



On the Researches of Mixed User Grouping NOMA Systems

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Abstract. This paper introduces the concept of a mixed grouping strategies for NOMA systems, analyzes its advantages over traditional OMA and fixed grouping strategies for NOMA systems, and proposes two feasible mixed grouping algorithms based on genetic algorithm and simulated annealing algorithm to improve system channel capacity. Through simulation verification, we find that the NOMA system with mixed grouping exhibits significant advantages in terms of system capacity and fairness. Furthermore, compared to brute force search algorithms, the proposed mixed grouping algorithms can significantly reduce computational complexity with only minor performance loss. Therefore, it can be concluded that the proposed mixed grouping algorithms strikes a good balance between performance and computational complexity. Additionally, this paper further explores the potential for implementing more complex NOMA system mixed grouping algorithms using machine learning methods in future scenarios.

Keywords: Non-orthogonal multiple access (NOMA) · Mixed grouping strategies · Genetic algorithms · Simulated annealing algorithms

1 Introduction

Non-orthogonal Multiple Access (NOMA) [1,2] is a promising wireless access technology for next generation cellular communication systems. Compared to

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traditional orthogonal multiple access (OMA), NOMA offers several advantages, including higher spectral efficiency, increased channel capacity, and improved user fairness. General NOMA includes power domain NOMA (PD-NOMA) and code domain NOMA (such as Spatial Division Multiple Access, SDMA), while narrow NOMA specifically refers to PD-NOMA. The focus of this study is on narrow NOMA, i.e., PD-NOMA.

In comparison to traditional OMA techniques like Time Division Multiple Access (TDMA), NOMA provides higher bandwidth utilization and can achieve greater channel capacity while supporting more users. This makes it well suited for meeting the requirements of enhanced mobile broadband (eMBB) high capacity and massive machine type communication (mMTC) multiple connections in the future communication systems. In the context of NOMA, power domain is fully utilized by serving multiple users with the same resource block at the same time frequency resource.

At the transmitter end, superposition coding (SC) [3] is used in NOMA; at the receiver end, successive interference cancellation (SIC) [4, 5] is employed. Due to SIC's characteristics, receiving users need to decode other large signal users' data first before decoding their own data. This introduces decoding delay and error propagation issues where errors from one user may affect others within the group leading to degraded performance. Given these challenges associated with error propagation and decoding delay in multi-user scenarios under a typical two-user grouping approach that limits research scope due to complexity concerns related to multi-user scenarios or mixed grouping based studies are lacking.

In our previous studies [6], we can observe that different user grouping strategies perform differently in various scenarios. Therefore, there must exist a grouping strategy that maximizes system capacity. This strategy is likely to be hybrid, combining OMA with NOMA. Furthermore, NOMA is not limited to consistent group sizes and can include two-user NOMA, three-user NOMA, or even more users in NOMA. However, finding the optimal user grouping is extremely challenging because for a system with N users, there are $N!$ possible ways to divide the users arbitrarily. For systems with a large number of users, it is impractical to obtain the best user grouping strategy through brute force search. Henceforth, we need to identify an algorithm with lower computational complexity and better performance that approaches optimality. In this paper, we employ genetic algorithms and simulated annealing-two classic algorithms for optimizing planning-to determine the optimal user grouping strategy.

2 Optimal Fixed Grouping

Before delving into the study of mixed user grouping, we first investigate the optimal fixed user grouping. We assume that there are a total of N users in the NOMA system, evenly distributed into M groups, with each group containing K users; thus, it is evident that $N = MK$. Herein, $\pi_{m,k}$ denotes the k th user in the m th group of the NOMA system, π_m represents the union of all K users in the m th group, $\beta_{m,k}$ signifies the power allocation coefficient for the k th user in the

m th group, and $\gamma_{\pi_{m,k}}$ indicates signal-to-noise ratio (SNR) for all K users within m -th group. Consequently, we derive formulas for capacity within a multi-cluster NOMA system:

$$C_{sum} = K \sum_{m=1}^M \sum_{k=1}^K \log\left(1 + \frac{\beta_{\pi_{m,k}} \gamma_{\pi_{m,k}}}{1 + \sum_{k'=1}^{k-1} \beta_{\pi_{m,k'}} \gamma_{\pi_{m,k}}}\right) \quad (1)$$

