GTRS Based Joint Time Synchronization and Localization in Wireless Sensor Networks

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Abstract. In this paper, we investigate the joint time synchronization and localization in wireless sensor networks. Specially, based on time-of-arrival (TOA), we consider a squared-range-based least squares formulation problem and propose a generalized trust region subproblem (GTRS) algorithm based joint time synchronization and localization, which can guarantee an optimal solution to the joint time synchronization and localization problem. Sufficient experiments results show that the estimation accuracy of the proposed algorithm outperforms the traditionalunconstraint linear least squares (ULLS) and nearly coincides with the Cramer-Rao lower bound (CRLB).

Keywords: Optimal solution · Joint time synchronization and localization · Generalized trust region sub-problems (GTRS)

1 Introduction

Source localization with wireless sensor networks has attracted significant attentions owing to the enormous number of applications and services, including vehicle navigation, target detection and indoor positioning, etc. [1]. Hence high accuracy localization methods have been widely investigated. Traditionally, source localization methods include time of arrival (TOA), time difference of arrival (TDOA), received signal strength (RSS) and angle of arrival (AOA) [2,3]. Among them, TOA based localization methods have attracted intensive interests [4–12]. In [4] TOA based localization was formulated as a maximum likelihood (ML) problem. As it is difficult to solve this ML problem directly, suboptimal methods such as the Taylor search method [4], and the Gauss-Newton search method [5] were proposed. [6] translated nonlinear constraints of an ML solution into a set of linear constraint equations and obtained an approximate solution to

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the ML problem. In addition, [7] and [8] relaxed the ML problem to a semi-definite programming (SDP) problem, which cannot guarantee an optimal solution. Another kind of localization methods extensively used formulate the localization problem into a nonlinear least squares (NLS) problem, for example the squared-range-based least square (SR-LS) [9]. In order to solve the SR-LS problem, [10] proposed a constrained weighted least squares (CWLS) algorithm using the Lagrange multiplier method to obtain a suboptimal solution to the SR-LS problem. Besides, a weighted linear least squares algorithm (WLS) only for a linear anchor node distribution was proposed in [11], which provided a suboptimal solution to the SR-LS problem. Furthermore, [12] proposed a generalized trust region subproblem (GTRS) based algorithm, which guaranteed an optimal solution to the SR-LS problem.

However, all works above assumed perfect synchronization among various sensors. Traditionally, synchronization and localization have been treated separately. In practice, sensors are asynchronous in time with clock bias [13]. In this case, even there is a small error in the synchronization, it can propagate into the localization step and be amplified, which may significantly degrade localization accuracy. Consequently, joint time synchronization and localization has aroused wide interests recently [14–16]. The author in [14] formulated the joint time synchronization and localization as an ML problem, and proposed an unconstraint linear least squares (ULLS) algorithm, which provided a suboptimal solution to the ML problem. A suboptimal WLS algorithm for joint time synchronization and localization was proposed in [15]. The authors in [16] relaxed the ML problem into a SDP problem, which can not always guarantee to obtain the optimal solution.

Against this background, we investigate the joint time synchronization and localization problem. Specially, we formulate it as a SR-LS problem and propose a novel GTRS based algorithm, which can guarantee to the optimal solution to the SR-LS problem.

Notation: The lower case represents a scalar, while the upper and the lower boldface represent a matrix and a vector, respectively. $\boldsymbol{I}_n$ represents the unit matrix of $n \times n$, and $\mathbf{0}_{n\times k}$ represents the all-zero matrix of $n \times k$. $\parallel \boldsymbol{x} \parallel$ denotes the 2-norm of the vector $\boldsymbol{x}$. Denote $\boldsymbol{\nabla}^n_{\boldsymbol{x}} y(\boldsymbol{x})$ as the n order derivative of function $y(\boldsymbol{x})$ on vector $\boldsymbol{x}$ and denote $y'(z)$ as the first order derivative of function $y(z)$ on scalar z. $\boldsymbol{A} \succ 0$ ($\boldsymbol{A} \succeq 0$) indicates that $\boldsymbol{A}$ is a positive definite matrix (a semi-definite matrix).

