)$$

PROOF. See the Appendix.

Condition (20) may be interpreted as a light-traffic condition. However, it is important to observe that this condition holds more generally when the system supports many "small" users. Recall from Theorem 2(iii) that the better-equilibrium probabilities always satisfy $\sum_i p_i < 1$. When each p_i is small, the latter sum well approximates the sum in (20). Indeed, we note that in this case the price-taking approach is more easily justified, as the effect of each individual on its average rate per attempt R_i becomes negligible.

5. TRADEOFFS IN DETERMINING THE DATA TRANSMISSION PERIOD

In this section we address the issue of selecting the data transmission period T_2, from the system (or base-station) point of view. The relevant measures that may influence this design decision are *capacity, power* and *delay.* We separately consider each of the three, and then point at the tradeoff while setting T_2.

5.1 Capacity

Denote by $\vec{\rho} = (\rho_1, \ldots, \rho_n)$ the vector of throughput demands, and recall that Ω is the set of *feasible* throughput vectors $\vec{\rho}$ for which there exists at least one equilibrium point. Figure 2 illustrates the set of feasible throughput demands for a simple two-user case. As noted before, Ω is a closed and non-empty set. Further, it can be seen to have the following *ray continuity* property (cf. [10]): If $\vec{\rho}$ is feasible, then so is $\alpha\vec{\rho}$ for all $0 \le \alpha \le 1$.

Consider two different transmission periods T_2 and $\tilde{T}_2$, with $\tilde{T}_2 > T_2$. We informally use the term "capacity" to denote the extent to which different throughput vectors can be accommodated by the network. The next proposition suggests that the capacity of the network increases as the transmission period increases. Moreover, we are able to quantify this increase in terms of the upper boundary.

PROPOSITION 6. *Consider some data transmission period T_2 and let $\tilde{T}_2 = \beta T_2$ with $\beta > 1$. Suppose $\vec{\rho}$ is feasible for the T_2-system, and let $\rho = \sum_i \rho_i$. Then $\tilde{\vec{\rho}} = \gamma\vec{\rho}$ is feasible for the $\tilde{T}_2$-system, where*

$$\gamma = \frac{\beta}{1 + (\beta-1)\rho}\,.$$

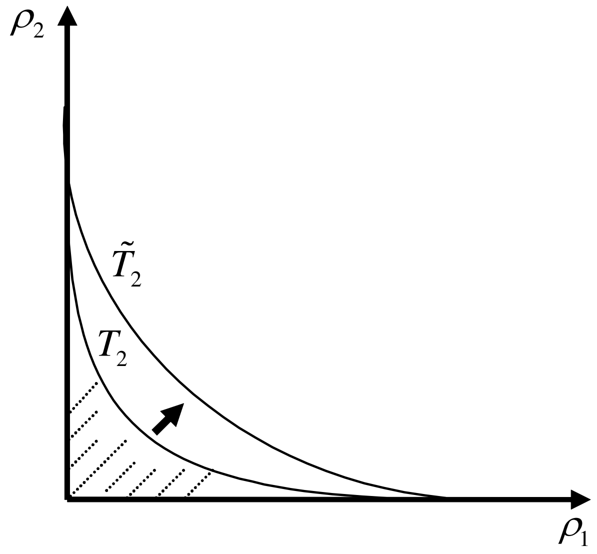

Figure 2: The set of feasible throughput demands for a two user network with two different transmission periods $\tilde{T}_2 > T_2$. The upper-boundary increases due to the increase of the data transmission period.

PROOF. Let $\mathbf{p}$ denote the equilibrium strategy vector in the T_2-system. Then $\mathbf{p}$ satisfies the modified equilibrium equations (7). Fixing $\mathbf{p}$, it is easily verified that the same equation is satisfied with $\tilde{T}_2$ and $\vec{\tilde{\rho}}$ on the right hand side. □

The factor γ represents the gain in capacity that is obtained by increasing T_2 by a factor of β. This gain is more significant at light loads ($\rho << 1$), and reaches saturation as β increases (and ρ approaches 1).