Therefore, in the absence of specific targets such as minimum data rate and user fairness, and under the assumption of a fixed number of NOMA groups, the objective function for NOMA grouping that simply pursues maximum channel capacity is:

$$\pi^* = \arg \max_{\pi} C_{sum}, \quad (2)$$

The symbol π^* denotes the optimal fixed user grouping scheme, while the expression for the total capacity C_{sum} is given by

$$C_{sum} = K \sum_{m=1}^M \sum_{k=1}^K \log\left(1 + \frac{\beta_{\pi_{m,k}} \gamma_{\pi_{m,k}}}{1 + \sum_{k'=1}^{k-1} \beta_{\pi_{m,k'}} \gamma_{\pi_{m,k}}}\right). \quad (3)$$

Due to our adoption of a fixed power allocation scheme, and the consistent power allocation scheme for users within each cluster, we have

$$\beta_{\pi_{m,k}} = \beta_k = \delta \sum_{k'=1}^{k-1} \beta_{\pi_{m,k'}} = \delta \sum_{k'=1}^{k-1} \beta_{k'}, \quad (4)$$

Thus, the formula for capacity can be reformulated as:

$$\begin{aligned} C_{sum} &= K \sum_{m=1}^M \sum_{k=2}^K \log\left(\frac{\delta + (1 + \delta)\beta_k \gamma_{\pi_{m,k}}}{\delta + \beta_k \gamma_{\pi_{m,k}}}\right) + K \sum_{m=1}^M \log(1 + \beta_1 \gamma_{\pi_{m,1}}) \\ &= K \log\left(\prod_{m=1}^M \prod_{k=2}^K \frac{\delta + (1 + \delta)\beta_k \gamma_{\pi_{m,k}}}{\delta + \beta_k \gamma_{\pi_{m,k}}} (1 + \beta_1 \gamma_{\pi_{m,1}})\right). \end{aligned} \quad (5)$$

Therefore, the objective function for the optimal group assignment problem becomes

$$\pi^* = \arg \max_{\pi} C'_{sum}, \quad (6)$$

where

$$C'_{sum} = \prod_{m=1}^M \prod_{k=2}^K \frac{\delta + (1 + \delta)\beta_k \gamma_{\pi_{m,k}}}{\delta + \beta_k \gamma_{\pi_{m,k}}} (1 + \beta_1 \gamma_{\pi_{m,1}}). \quad (7)$$

Theorem 1. *The optimal fixed grouping scheme for an NOMA system with N users, divided into M groups, each containing K users, to maximize the sum system capacity is as follows:*

$$\pi_{m,k}^* = M(k - 1) + m. \tag{8}$$

The above scheme can be succinctly described as follows: Firstly, users are sorted in descending order of channel gain. The user with the highest channel gain is designated as user 1, while the user with the lowest channel gain is denoted as user K . Despite the fact that, due to small scale fading considerations, the distance from user 1 to the base station may not necessarily be shorter than that from user K to the base station, we still conventionally refer to user 1 as the near end user and user K as the far end user. Once all users have been sorted, users 1, users $M + 1$ through to users $M(K - 1) + 1$ form the first group, with individual IDs ranging from 1 to K serving as a basis for power allocation. Users 2 through $M + 2$ up to $M(K - 1) + 2$ constitute subsequent groups until all users have been allocated into exactly M groups.