2 System Model

The system considered in this paper is showed in Fig. 1, where there are m anchor nodes and one source node. For the i^{th} anchor node, location vector $\boldsymbol{p}^i \in \mathbb{R}^{\mathrm{n}}$ and clock bias τ_i [15] ($i = 1, 2 \cdots m$) are known. For source node, location vector $\boldsymbol{p}^i \in \mathbb{R}^{\mathrm{n}}$ and clock bias τ are unknown. The i^{th} TOA measurement is given by [15]

$$T_i = \tau - \tau_i + \frac{\parallel \boldsymbol{p} - \boldsymbol{p}_i \parallel}{c} + n_i \quad (i{=}\,1, 2 \cdots m) \tag{1}$$

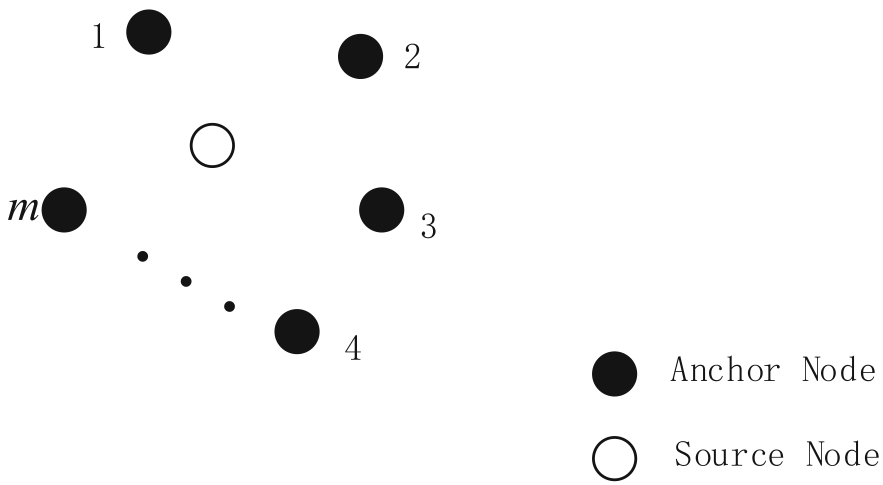

Fig. 1. Localization system

where n_i is the measurement noise at the i^{th} anchor node, which is independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and variance σ^2, while c is the propagation speed of electromagnetic waves. In this paper, for simplicity we assume $c = 1$, then the Eq. (1) can be written as [15]

$$T_i = \tau - \tau_i + \| \boldsymbol{p} - \boldsymbol{p}_i \| + n_i \quad (i = 1, 2 \cdots m). \tag{2}$$

Based on the Eq. (2), we want to find optimal $\boldsymbol{p}^*$ and τ^* to the following problem P_1.

$$P_1 : \min_{\tau \in \mathbb{R}, \boldsymbol{p} \in \mathbb{R}^n} \sum_{i=1}^{m} ((T_i - \tau + \tau_i)^2 - \| \boldsymbol{p} - \boldsymbol{p}_i \|^2)^2, \tag{3}$$

which is a SR-LS problem. Denote $\alpha = \| \boldsymbol{p} \|^2 - \tau^2$, problem P_1 can be equivalently transformed into

$$P_2 : \min_{\boldsymbol{y} \in \mathbb{R}^{n+2}} \left\{ \| \boldsymbol{A}\boldsymbol{y} - \boldsymbol{b} \|^2 : \boldsymbol{y}^T \boldsymbol{D} \boldsymbol{y} + 2 \boldsymbol{f}^T \boldsymbol{y} = 0 \right\}, \tag{4}$$

which is a constrained optimization problem, where

$$\boldsymbol{A} = \begin{bmatrix} -2\boldsymbol{p}_1{}^T & 2(\tau_1 + T_1) & 1 \\ \vdots & \vdots & \vdots \\ -2\boldsymbol{p}_m{}^T & 2(\tau_m + T_m) & 1 \end{bmatrix} \in \mathbb{C}^{m \times (n+2)}, \boldsymbol{y} = \begin{bmatrix} \boldsymbol{p}^T \; \tau \; \alpha \end{bmatrix}^T \in \mathbb{C}^{(n+2) \times 1}$$

