Optical Solitons and Their Numerical Simulations of Coupled Nonlinear Schrödinger's Equation in a Cascaded System

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Abstract. This work focuses on the coupled nonlinear Schrödinger's equation which appears in a cascaded three-level atomic system. By using the trial solution technique and the symbolic computation method, the exact bright-dark soliton, dark-bright soliton and singular soliton solutions are obtained. The propagation properties of the above solitons are simulated.

Keywords: Coupled nonlinear Schrödinger's equation · Bright-dark soliton · Dark-bright soliton · Singular soliton · Numerical simulations

1 Introduction

Nonlinear phenomena, which widely exists in various scientific fields, are usually characterized by nonlinear partial differential equations called governing equations (GEs). It is a very important topic to study their exact analytical solutions of these GEs since these solutions can help one easily research the dynamical behaviors and nonlinear phenomena. The nonlinear Schrödinger equation, as one of the most famous governing equations, is a significant mathematical model in different areas such as nonlinear optics [1–8], finance [9], biophysics [10] and so on. Moreover the Schrödinger equation has many mathematical features and has been widely applied in nonlinear optical communications. Along with the growing interest in nonlinear phenomena, coupled nonlinear Schrödinger's (CNLS) equations have also attracted a lot of attentions [11–13]. The CNLS equations can be used to describe the interaction among the modes in the case of birefringent or other two-mode fibers [14,15] and the solitary waves in CNLS equations called

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vector solitons have more rich phenomena and complex dynamics. In the past decades, the related research achievements on CNLS equations have emerged in endlessly [16–24].

In this paper, we focus on the propagation of two intense optical beam of different frequencies in a three-level atomic system in the cascade configuration. This cascade system is governed by a coupled nonlinear Schrödinger's equation. The coupled Schrödinger's equation is such an equation whose unknown functions are interrelated. We try to construct exact and explicit optical soliton solutions by means of the trial solution technique and the symbolic computation method [25,26]. As a result, exact coupled bright-dark soliton, dark-bright soliton and singular soliton solutions are obtained and the propagation properties of these solitons are simulated.

2 Governing Equation

We consider two coupled optical beams of different frequencies propagating in the cascaded three-level atomic system governed by the following coupled nonlinear Schrödinger's equation [27]

$$\mathrm{i}a_1 q_{1,\,t} + b_1 q_{1,\,xx} + c_1 |q_2|^2 q_1 = 0, \tag{1a}$$

$$\mathrm{i}a_2 q_{2,\,t} + b_2 q_{2,\,xx} + (c_2 |q_1|^2 + d_2 |q_2|^2) q_2 = 0, \tag{1b}$$

where $q_1(x,t)$ and $q_2(x,t)$ are the dependent variables representing the complex-valued wave profile. Moreover a_j, b_j and c_j $(j = 1, 2)$ respectively are the temporal evolution coefficients, the group velocity dispersion coefficients and the cross-phase modulation while d_2 is the self-phase modulation [28]. Konar et al. [27] studied the existence and stability of soliton solutions to (1) and derived an existence curve of stable soliton pairs. Recently, Bhrawy et al. [28] obtained some bright-bright and dark-dark soliton solutions by applying the ansatz method.

The goal in this paper is to find the bright-dark soliton, dark-bright soliton and singular soliton to (1), which belong to different type solitons and have different dynamical behaviors from the results in [28].

To find the exact solutions of (1), we make the wave transformation

$$q_1(x,t) = Q_1(\xi)\,\mathrm{e}^{\mathrm{i}\eta_1}, \tag{2a}$$

$$q_2(x,t) = Q_2(\xi)\,\mathrm{e}^{\mathrm{i}\eta_2}, \tag{2b}$$

where $Q_1(\xi)$ and $Q_2(\xi)$ stand for amplitude components of the traveling waves. η_1 and η_2 in (2) are phase components of the traveling waves given by

$$\eta_j = -k_j x + \omega_j t + \phi_j,\ j = 1, 2, \tag{3}$$

where k_j, ω_j and ϕ_j $(j = 1, 2)$ are wave numbers, frequencies and phase constants. And ξ in (2) is the wave variable defined by