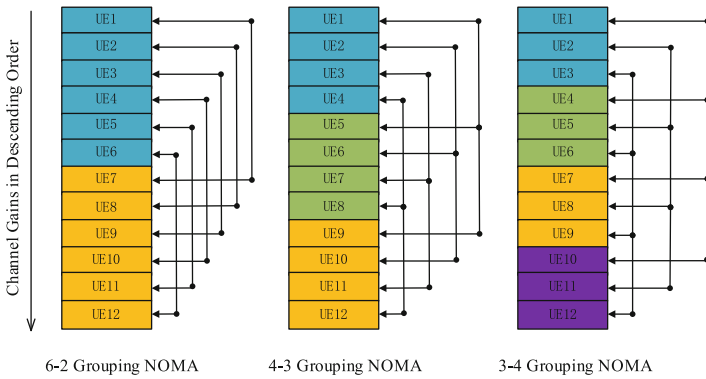


Fig. 1. Three different user grouping strategies in NOMA with 12 users. UE stands for user equipment. The users connected by the arrow lines are grouped into the same cluster. The users in the same color hold the same position in their own clusters.

Figure 1 illustrates various grouping schemes for a NOMA system with 12 users. When comparing different user grouping NOMA systems, we assume that the total power transmitted by the base station is constant. Additionally, within each grouping scheme, the power allocated to each group is also uniform; however, the power distribution among users within each group follows our predetermined fixed power allocation scheme.

3 Mixed User Grouping Algorithms

3.1 Genetic Algorithms

Genetic Algorithms (GAs) were introduced in 1975 by Professor Holland and his research team at the University of Michigan, USA, as a comprehensive

theoretical framework and methodology [7]. GAs mimic the natural selection and evolutionary mechanisms of “survival of the fittest” in nature to construct artificial system models [8]. Over several decades, significant progress has been made in both theoretical and applied research. The first International Conference on Genetic Algorithms (ICGA’85) was held at Carnegie Mellon University, USA in 1985, followed by the inauguration of IEEE Transactions on Evolutionary Computation in 1997 [9], signifying the maturation of GA research as a parallel, random, and adaptive high performance computing and modeling method. Genetic algorithms are a potent and widely applicable random search optimization technique that is highly effective for many problems that traditional methods struggle to solve. As such, genetic algorithms have become a focal point for research across various fields including management science, operations research, industrial engineering, and systems engineering. Given that most engineering optimization problems entail complex constraints which simple genetic algorithms often cannot effectively address; numerous scholars have endeavored to enhance GA through improvements to coding methods, control parameters selection techniques, and crossover strategies. Furthermore, integrating genetic algorithms with other optimization approaches has emerged as an area of interest among researchers in recent years.

Based on genetic algorithms, we proposed a mixed user grouping algorithm as follows:

Algorithm 1: Genetic Algorithms

Input: Number of iterations T ; Population size K ; Number of system users N

Output: Optimal Grouping optimalGroup

- 1 Generate an initial set of grouping schemes, including OMA and traditional NOMA schemes: **group=groupInitialize(K)**
 - 2 Calculate the system capacity of each grouping scheme:
capacity=calculateCapacity(group)
 - 3 According to the calculated system capacity, remove the inappropriate part of the grouping schemes: **group=eliminate(group,capacity)**
 - 4 Records the current optimal group: **currentOptimal=group**
 - 5 **while** $tt \neq T$ **do**
 - 6 Some grouping schemes are eliminated based on probability:
group=select(group)
 - 7 Cross to form a new grouping scheme: **group=crossing(group)**
 - 8 Mutate to form new groups: **group=mutation(group)**
 - 9 Replace the worst option with the current best option:
group=elitist(group)
 - 10 Records the current optimal solution: **currentOptimal=group**
 - 11 **end**
 - 12 In this case, the current optimal scheme is the optimal scheme obtained by the algorithm: **optimalGroup=currentOptimal;**
-