$$\boldsymbol{b} = \begin{bmatrix} (\tau_1 + T_1)^2 - \| \boldsymbol{p}_1 \|^2 \\ \vdots \\ (\tau_m + T_m)^2 - \| \boldsymbol{p}_m \|^2 \end{bmatrix} \in \mathbb{C}^{m \times 1}, \boldsymbol{D} = \begin{bmatrix} \boldsymbol{I}_n & & \\ & -1 & \\ & & 0 \end{bmatrix} \in \mathbb{C}^{(n+2) \times (n+2)}$$

$$\boldsymbol{f} = \begin{bmatrix} \boldsymbol{0}_{(n+1)\times 1} \\ -0.5 \end{bmatrix} \in \mathbb{C}^{(n+2)\times 1}$$

Assume that $\boldsymbol{A}^T\boldsymbol{A}$ is an invertible matrix. One common way to get an approximate solution to problem P_2 is to remove the quadratic constraint, so that problem P_2 is reduced to

$$P_3 : \min_{\boldsymbol{y}\in\mathbb{R}^{n+2}} (\| \boldsymbol{A}\boldsymbol{y} - \boldsymbol{b}\|^2). \tag{5}$$

which is a ULLS problem. The optimal solution to problem P_3 is given by [17]

$$\hat{\boldsymbol{y}} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{b}. \tag{6}$$

However it may not be the optimal solution to problem P_2. In order to solve problem P_2, a novel GTRS based algorithm is proposed in this paper, which can guarantee an optimal solution to problem P_2.

3 GTRS Based Joint Time Synchronization and Localization

Denote

$$c(\boldsymbol{y}) = \boldsymbol{y}^T\boldsymbol{D}\boldsymbol{y} + 2\boldsymbol{f}^T\boldsymbol{y}, \tag{7}$$

$$q(\boldsymbol{y}) = \| \boldsymbol{A}\boldsymbol{y} - \boldsymbol{b}\|^2, \tag{8}$$

Problem P_2 can be equivalently expressed as

$$P_4 : \min_{\boldsymbol{y}\in\mathbb{R}^{n+2}} \{q(\boldsymbol{y}) : c(\boldsymbol{y}) = 0\}. \tag{9}$$

Theorem 1: If $c(\boldsymbol{y})$ and $q(\boldsymbol{y})$ are quadratic and continuous functions on $\mathbb{R}^{n+2}$ and $\min\{c(\boldsymbol{y})\} < 0 < \max\{c(\boldsymbol{y})\}$, $\boldsymbol{y}$ is an optimal solution for problem P_4 if and only if there is a λ that satisfies the following equations

$$(\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D})\boldsymbol{y} = \boldsymbol{A}^T\boldsymbol{b} - \lambda\boldsymbol{f}, \tag{10}$$

$$\boldsymbol{y}^T\boldsymbol{D}\boldsymbol{y} + 2\boldsymbol{f}^T\boldsymbol{y} = 0, \tag{11}$$

$$\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D} \succeq \boldsymbol{0}. \tag{12}$$

.

Proof. As $c(\boldsymbol{y})$ is a quadratic and continuous function and $\min\{c(\boldsymbol{y})\} < 0 < \max\{c(\boldsymbol{y})\}$, based on [18, Theorem 3.2], $\boldsymbol{y}$ is an optimal solution for problem P_4, if and only if there is a λ satisfying the following equations

$$\nabla_{\boldsymbol{y}} q(\boldsymbol{y}) + \lambda\nabla_{\boldsymbol{y}} c(\boldsymbol{y}) = \boldsymbol{0}, \tag{13}$$

$$c(\boldsymbol{y}) = 0, \tag{14}$$

$$\nabla^2_{\boldsymbol{y}} q(\boldsymbol{y}) + \lambda\nabla^2_{\boldsymbol{y}} c(\boldsymbol{y}) \succeq \boldsymbol{0}. \tag{15}$$

Following the rule of derivatives of vector [19, Chapter 2.4], we have

$$\begin{array}{ll} \nabla_{\boldsymbol{y}} q(\boldsymbol{y}) = 2\boldsymbol{A}^T(\boldsymbol{A}\boldsymbol{y} - \boldsymbol{b}), & \nabla^2_{\boldsymbol{y}} q(\boldsymbol{y}) = 2\boldsymbol{A}^T\boldsymbol{A}, \\ \nabla_{\boldsymbol{y}} c(\boldsymbol{y}) = 2\boldsymbol{D}\boldsymbol{y} + 2\boldsymbol{f}, & \nabla^2_{\boldsymbol{y}} c(\boldsymbol{y}) = 2\boldsymbol{D}. \end{array} \tag{16}$$

Based on Eqs. (16), Eqs. (13), (14) and (15) can be reduced to (10), (11) and (12), respectively.