$$\xi = x - vt + \xi_0, \tag{4}$$

where v and ξ_0 respectively are wave velocity and mean positions of $Q_1(\xi)$ and $Q_2(\xi)$. From Eqs. (2)–(4), it is clear that

$$\mathrm{i}\, q_{1,\,t} = -\left(\omega_1 Q_1 + \mathrm{i}\, v Q_1'\right) \mathrm{e}^{\mathrm{i}\eta_1}, \tag{5a}$$

$$q_{1,\,xx} = \left(Q_1'' - k_1^2 Q_1 - 2\mathrm{i}\, k_1 Q_1'\right) \mathrm{e}^{\mathrm{i}\eta_1}, \tag{5b}$$

$$\mathrm{i}\, q_{2,\,t} = -\left(\omega_2 Q_2 + \mathrm{i}\, v Q_2'\right) \mathrm{e}^{\mathrm{i}\eta_2}, \tag{5c}$$

$$q_{2,\,xx} = \left(Q_2'' - k_2^2 Q_2 - 2\mathrm{i}\, k_2 Q_2'\right) \mathrm{e}^{\mathrm{i}\eta_2}, \tag{5d}$$

where the primes denote the derivatives with respect to ξ. Substituting (5) into (1) and decomposing it into real and imaginary parts, it follows from the real parts of the two components that

$$b_1 Q_1'' - \left(a_1\omega_1 + b_1 k_1^2\right) Q_1 + c_1 Q_2^2 Q_1 = 0, \tag{6a}$$

$$b_2 Q_2'' - \left(a_2\omega_2 + b_2 k_2^2\right) Q_2 + \left(c_2 Q_1^2 + d_2 Q_2^2\right) Q_2 = 0. \tag{6b}$$

From the imaginary parts of the two components, we can derive the wave velocity as

$$v = -\frac{2b_1 k_1}{a_1}, \tag{7}$$

and

$$v = -\frac{2b_2 k_2}{a_2}. \tag{8}$$

From (7) and (8), we obtain the following constraint relation

$$a_1 b_2 k_2 = a_2 b_1 k_1, \quad a_1 a_2 \neq 0. \tag{9}$$

From (9), we have

$$k_2 = \frac{a_2 b_1 k_1}{a_1 b_2}, \tag{10}$$

which shows that the ratio of the wave numbers k_1 and k_2 depend on the evolutions and the dispersions. To consider the exact solutions to (1), we must only discuss the real part equations given by (6) under the constraint relation (9).

3 Bright-Dark Soliton Solution and Its Numerical Simulations

3.1 Bright-Dark Soliton Solution

In this subsection, we will construct the bright-dark soliton solution of (1). By using the following transformations

$$Q_1(\xi) = A_1 \operatorname{sech}^{p_1}(B\xi), \tag{11a}$$

$$Q_2(\xi) = A_2 \tanh^{p_2}(B\xi), \tag{11b}$$

where A_1 and A_2 are the amplitudes of the solitons while B is the inverse width of the solitons. Substituting (11) into (6) and balancing the terms Q_1'' between

$Q_2^2 Q_1$ in (6a) along with Q_2'' between $Q_1^2 Q_2$ in (6b), we obtain that $p_1 = p_2 = 1$ and therefore yield the following equations:

$$\left(c_1 A_2^2 + b_1 B^2 - a_1\omega_1 - b_1 k_1^2\right) - \left(c_1 A_2^2 + 2b_1 B^2\right) \operatorname{sech}^2(B\xi) = 0, \quad (12a)$$

$$\left(d_2 A_2^2 - a_2\omega_2 - b_2 k_2^2\right) + \left(c_2 A_1^2 - d_2 A_2^2 - 2b_2 B^2\right) \operatorname{sech}^2(B\xi) = 0. \quad (12b)$$