3.2 Simulated Annealing Algorithms

The Simulated Annealing (SA) algorithm, proposed by Metropolis et al. [10] in 1953, is a heuristic random search process based on the Monte Carlo iterative solution method. In 1983, Kirkpatrick et al. [11] introduced the annealing process into the field of combinatorial optimization and proposed the SA algorithm for finding the global optimal solution of combinatorial optimization problems. SA is a general optimization algorithm with theoretical probabilistic global optimization performance and has been widely applied in engineering fields such as computer aided design, image processing, production scheduling, machine learning, signal processing, and neural networks [12]. Under conditions where the initial temperature is sufficiently high and the temperature decreases slowly enough, SA can probabilistically converge to the global optimal value with probability 1. The performance of SA is closely related to factors such as initial temperature, design of cooling control function, and termination conditions of the algorithm.

Based on simulated annealing algorithms, we proposed a mixed user grouping algorithm as follows:

Algorithm 2: Simulated Annealing Algorithm

Input: Initial temperature T_0 ; Number of iterations T ; Coefficient of decline α

Output: optimalGroup

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1 while  $t \leq T$  do
2   Generate an initial grouping scheme A: groupA=groupInitialize
3   Calculate the system capacity of scheme A:
   capacityA=calculateCapacity(groupA)
4   Generate grouping scheme B randomly near scheme A
5   Calculate the system capacity of scheme B:
   capacityB=calculateCapacity(groupB)
6   if  $capacityA > capacityB$  then
7     | groupA=groupB;
8   else
9     | Calculate  $C_t = \frac{1}{T_0 \times \alpha^{tt}}$  Calculate the accepting probability of B
   |  $p_t = \exp(-(capacityB - capacityA) \times C_t)$ 
10    | Generate a random number  $r = \text{rand}(1)$ 
11    | if  $r < p_t$  then
12    | | groupA=groupB;
13    | end
14  end
15 end
16 In this case, the current optimal scheme is the optimal scheme obtained by the
   algorithm: optimalGroup=groupA;

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4 Simulation Results and Discussions

Our simulation still adopts the system model from the previous chapter. However, in order to compare our algorithm with the theoretically optimal user grouping algorithm, i.e., the brute force search algorithm, we set the total number of system users to 6 in this chapter. Additionally, we use more dimensions to evaluate the overall system performance, including user fairness [13], which is defined as

$$F = \frac{(\sum_{k=1}^K R_k)^2}{K \sum_{k=1}^K R_k^2} \tag{9}$$

In the given context, K represents the number of users, and R_k denotes the data rate of user k . It is evident that F is a positive value less than 1, signifying the overall disparity in data rates among users in the system. A larger value of F indicates a smaller gap in data rates among users within the system, thereby reflecting better fairness in the system.

Furthermore, in order to better observe the optimal grouping algorithm, the power allocation factor is no longer constant. Additionally, we have introduced a new variable, the power residual factor ϵ [14], which represents the residual power from upper layer users that spills over to the lower layer due to imperfect successive interference cancellation (SIC), leading to degradation of system performance (Fig. 3).

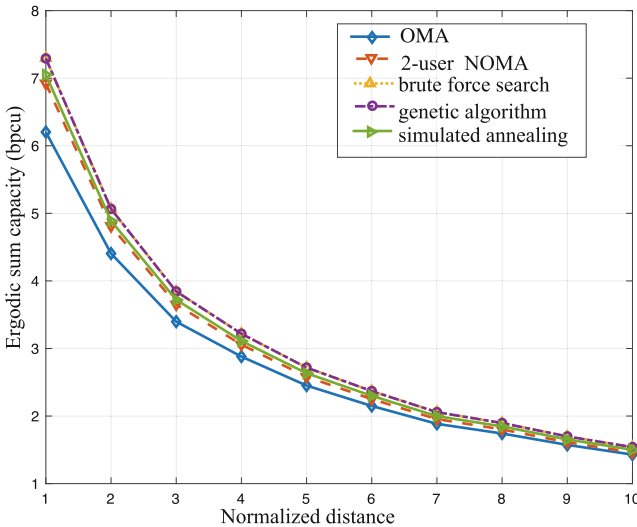


Fig. 2. Ergodic sum capacity with different user grouping algorithms; where the power allocation is 1/2, i.e., the upper user takes the 1/2 of the remaining power; assuming perfect SIC, power residual $\epsilon = 0$