Remark 1: $c(\boldsymbol{y})$ is a continuous and quadratic function.

Proof. Obviously, $c(\boldsymbol{y})$ is a continuous function on $\mathbb{R}^{n+2}$. Furthermore, as $\boldsymbol{D}$ is a nonzero matrix, ${\nabla_{\boldsymbol{y}}}^2 c(\boldsymbol{y}) \neq \boldsymbol{0}$ holds. Thus $c(\boldsymbol{y})$ is a quadratic function.

Remark 2: The inequality $\min\{c(\boldsymbol{y})\} < 0 < \max\{c(\boldsymbol{y})\}$ is always satisfied.

Proof. We can chose $\boldsymbol{y}_1 = \begin{bmatrix} \boldsymbol{0}_{1\times(n+1)} & 1 \end{bmatrix}^T$ and $\boldsymbol{y}_2 = \begin{bmatrix} \boldsymbol{0}_{1\times(n+1)} & -1 \end{bmatrix}^T$. Hence, $c(\boldsymbol{y}_1) = -1 < 0$ while $c(\boldsymbol{y}_2) = 1 > 0$. Consequently, $\min\{c(\boldsymbol{y})\} \leq c(\boldsymbol{y}_1) = -1 < 0$ and $\max\{c(\boldsymbol{y})\} \geq c(\boldsymbol{y}_2) = 1 > 0$. Hence $\min\{c(\boldsymbol{y})\} < 0 < \max\{c(\boldsymbol{y})\}$.

Remark 3: In order to satisfy the constraint $\boldsymbol{A}^T\boldsymbol{A}$ is an invertible matrix, m must satisfy $m \geq 4$.

Proof. As $\boldsymbol{A} \in \mathbb{C}^{m\times(n+2)}$, $\text{Rank}(\boldsymbol{A}^T\boldsymbol{A}) \leq \min(m, n+2)$. As $n \geq 2$, $n+2 \geq 4$, in order to satisfy the constraint $\boldsymbol{A}^T\boldsymbol{A}$ is an invertible matrix, m must satisfy $m \geq 4$.

Theorem 2: If there exists a λ^* satisfying (10), (11) and (12), λ^* is the unique solution of

$$\varphi(\lambda) = 0, \lambda \in I, \tag{17}$$

where

$$\varphi(\lambda) = \tilde{\boldsymbol{y}}(\lambda)^T \boldsymbol{D} \tilde{\boldsymbol{y}}(\lambda) + 2\boldsymbol{f}^T \tilde{\boldsymbol{y}}(\lambda), \tag{18}$$

where $\tilde{\boldsymbol{y}}(\lambda)$ is the optimal solution of problem P_4 and $I = \left[-\frac{1}{\lambda_1}, -\frac{1}{\lambda_{n+2}}\right]$.

Proof. Denote $\phi(i), (i = 1, 2 \cdots n+2)$ as the i^{th} eigenvalue of matrix $\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D}$. In order to satisfy (12), $\phi(i) \geq 0$ must hold.

As

$$\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D} = ((\boldsymbol{A}^T\boldsymbol{A})^{\frac{1}{2}})^T(\boldsymbol{I}_{n+2} + \lambda(\boldsymbol{A}^T\boldsymbol{A})^{-\frac{1}{2}}\boldsymbol{D}(\boldsymbol{A}^T\boldsymbol{A})^{-\frac{1}{2}})(\boldsymbol{A}^T\boldsymbol{A})^{\frac{1}{2}} \tag{19}$$