5.2 Power

An important factor in determining the data transmission period is the total power investment at equilibrium.

PROPOSITION 7. *By increasing T_2, the average power investment at the better equilibrium decreases for every user.*

PROOF. See the Appendix.

We emphasize that this result is not trivial, as it is ensured in the better equilibrium only and does not necessarily hold for the other equilibrium point.

5.3 Delay

In many networking applications, part of the traffic sent is required in *real-time*, in the sense that if the elapsed time between two data transmissions is too large, the performance of the application is severely damaged. Voice or video could be good examples for such applications. Up until now, we have focused on power and throughout as the QoS measures. In the context of determining T_2, we are also be interested in *delay*, defined as the time between two consecutive useful transmissions (of length T_2) of the user. The following theorem suggest that the delay of each user increases linearly in T_2.

THEOREM 8. *(i) The average delay in an equilibrium* $\mathbf{p}$ *is independent of the equilibrium probabilities and given by*

$$D_i = T_2/\rho_i. \tag{21}$$

Consequently, the delay of each user increases linearly in T_2.

PROOF. Denote by f_i the average frequency of useful transmissions for user i. Then $f_i T_2 = \rho_i$. Noting that $D_i = 1/f_i$ concludes the proof. □

5.4 Design Tradeoff

As may be expected, an increase in the data transmission interval T_2 (relative to the reservation control interval T_1) should lead to improved network performance in terms of both capacity and overall transmission power. As verified above, these relations indeed hold for the better equilibrium point. However, such increase also results in an undesired increase in delay.

In most protocols, it is expected that the data transmission period T_2 would be set off-line, or be modified over long time scales. A simple approach would be to choose T_2 based on (single-packet) delay constraints. In other cases, a throughput-delay tradeoff may be considered. The equilibrium model and estimates developed above enable to formulate this tradeoff in mathematical terms, and obtain corresponding optimal values for T_2. A detailed formulation of this problem and its solution will not be treated within this paper.

6. CONCLUSION

The motivation for using reservation mechanisms in random access control systems comes from their ability to reduce the congestion overhead, and thus increase capacity. However, it is not clear a-priory whether self-interested user behavior would exploit or misuse this additional feature. In this paper we have shown that the power investment at one of the equilibrium point (the better one) decreases with the increase of the data transmission period. Yet, supplementary mechanisms are required to lead the system to the better equilibrium. We proposed two such mechanisms, which correspond to different technological abilities of the mobiles. Additional mechanisms could obviously be designed and investigated, for sustaining the better working point, but also for detecting and avoiding the worse one.

Interesting extensions to the network model include channel fading, power control, and more general packet-reception and capture models (as in [8], [9] and [7], respectively). The scope of the current user-model may be enhanced as well, by incorporating stochastic packet arrivals, and studying non-stationary strategies for collision resolution within a game theoretic framework (such as 802.11's back-off mechanism). It is hoped that furthering our understanding of decentralized medium access control with selfish end-users will help to devise more flexible and robust systems in existing and emerging wireless networks.

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APPENDIX

Proofs for Sections 4 and 5.2

Theorem 4 (convergence of the best-response dynamics) may be deduced from the convergence results in [10] by utilizing the modified equilibrium equations (7). We shall provide here a direct proof, both for completeness, and as it sets the stage for the proof of Theorem 5 (naive best-response).

The following monotonicity properties will be required. We assume throughput that the strategy vector $\mathbf{p}$ has all elements in $[0, 1]$.