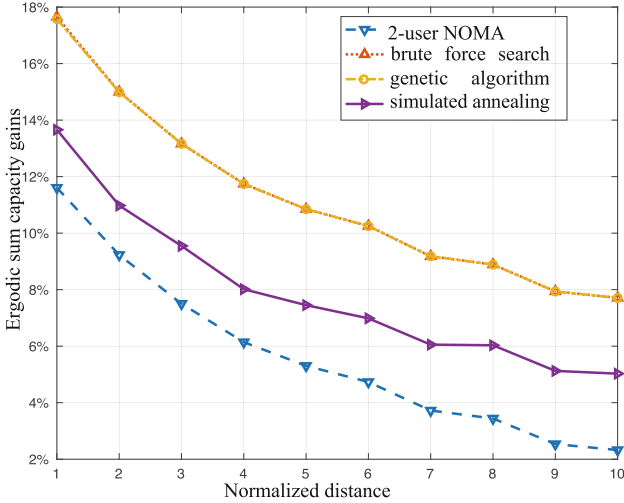


Fig. 3. Ergodic sum capacity gains compared to OMA with different user grouping algorithms; where the power allocation is 1/2, i.e., the upper user takes the 1/2 of the remaining power; assuming perfect SIC, power residual $\epsilon = 0$

In Fig. 1 and Fig. 2, it is evident that the theoretically optimal user grouping, i.e., the brute force search algorithm, outperforms the OMA system significantly in theory, with relative gains ranging from 8% to 18%. It also demonstrates a significant improvement over the two-user NOMA system, thereby validating the correctness and necessity of NOMA’s hybrid user grouping. From another perspective, it can be observed that the genetic algorithm’s performance approaches theoretical optimality due to two main reasons: Firstly, there are relatively few total users (only 6), allowing for rapid convergence and better performance of the genetic algorithm. Secondly, our genetic algorithm utilizes OMA, two-user NOMA, and three-user NOMA as initial populations; hence its performance is undoubtedly superior to these three grouping methods and even approaches optimal performance.

Regardless of this comparison between algorithms’ performances in terms of proximity to optimality or efficiency in computation complexity reduction, the genetic algorithm has proven its superiority and should be considered as a primary choice for determining an optimal grouping strategy. The simulated annealing algorithm exhibits inferiority compared to the genetic algorithm but surpasses both OMA and two-user NOMA systems. This observation underscores its value as a simple yet low complexity computational method.

It is important to note that in this study, the simulated annealing algorithm employed a single point annealing approach with random initial groupings, which reduced computational complexity while compromising on overall performance. If a multiple point annealing approach were adopted instead, its performance would likely improve further.

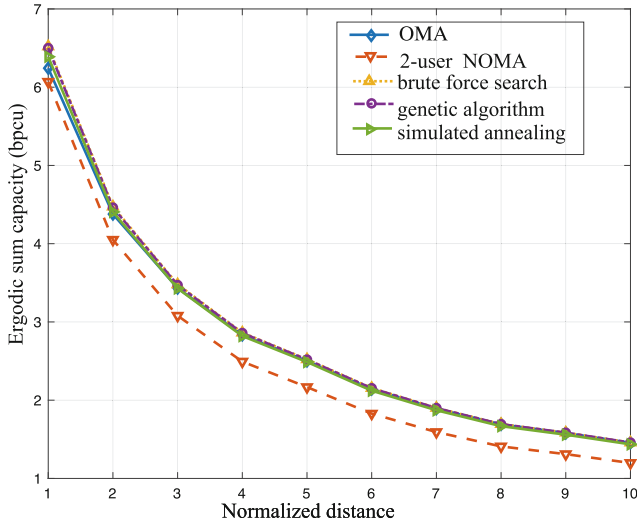


Fig. 4. Ergodic sum capacity with different user grouping algorithms; where the power allocation is 2/3, i.e., the upper user takes the 2/3 of the remaining power; assuming perfect SIC, power residual $\epsilon = 0$