matrices $\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D}$ and $\boldsymbol{I}_{n+2} + \lambda(\boldsymbol{A}^T\boldsymbol{A})^{-\frac{1}{2}}\boldsymbol{D}(\boldsymbol{A}^T\boldsymbol{A})^{-\frac{1}{2}}$ are congruent [20]. Hence these two matrices have the same inertial index [20], i.e. they have same number of positive, zero and negative eigenvalues. Denote $\psi_i, (i = 1, 2 \cdots n+2)$ as the i^{th} eigenvalue of $\boldsymbol{I}_{n+2} + \lambda(\boldsymbol{A}^T\boldsymbol{A})^{-\frac{1}{2}}\boldsymbol{D}(\boldsymbol{A}^T\boldsymbol{A})^{-\frac{1}{2}}$. Hence in order to satisfy $\phi(i) \geq 0$, $\psi(i) \geq 0$ must hold. Denote $\lambda_i, (i = 1, 2 \cdots n+2)$, arranged in the non-increasing order, as the i^{th} eigenvalue of matrix $(\boldsymbol{A}^T\boldsymbol{A})^{-\frac{1}{2}}\boldsymbol{D}(\boldsymbol{A}^T\boldsymbol{A})^{-\frac{1}{2}}$. Clearly, $\psi_i = 1 + \lambda\lambda_i, (i = 1, 2 \cdots n+2)$. Furthermore, as matrices $(\boldsymbol{A}^T\boldsymbol{A})^{\frac{1}{2}}\boldsymbol{D}(\boldsymbol{A}^T\boldsymbol{A})^{-\frac{1}{2}}$ and $\boldsymbol{D}$ are congruent, these two matrices have the same inertial index as well. As the matrix $\boldsymbol{D}$ has n positive eigenvalues, one zero eigenvalue and one negative eigenvalue, λ_i satisfies following inequality.

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n > \lambda_{n+1} = 0 > \lambda_{n+2}. \tag{20}$$

In order to satisfy $\psi(i) \geq 0, (i = 1, 2 \cdots n+2)$, there are three different scenarios to consider.

1. For $i \in [1, n]$, according to (20), $\lambda_i > 0$. Hence we have

$$\psi_i \geq 0 \Rightarrow \lambda \geq -\frac{1}{\lambda_i} \Rightarrow \lambda \geq -\frac{1}{\lambda_1}, i \in [1, n]; \tag{21}$$

2. For $i = n+1$, according to (20), $\lambda_{n+1} = 0$. Hence we have

$$\psi_{n+1} = 1 > 0 \Rightarrow \lambda \in R; \tag{22}$$

3. For $i = n+2$, according to (20), $\lambda_{n+2} < 0$. Hence we have

$$\psi_{n+2} \geq 0 \Rightarrow \lambda \leq -\frac{1}{\lambda_{n+2}}. \tag{23}$$

Denote I is the interval of λ. Consequently, if λ satisfies (12), $I = \left[-\frac{1}{\lambda_1}, -\frac{1}{\lambda_{n+2}}\right]$.

As λ satisfies (10), (11) and (12), according to Theorem 1, there exists an optimal solution $\boldsymbol{y}$ denoted as $\tilde{\boldsymbol{y}}(\lambda)$ for problem P_4. Next, we will show that $\varphi(\lambda) = 0$ has a unique solution within the interval I. As $\tilde{\boldsymbol{y}}(\lambda)$ satisfies (10), we have

$$(\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D})\tilde{\boldsymbol{y}}(\lambda) = \boldsymbol{A}^T\boldsymbol{b} - \lambda\boldsymbol{f} \tag{24}$$

Following the rule of derivatives of scalars [19, Chapter 2.4], taking the derivative of (24) on λ at both sides, we have

$$-\left(\boldsymbol{D}\tilde{\boldsymbol{y}}(\lambda) + \boldsymbol{f}\right) = (\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D})\tilde{\boldsymbol{y}}'(\lambda). \tag{25}$$

As $\tilde{\boldsymbol{y}}(\lambda)$ satisfies (11), we have

$$\tilde{\boldsymbol{y}}(\lambda)^T\boldsymbol{D}\tilde{\boldsymbol{y}}(\lambda) + 2\boldsymbol{f}^T\tilde{\boldsymbol{y}}(\lambda) = 0. \tag{26}$$