Making all the constant terms and the coefficients of $\operatorname{sech}^2(B\xi)$ in (12) equal to zero, we get a series of algebraic equations

$$c_1 A_2^2 + b_1 B^2 - a_1\omega_1 - b_1 k_1^2 = 0, \quad (13a)$$

$$c_1 A_2^2 + 2b_1 B^2 = 0, \quad (13b)$$

$$d_2 A_2^2 - a_2\omega_2 - b_2 k_2^2 = 0, \quad (13c)$$

$$c_2 A_1^2 - d_2 A_2^2 - 2b_2 B^2 = 0. \quad (13d)$$

Solving (13) gives

$$A_1 = \sqrt{\frac{2(b_1 d_2 - b_2 c_1)}{-c_1 c_2}}\, B, \quad (14a)$$

$$A_2 = \sqrt{\frac{2b_1}{-c_1}}\, B, \quad (14b)$$

$$\omega_1 = -\frac{b_1}{a_1}\,(k_1^2 + B^2), \quad (14c)$$

$$\omega_2 = -\frac{2b_1 d_2}{a_2 c_1}\, B^2 - \frac{b_2}{a_2}\, k_2^2, \quad (14d)$$

with the constraint conditions $b_1 c_1 < 0$ and $c_1 c_2(b_1 d_2 - b_2 c_1) < 0$.

Substituting (11) and (14) into (2), we get a bright-dark soliton solution for (1)

$$q_1(x,t) = \sqrt{\frac{2(b_1 d_2 - b_2 c_1)}{-c_1 c_2}}\, B \operatorname{sech}\left[B\,(x - vt + \xi_0)\right]\, e^{\mathrm{i}\,(-k_1 x + \omega_1 t + \phi_1)}, \quad (15a)$$

$$q_2(x,t) = \sqrt{\frac{2b_1}{-c_1}}\, B\, \tanh\left[B\,(x - vt + \xi_0)\right]\, e^{\mathrm{i}\,(-k_2 x + \omega_2 t + \phi_2)}. \quad (15b)$$

3.2 Numerical Simulations of Propagation Properties of the Bright-Dark Soliton

In this subsection, we will graphically discuss the numerical simulations of the propagation properties of the bright-dark soliton for (1). Based on (15), we have

$$\chi_1(x,t) = \sqrt{\frac{2(b_1 d_2 - b_2 c_1)}{-c_1 c_2}}\, B \operatorname{sech}\left[B\,(x - vt + \xi_0)\right], \quad (16a)$$

$$\chi_2(x,t) = \sqrt{\frac{2b_1}{-c_1}}\, B\, \tanh\left[B\,(x - vt + \xi_0)\right], \quad (16b)$$

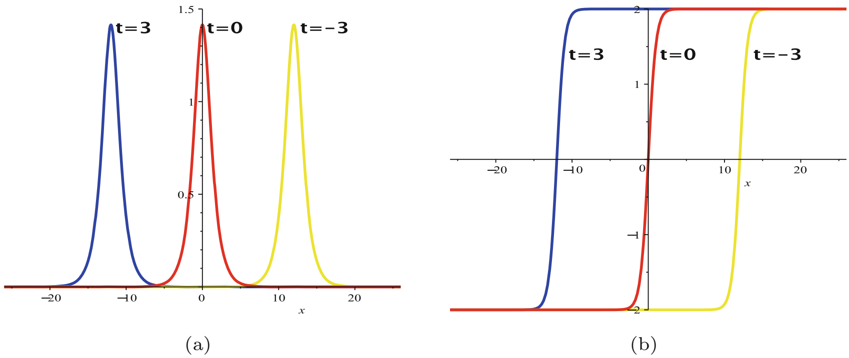

Fig. 1. The propagation of the bright-dark soliton wave (15) of (1) by choosing suitable parameters: $b_1 = 2$, $B = d_2 = c_2 = a_1 = k_1 = 1$, $c_1 = b_2 = -1$, $\xi_0 = 0$. (a) Wave propagation pattern of $\chi_1(x,t)$ along the x axis. (b) Wave propagation pattern of $\chi_2(x,t)$ along the x axis.

where χ_1 and χ_2 denote the amplitudes for the components q_1 and q_2 respectively. χ_1 achieves the maximum $\sqrt{\frac{2(b_1 d_2 - b_2 c_1)}{-c_1 c_2}}\,|B|$ if and only if $x - vt + \xi_0 = 0$ and tends to the minimum if $x - vt + \xi_0 \to \pm\infty$. And χ_2 tends to the maximum $\sqrt{\frac{2b_1}{-c_1}}\,|B|$ as $x - vt + \xi_0 \to +\infty$ and the minimum $-\sqrt{\frac{2b_1}{-c_1}}\,|B|$ as $x - vt + \xi_0 \to -\infty$. Thus q_1 is bright while q_2 is dark. The bright-dark soliton maintains its shape, velocity and amplitude during the propagation. The propagation of the bright-dark soliton is presented in Fig. 1.