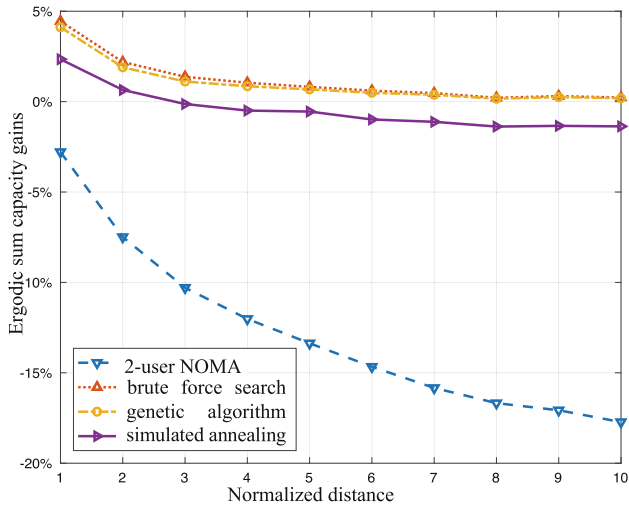


Fig. 5. Ergodic sum capacity gains compared to OMA with different user grouping algorithms; where the power allocation is 2/3, i.e., the upper user takes the 2/3 of the remaining power; assuming perfect SIC, power residual $\epsilon = 0$

The power allocation scheme used in Fig.4 and Fig.4 differs from our previous approach, employing a 2/3 power allocation strategy where the upper layer users are allocated 2/3 of the total power. This has resulted in a dramatic shift in the scenario. Under this power allocation scheme, both the system and capacity of the two-user NOMA system are notably smaller than those of the OMA system, whereas the opposite holds true for the 1/2 power allocation scheme. The disparity is primarily due to the fact that in the 2/3 power allocation scheme, most of the power is allocated to users with poor channel conditions, while those with better channel conditions receive insufficient power, leading to reduced system and capacity performance. The gap between them may even reach close to 20%. Despite yielding less than a 5% improvement, it is worth noting that both system and capacity achieved through genetic algorithm still surpass those of OMA systems.

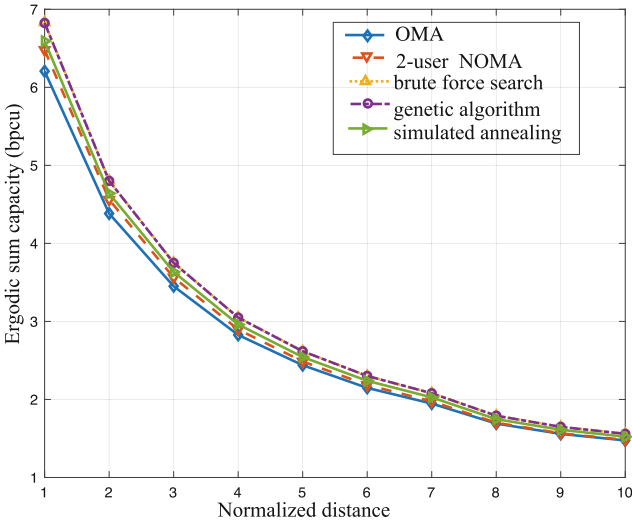


Fig. 6. Ergodic sum capacity with different user grouping algorithms; where the power allocation is 2/3, i.e., the upper user takes the 2/3 of the remaining power; power residual $\epsilon = 0.1$

Figure 5 and Fig. 6 also presents a scenario that has not been previously considered, wherein imperfect successive interference cancellation (SIC) results in residual power for upper layer users, causing interference with the decoding of lower layer users and directly impacting system performance. As depicted in the figure, this situation highlights the drawbacks of NOMA’s multi-user resource block sharing. While its performance remains superior to OMA, its advantage is significantly diminished. In contrast, a system employing genetic algorithms can still maintain a relative gain of 5% (Fig. 8).