Denote

$$\varphi(\lambda) = \tilde{\boldsymbol{y}}(\lambda)^T\boldsymbol{D}\tilde{\boldsymbol{y}}(\lambda) + 2\boldsymbol{f}^T\tilde{\boldsymbol{y}}(\lambda). \tag{27}$$

Obviously, $\varphi(\lambda)$ is a continuous function of λ. Taking the derivative of (27) on λ at both sides, we have

$$\varphi'(\lambda) = (\nabla_{\tilde{\boldsymbol{y}}(\lambda)}\varphi(\lambda))^T\tilde{\boldsymbol{y}}'(\lambda). \tag{28}$$

Following the rule of derivatives of vectors [19, Chapter 2.4], we can obtain

$$\nabla_{\tilde{\boldsymbol{y}}(\lambda)}\varphi(\lambda) = 2\boldsymbol{D}\tilde{\boldsymbol{y}}(\lambda) + 2\boldsymbol{f} = -2(\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D})\tilde{\boldsymbol{y}}'(\lambda), \tag{29}$$

where the second equality is based on (25). Upon substituting (29) into (28), we can obtain

$$\varphi'(\lambda) = -2\tilde{\boldsymbol{y}}'(\lambda)^T(\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D})\tilde{\boldsymbol{y}}'(\lambda). \tag{30}$$

Furthermore, for

$$\lambda \in \left(-\frac{1}{\lambda_1}, -\frac{1}{\lambda_{n+2}}\right), \tag{31}$$

$\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D}$ is a positive definite matrix, hence $\varphi'(\lambda) < 0$ and $\varphi(\lambda)$ is strictly monotonically decreasing. Note that here we strictly limit $\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D}$ to be a positive definite matrix. The reason is that for tens of thousands of experiments, the unique solution to $\varphi(\lambda) = 0$ has never been the endpoints of interval I, so we assume $\boldsymbol{A}^T\boldsymbol{A} + \lambda\boldsymbol{D} \succ \boldsymbol{0}$. The same phenomenon is also observed in [12].

Moreover, it is very difficult to theoretically prove there exist two points λ_a and λ_b, which satisfy $\varphi(\lambda_a) < 0$ and $\varphi(\lambda_b) > 0$. However during our experimental simulations, we can always find two points λ_a and λ_b in interval I satisfying $\varphi(\lambda_a) < 0$ and $\varphi(\lambda_b) > 0$. Therefore, we can use bisection method to find the unique solution λ^* to $\varphi(\lambda) = 0$ [12].

In summary, the GTRS based joint time synchronization and localization is presented in Table 1.

Table 1. The GTRS based joint time synchronization and localization

1)	Calculate the interval I
2)	Find two points λ_a and λ_b in interval I to satisfy $\varphi(\lambda_a) < 0$ and $\varphi(\lambda_b) > 0$
3)	Use bisection method to find the unique solution λ^* of $\varphi(\lambda) = 0$
4)	Substitute the unique solution λ^* into $\tilde{\boldsymbol{y}}(\lambda^*) = (\boldsymbol{A}^T\boldsymbol{A} + \lambda^*\boldsymbol{D})^{-1}(\boldsymbol{A}^T\boldsymbol{b} - \lambda^*\boldsymbol{f})$ and get the optimal $\boldsymbol{p}^*$ and τ^*

4 Simulation Results

In this section, we demonstrate the performance of the proposed GTRS based joint time synchronization and localization. We simulate a localization system consisting of five anchor nodes and one source node. As showed in Fig. 2, the coordinate $\boldsymbol{p}_i$ of the i^{th} anchor node is generated randomly following a uniform distribution in the region of $[-100, -50] \times [-100, -50]$ and the coordinate $\boldsymbol{p}$ of the source node is randomly placed following a uniform distribution in the region of $[10, 20] \times [10, 20]$. The clock bias τ_i and τ are drawn from one-dimensional uniform distribution in the region of $[0, 1]$. Besides, σ^2 is set to be $10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}$ and 1, respectively.