4 Dark-Bright Soliton Solution and Its Numerical Simulations

4.1 Dark-Bright Soliton Solution

For dark-bright soliton, the hypothesis for Q_1 and Q_2 will be

$$Q_1(\xi) = A_1 \tanh^{p_1}(B\xi), \tag{17a}$$

$$Q_2(\xi) = A_2 \operatorname{sech}^{p_2}(B\xi). \tag{17b}$$

Substituting (17) into (6) and balancing the terms Q_1'' between $Q_2^2 Q_1$ in (6a) along with Q_2'' between $Q_1^2 Q_2$ in (6b), we get $p_1 = p_2 = 1$ and therefore obtain the equations

$$(a_1\omega_1 + b_1 k_1^2) - \left(c_1 A_2^2 - 2b_1 B^2\right) \operatorname{sech}^2(B\xi) = 0, \tag{18a}$$

$$(c_2 A_1^2 + b_2 B^2 - a_2\omega_2 - b_2 k_2^2) - \left(c_2 A_1^2 - d_2 A_2^2 + 2b_2 B^2\right) \operatorname{sech}^2(B\xi) = 0. \tag{18b}$$

Setting the constant terms and the coefficients of $\mathrm{sech}^2(B\xi)$ in (18) equal to zero gives the algebraic equations

$$a_1\omega_1 + b_1k_1^2 = 0, \tag{19a}$$
$$c_1A_2^2 - 2b_1B^2 = 0, \tag{19b}$$
$$c_2A_1^2 + b_2B^2 - a_2\omega_2 - b_2k_2^2 = 0, \tag{19c}$$
$$c_2A_1^2 - d_2A_2^2 + 2b_2B^2 = 0. \tag{19d}$$

From (19), we have

$$A_1 = \sqrt{\frac{2(b_1d_2 - b_2c_1)}{c_1c_2}}\,B, \tag{20a}$$
$$A_2 = \sqrt{\frac{2b_1}{c_1}}\,B, \tag{20b}$$
$$\omega_1 = -\frac{b_1}{a_1}k_1^2, \tag{20c}$$
$$\omega_2 = \frac{2b_1d_2}{a_2c_1}\,B^2 - \frac{b_2}{a_2}\,(k_2^2 + B^2), \tag{20d}$$

with the constraint relations $b_1c_1 > 0$ and $c_1c_2(b_1d_2 - b_2c_1) > 0$.

Substituting (20) and (17) into (2), we get the dark-bright soliton solution for (1)

$$q_1(x,t) = \sqrt{\frac{2(b_1d_2 - b_2c_1)}{c_1c_2}}\,B\,\tanh\left[B\,(x - vt + \xi_0)\right]\,\mathrm{e}^{\mathrm{i}\,(-k_1x+\omega_1t+\phi_1)}, \tag{21a}$$
$$q_2(x,t) = \sqrt{\frac{2b_1}{c_1}}\,B\,\mathrm{sech}\left[B\,(x - vt + \xi_0)\right]\,\mathrm{e}^{\mathrm{i}(-k_2x+\omega_2t+\phi_2)}. \tag{21b}$$