LEMMA 3. *The effective throughput $r_i(\mathbf{p})$ of user i is (i) strictly increasing in p_i, and (ii) strictly decreasing in p_j for each $j \neq i$.*

PROOF. Recalling equations (1) and (2), it follows after some algebra that

$$r_i(\mathbf{p}) = \left(\frac{T_1}{q_i T_2} + \sum_j \frac{h(p_j)}{h(p_i)} \right)^{-1}, \tag{22}$$

where $h(p) = \frac{p}{1-p}$. Since $h(p)$ is strictly increasing in p, the second summand in (22) obviously strictly decreases in p_i and strictly increases in p_j. Recalling again that $q_i = p_i \prod_{j \neq i}(1 - p_j)$, the first summand in (22) has the same monotonicity properties. Taking the inverse establishes the required properties. □

Let $\mathrm{BR}_i(\mathbf{p})$ denote the best-response set of user i to a strategy vector $\mathbf{p}$, that is, $\mathrm{BR}_i(\mathbf{p})$ is the set of probabilities $\bar{p}_i \in [0, 1]$ that satisfy the throughput requirement (13) for that user. When $\mathrm{BR}_i(\mathbf{p})$ is a singleton we shall use the same notation to denote its value.

LEMMA 4. *For each user i,*
(i) The best-response set $\mathrm{BR}_i(\mathbf{p})$ is either a singleton or else empty.
(ii) The best-response is monotone increasing in $\mathbf{p}$. That is, if $\mathbf{p}^1 \leq \mathbf{p}^2$ (component-wise) and $\mathrm{BR}_i(\mathbf{p}^2) \neq \emptyset$, then $\mathrm{BR}_i(\mathbf{p}^1) \neq \emptyset$, and $\mathrm{BR}_i(\mathbf{p}^1) \leq \mathrm{BR}_i(\mathbf{p}^2)$.

PROOF. Both claims follow directly from the definition of the best-response and the monotonicity properties of $r_i(\mathbf{p})$ that were established in Lemma 3. □

The following notation will be used in the remainder of this section, in reference to the user strategy updates in the best-response dynamics. Let the update times of each user i be given by an increasing sequence $\{t_i^k\}$, $k = 1, 2, 3, \ldots$. Also, let

$$\{t^k\} = \{t_1^k\} \cup \{t_2^k\} \cup \ldots \{t_n^k\} \quad , \quad k = 1, 2, \ldots.$$

Note that at each t^k at least one user updates its transmission probability. We shall use the notation p_i^k for the transmission probability of user i at time t^k (similarly, $\mathbf{p}^k$ is the transmission probability vector at time t^k), with initial conditions $\mathbf{p}^0$.

Let Assumption 1 hold. We assume throughout that the throughput requirement vector ρ is feasible, and let $\mathbf{p}^a$ denote the respective better equilibrium point.

Proof of Theorem 4: The result follows by induction. We first show that $\mathbf{p}^k$ is bounded above by $\mathbf{p}^a$. Indeed, $0 = \mathbf{p}^0 \leq \mathbf{p}^a$ by assumption, and $\mathbf{p}^k \leq \mathbf{p}^a$ implies that $\mathbf{p}^{k+1} \leq \mathbf{p}^a$ (as follows from the monotonicity of the best response, Lemma 4). It follows that $\mathbf{p}^k \leq \mathbf{p}^a$ for all k.

We next establish that the sequence $\{\mathbf{p}^k\}$ is monotone increasing. Recall that $r_i(\mathbf{p}^0) \leq \rho_i$ holds by the slow-start assumption. Since $r_i(\mathbf{p})$ is increasing in p_i, it follows that the best response of any user to $\mathbf{p}^0$ must result in increasing its transmission probability. Therefore $\mathbf{p}^0 \leq \mathbf{p}^1$. Furthermore, if $\mathbf{p}^{k-1} \leq \mathbf{p}^k$, then $\mathbf{p}^k \leq \mathbf{p}^{k+1}$ (again by Lemma 4). It follows that $\{\mathbf{p}^k\}$ is a monotone increasing sequence, bounded above by $\mathbf{p}^a$. Thus, $\mathbf{p}^k$ converges to a finite limit $\mathbf{p}^\infty \leq \mathbf{p}^a$. Since Assumption 1(ii) (frequent update) is in effect, a standard continuity argument can be used to show that this limit is a fixed point of the best response map, namely an equilibrium point. But since $\mathbf{p}^a$ is the better (i.e., smallest) equilibrium point, it follows that $\mathbf{p}^\infty = \mathbf{p}^a$. □

We turn next to the proof of Theorem 5. Recall the definition of the naive best-response in equation (17), namely $\bar{p}_i = \rho_i / R_i(\mathbf{p})$. The following properties of R_i will be instrumental in our proof.