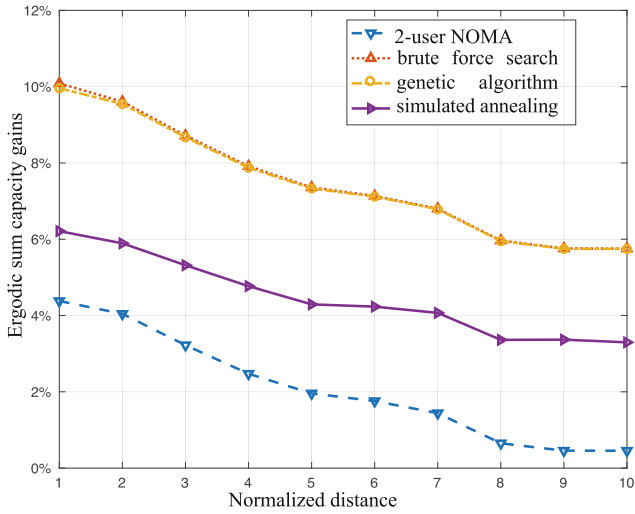


Fig. 7. Ergodic sum capacity gains compared to OMA with different user grouping algorithms; where the power allocation is 1/2, i.e., the upper user takes the 1/2 of the remaining power; power residual $\epsilon = 0.1$

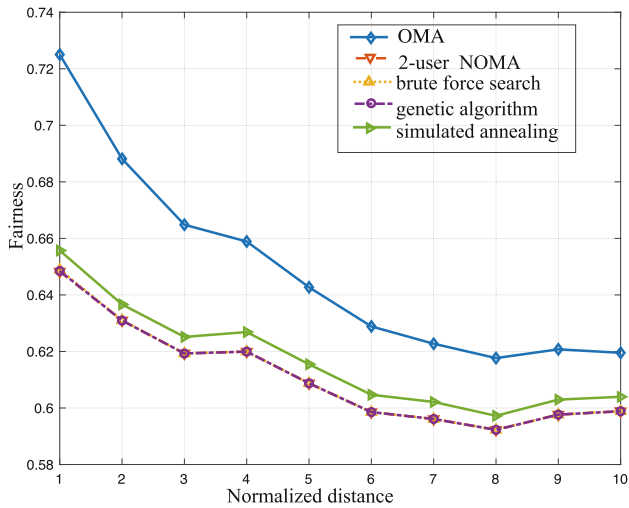


Fig. 8. System fairness with different user grouping algorithms; where the power allocation is 1/2, i.e., the upper user takes the 1/2 of the remaining power; assuming perfect SIC, power residual $\epsilon = 0$

Fairness is also an important yet previously unmentioned metric for evaluating system performance in this paper. Figure 7 presents a different perspective on the notion that “NOMA fairness is better”. Under the 1/2 power allocation scheme, OMA exhibits better fairness performance than NOMA,

while at the same time NOMA’s system and capacity outperform OMA’s. This indicates that achieving a balance between fairness and efficiency is challenging. Similarly, genetic algorithms and simulated annealing algorithms are designed to optimize system capacity, leading to a relative decrease in fairness, which is considered normal.