4.2 Numerical Simulations of Propagation Properties of the Dark-Bright Soliton

In this subsection, we discuss the numerical simulations of the propagation properties of the dark-bright soliton for (1). It follows from (21) that

$$\delta_1(x,t) = \sqrt{\frac{2(b_1d_2 - b_2c_1)}{c_1c_2}}\,B\,\tanh\left[B\,(x - vt + \xi_0)\right], \tag{22a}$$
$$\delta_2(x,t) = \sqrt{\frac{2b_1}{c_1}}\,B\,\mathrm{sech}\left[B\,(x - vt + \xi_0)\right], \tag{22b}$$

where δ_1 and δ_2 denote the amplitudes for the components q_1 and q_2 in (21) respectively. δ_1 tends to the maximum $\sqrt{\frac{2(b_1d_2-b_2c_1)}{c_1c_2}}\,|B|$ as $x - vt + \xi_0 \to +\infty$ and the minimum $-\sqrt{\frac{2(b_1d_2-b_2c_1)}{c_1c_2}}\,|B|$ as $x - vt + \xi_0 \to -\infty$. And δ_2 achieves the maximum $\sqrt{\frac{2b_1}{c_1}}|B|$ if and only if $x - vt + \xi_0 = 0$ and tends to the minimum if $x - vt + \xi_0 \to \pm\infty$. Hence q_1 is dark and q_2 is bright. The dark-bright soliton maintains its shape, velocity and amplitude during the propagation. The propagation of the dark-bright soliton is shown in Fig. 2.

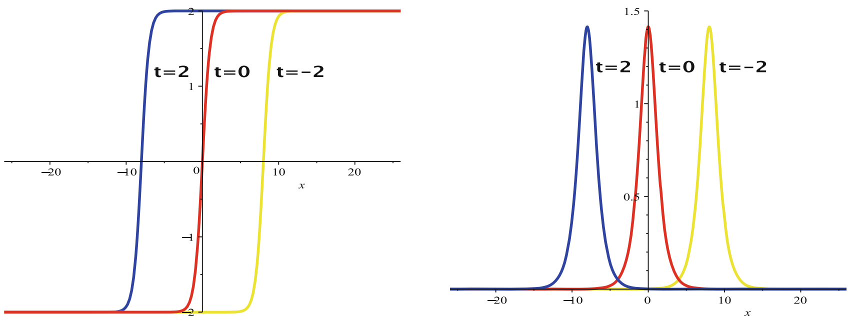

Fig. 2. The propagation of the dark-bright soliton wave (21) for (1) by choosing suitable parameters: $b_1 = 2$, $B = d_2 = c_1 = c_2 = b_2 = a_1 = k_1 = 1$, $\xi_0 = 0$. (a) Wave propagation pattern of $\delta_1(x,t)$ along the x axis. (b) Wave propagation pattern of $\delta_2(x,t)$ along the x axis.

5 Singular Soliton Solutioin and Its Numerical Simulations

5.1 Singular Soliton Solutioin

For singular soliton solutions, we assume that the solutions of (1) take the forms

$$Q_1 = A_1 \coth^{p_1}(B\xi), \tag{23a}$$
$$Q_2 = A_2 \coth^{p_2}(B\xi). \tag{23b}$$

Substituting (23) into (6) and balancing the terms Q_1'' between $Q_2^2 Q_1$ in (6a) along with Q_2'' between $Q_1^2 Q_2$ in (6b) gives $p_1 = p_2 = 1$. Substituting (23) with $p_1 = p_2 = 1$ into (6) again respectively yields

$$(c_1 A_2^2 + 2b_1 B^2) - (2b_1 B^2 + a_1\omega_1 + b_1 k_1^2)\tanh^2(B\xi) = 0, \tag{24a}$$
$$(c_2 A_1^2 + d_2 A_2^2 + 2b_2 B^2) - (2b_2 B^2 + a_2\omega_2 + b_2 k_2^2)\tanh^2(B\xi) = 0. \tag{24b}$$

Making all the constant terms and the coefficients of $\tanh^2(B\xi)$ in (24) equal to zero gives the following algebraic equations

$$c_1 A_2^2 + 2b_1 B^2 = 0, \tag{25a}$$

$$2b_1 B^2 + a_1\omega_1 + b_1 k_1^2 = 0, \tag{25b}$$

$$c_2 A_1^2 + d_2 A_2^2 + 2b_2 B^2 = 0, \tag{25c}$$

$$2b_2 B^2 + a_2\omega_2 + b_2 k_2^2 = 0. \tag{25d}$$

It follows from (25) that

$$A_1 = \sqrt{\frac{2(b_1 d_2 - b_2 c_1)}{c_1 c_2}}\, B, \tag{26a}$$

$$A_2 = \sqrt{\frac{2b_1}{-c_1}}\, B, \tag{26b}$$

$$\omega_1 = -\frac{b_1}{a_1}(k_1^2 + 2B^2), \tag{26c}$$

$$\omega_2 = -\frac{b_2}{a_2}(k_2^2 + 2B^2), \tag{26d}$$

with the constraint relations $b_1 c_1 < 0$ and $c_1 c_2(b_1 d_2 - b_2 c_1) > 0$.