LEMMA 5. *The function $R_i(\mathbf{p})$ is:*
(i) Strictly decreasing in p_j for each $j \neq i$.
(ii) Decreasing in p_i if (and only if) $\sum_{j\neq i} \frac{p_j}{1-p_j} \leq 1$.

PROOF. Part (i) follows immediately from Lemma 3, after observing that $R_i(\mathbf{p}) = r_i(\mathbf{p})/p_i$ (Eq. 17). As for part (ii), using (17) and (1) we can write $R_i(\mathbf{p})$ as

$$R_i(\mathbf{p})^{-1} = \frac{T_1}{T_2 \prod_{j\neq i}(1-p_j)} + p_i + (1-p_i)\sum_{j\neq i} \frac{p_j}{1-p_j}\,.$$

The derivative in p_i obviously equals $1 - \sum_{j\neq i} \frac{p_j}{1-p_j}$, which implies the required monotonicity. □

We note that condition (20) ensures that the required inequality in Lemma 5(ii) is satisfied for all i.

The next lemma establishes the key properties of the naive best response updates.

LEMMA 6. *Suppose $\mathbf{p}^a$ satisfies the condition in (20). Let $\mathbf{p}$ be a strategy vector that satisfies the slow-start requirements, namely: (a) $\mathbf{p} \leq \mathbf{p}^a$, (b) $r_i(\mathbf{p}) \leq \rho_i$ for each i.*
Let $\bar{\mathbf{p}}$ be obtained from $\mathbf{p}$ by letting some user i employ a naive best-response, namely $\bar{p}_i = \rho_i/R_i(\mathbf{p})$, and $\bar{p}_j = p_j$ for $j \neq i$. Then $\bar{p}_i \geq p_i$, and $\bar{\mathbf{p}}$ satisfies the slow-start requirements (a) and (b) above.

PROOF. Note first that $\mathbf{p} \leq \mathbf{p}^a$ implies that $\mathbf{p}$ satisfies (20) as well, since the function $\frac{x}{1-x}$ is strictly increasing in $x \in [0, 1]$. Recall that $r_i(\mathbf{p}) = p_i R_i(\mathbf{p})$. Since $r_i(\mathbf{p}) \leq \rho_i$, it follows that $\bar{p}_i = \rho_i/R_i(\mathbf{p}) \geq p_i$. Next, to establish the slow-start property (a), we need to show that $\bar{p}_i \leq p_i^a$. This is done by comparing $\bar{p}_i$ with the (exact) best response $\tilde{p}_i = BR_i(\mathbf{p})$ of user i to the strategy vector $\mathbf{p}$. Recall that $\tilde{p}_i$ satisfies (13), namely $r_i(\tilde{p}_i, \mathbf{p}_{-i}) = \rho_i$, or equivalently $\tilde{p}_i R_i(\tilde{p}_i, \mathbf{p}_{-i}) = \rho_i$. Now, it was argued in the proof of Theorem 4 that $p_i \leq \tilde{p}_i \leq p_i^a$. Therefore, by Lemma 5(ii), it follows that $R_i(\tilde{p}_i, \mathbf{p}_{-i}) \leq R_i(p_i, \mathbf{p}_{-i}) \equiv R_i(\mathbf{p})$. This, in turn, implies that

$$\bar{p}_i = \frac{\rho_i}{R_i(\mathbf{p})} \leq \frac{\rho_i}{R_i(\tilde{p}_i, \mathbf{p}_{-i})} = \tilde{p}_i \leq p_i^a\,.$$