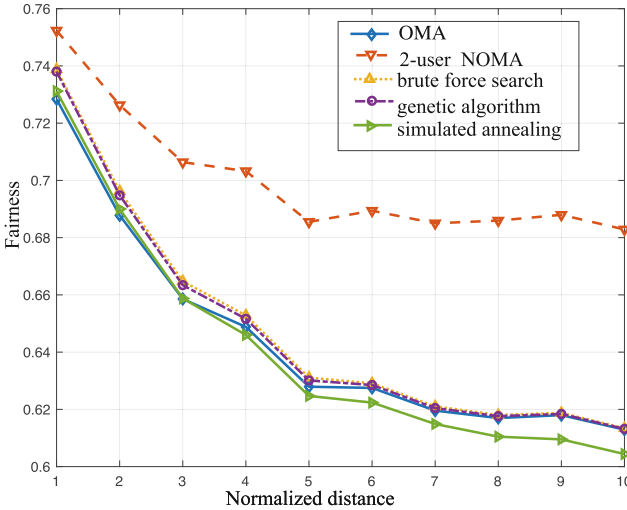


Fig. 9. System fairness with different user grouping algorithms; where the power allocation is 2/3, i.e., the upper user takes the 2/3 of the remaining power; assuming perfect SIC, power residual $\epsilon = 0$

However, when the power allocation scheme reaches 2/3, the fairness is reversed. At this point, the fairness of NOMA for two users is significantly superior to that of OMA and genetic algorithms. This comes at the expense of sacrificing overall system capacity, as previously mentioned. It is worth noting that in this scenario, the fairness of genetic algorithms also surpasses that of OMA. This applies to both system and capacity comparisons; in other words, both aspects are superior with genetic algorithms compared to OMA, although not by a large margin.

From Fig. 9, it can be observed that the fairness of NOMA users increases in the presence of residual power, while the efficiency of NOMA users, i.e., system and capacity, decreases. This is due to the fact that upper layer users typically have poorer channel conditions, resulting in inherently smaller channel capacities. Even with a higher allocation of power relative to lower layer users, their channel capacity cannot surpass that of the lower layer users. Although residual power has a minimal impact on upper layer users or even no impact at all, it leads to a reduction in the channel capacity of lower layer users, thereby narrowing the gap between upper and lower layer user capacities and passively enhancing system fairness (Fig. 10).

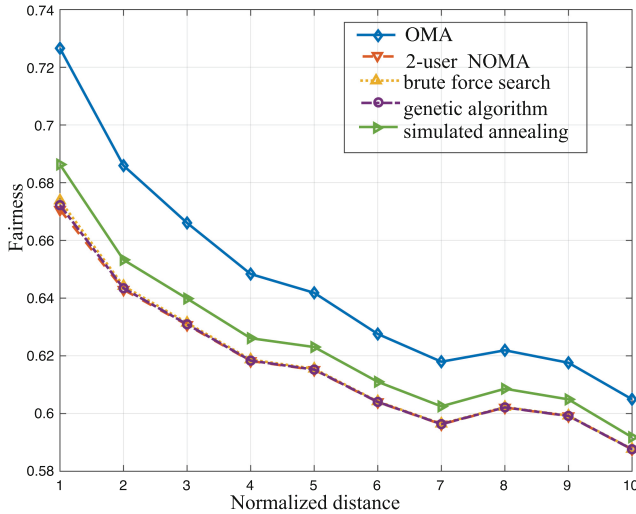


Fig. 10. System fairness with different user grouping algorithms; where the power allocation is $1/2$, i.e., the upper user takes the $1/2$ of the remaining power; power residual $\epsilon = 0.1$

5 Conclusions

This paper introduces two NOMA mixed user grouping algorithms, based on genetic algorithm and simulated annealing algorithm respectively, and compares them with traditional OMA and fixed user grouping of NOMA. Our analysis shows that the NOMA system performance using the mixed grouping algorithm is significantly superior to that of OMA and fixed grouping of NOMA. Furthermore, compared to the optimal grouping algorithm (i.e., brute force search algorithm), it incurs less performance loss while greatly reducing computational complexity. Therefore, adopting the mixed grouping algorithm can lead to substantial performance improvements with only a limited increase in computational complexity, indicating significant feasibility for application in future practical systems.

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