We thus obtain the singular soliton solution of (1)

$$q_1(x,t) = \sqrt{\frac{2(b_1 d_2 - b_2 c_1)}{c_1 c_2}}\, B \coth\left[B\left(x - vt + \xi_0\right)\right] \mathrm{e}^{\mathrm{i}\,(-k_1 x + \omega_1 t + \phi_1)}, \tag{27a}$$

$$q_2(x,t) = \sqrt{\frac{2b_1}{-c_1}}\, B \coth\left[B\left(x - vt + \xi_0\right)\right] \mathrm{e}^{\mathrm{i}(-k_2 x + \omega_2 t + \phi_2)}. \tag{27b}$$

5.2 Numerical Simulations of Propagation Properties of the Singular Soliton

In this subsection, we discuss the numerical simulations of the propagation properties of the singular soliton of (1). It follows from (27) that

$$\rho_1(x,t) = \sqrt{\frac{2(b_1 d_2 - b_2 c_1)}{c_1 c_2}}\, B \coth\left[B\left(x - vt + \xi_0\right)\right], \tag{28a}$$

$$\rho_2(x,t) = \sqrt{\frac{2b_1}{-c_1}}\, B \coth\left[B\left(x - vt + \xi_0\right)\right], \tag{28b}$$

where ρ_1 and ρ_2 denote the amplitudes for the components q_1 and q_2 in (27) respectively. Since $\rho_1 \to \infty$ and $\rho_2 \to \infty$ as $x - vt + \xi_0 \to 0$, namely, both ρ_1 and ρ_2 blow up in finite time. Therefore (27) is a singular soliton solution for (1) and also maintains its shape, velocity and amplitude during the propagation. The propagation of the singular soliton is presented in Fig. 3.

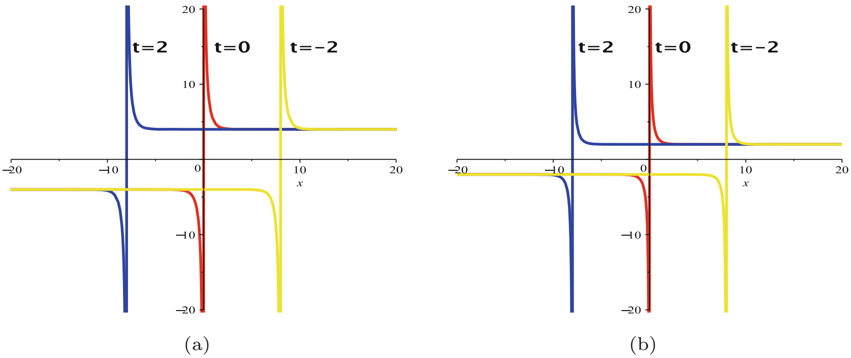

Fig. 3. The propagation of the singular soliton wave (27) for (1) by choosing suitable parameters: $b_1 = 2$, $B = d_2 = a_1 = k_1 = 1$, $c_1 = b_2 = -1$, $c_2 = -\frac{1}{8}$, $\xi_0 = 0$. (a) Wave propagation pattern of $\rho_1(x,t)$ along the x axis. (b) Wave propagation pattern of $\rho_2(x,t)$ along the x axis.

6 Conclusion

In this work, we investigate the coupled nonlinear Schrödinger equation and obtain the exact bright-dark soliton, dark-bright soliton and singular soliton solutions by employing the trial solution technique and the symbolic computation method. The propagation properties of these solutions are numerically simulated, which help one better understand their dynamical properties.

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