We finally need to establish the slow-start property (b). Noting that $\bar{p}_i \geq p_i$ (as shown above), it follows by Lemma 3(ii) and our choice of $\mathbf{p}$ that $r_j(\bar{\mathbf{p}}) \leq r_j(\mathbf{p}) \leq \rho_j$ for each $j \neq i$. Hence, it remains only to show that $r_i(\bar{\mathbf{p}}) \leq \rho_i$. But this readily follows from $\bar{p}_i \leq \tilde{p}_i$, which implies that $r_i(\bar{\mathbf{p}}) \equiv r_i(\bar{p}_i, \mathbf{p}_{-i}) \leq r_i(\tilde{p}_i, \mathbf{p}_{-i}) = \rho_i$. □

Proof of Theorem 5: Let $\{\mathbf{p}^k\}$ be the sequence of strategy vectors obtained by the naive best-response scheme. By our assumptions $\mathbf{p}^0$ satisfied the requirements of Lemma 6. It may be seen that the conclusions of this lemma hold true even if several users update their strategy simultaneously. Therefore $\mathbf{p}^1 \geq \mathbf{p}^0$, and $\mathbf{p}^1$ satisfies the requirements of Lemma 6 as well. Proceeding by induction, and using arguing as in the proof of Theorem 4, we may now establish that $\mathbf{p}^k$ converges to $\mathbf{p}^a$, as claimed. □

Proof of Proposition 7: Fix $\rho = (\rho_1, \ldots \rho_n)$ and consider two different transmission periods $\hat{T}_2$ and $\bar{T}_2$ such that $\hat{T}_2 > \bar{T}_2$. Then the equilibrium equations (7) for $\hat{T}_2$ and $\bar{T}_2$ are $q_i = \hat{\rho}_i$ and $q_i = \bar{\rho}_i$, where $\hat{\rho}_i < \bar{\rho}_i$ for every user i. We shall refer to the vectors $\hat{\rho} = (\hat{\rho}_1, \ldots, \hat{\rho}_n)$ and $\bar{\rho} = (\bar{\rho}_1, \ldots, \bar{\rho}_n)$ as modified demands (to distinguish these quantities from the actual demands ρ). The proof of the theorem then immediately follows from the next lemma.

LEMMA 7. *Let $\hat{\rho}$ and $\bar{\rho}$ be two modified demand vectors such that $\hat{\rho} < \bar{\rho}$ (component-wise), and let $\hat{\mathbf{p}}$ and $\bar{\mathbf{p}}$ denote the request probabilities at the respective better equilibria. Then $\hat{\mathbf{p}} < \bar{\mathbf{p}}$.*

PROOF. For the proof, we track the best response dynamics with parallel updates (where t_i^k does not depend on i), which are guaranteed to converge to an equilibrium point by Theorem 4. We next show that $\bar{\mathbf{p}}^{\mathbf{k}} \geq \hat{\mathbf{p}}^k$ for every k, thus also at the limit. Note that since $r_i(\bar{p}_i^1, \mathbf{0}) = \bar{\rho}_i \geq r_i(\hat{p}_i^1, \mathbf{0}) = \hat{\rho}_i$, then by the monotonicity of r_i, $\tilde{p}_i^1 \geq p_i^1$ for every i. At the next iteration, $r_i(\bar{p}_i^2, \bar{\mathbf{p}}_{-i}^1) = \bar{\rho}_i \geq r_i(\hat{p}_i^1, \hat{\mathbf{p}}_{-i}^1) = \hat{\rho}_i$. Since $\bar{\mathbf{p}}_{-i}^1 \geq \hat{\mathbf{p}}_{-i}^1$, it follows that $\bar{p}_i^2 \geq \hat{p}_i^2$ for every i. The same argument carries over to subsequent iteration, thus it is valid also at the limit. □

Proposition 7 follows immediately from this lemma by noting (12). □