In this simulation, our proposed GTRS based joint time synchronization and localization are compared with both ULLS algorithm in (6) and Cramer-Rao

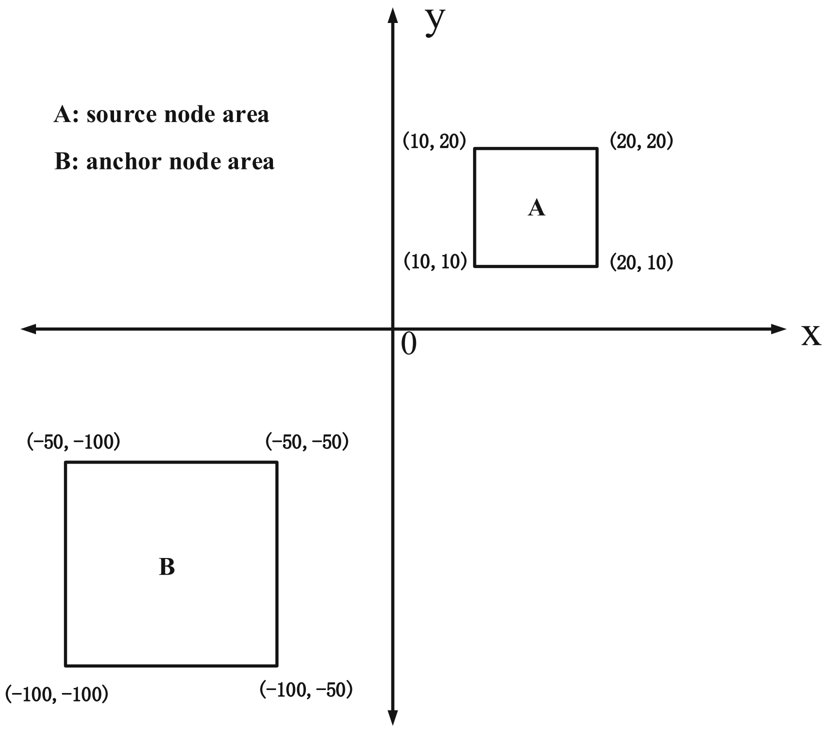

Fig. 2. Localization geometry

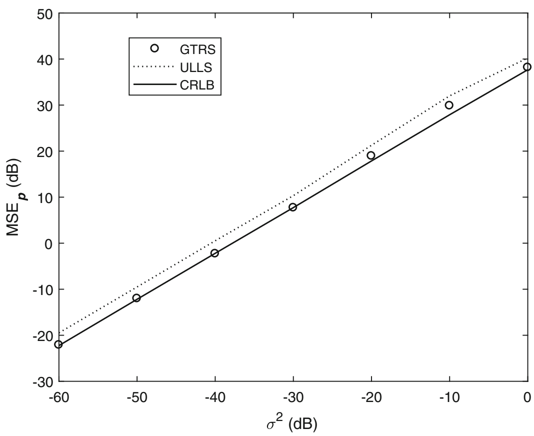

Fig. 3. MSE against σ^2 for source localization of the GTRS, ULLS and CRLB

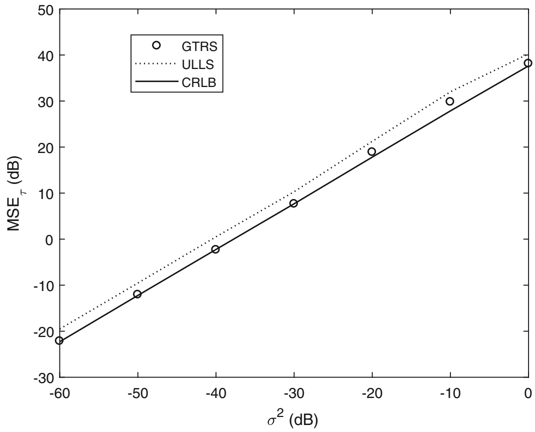

Fig. 4. MSE against σ^2 for clock bias of the GTRS, ULLS and CRLB

lower bound (CRLB) [21]. The metric of estimation performance for source node location and clock bias is mean square error (MSE). The MSE of source node coordinate is denoted as $\mathrm{MSE}_{\boldsymbol{p}} = \frac{1}{L}\sum_{l=1}^{L} \| \bar{\boldsymbol{p}}^l - \boldsymbol{p}^l\|^2$ and the MSE of source node clock bias is denoted as $\mathrm{MSE}_\tau = \frac{1}{L}\sum_{l=1}^{L} \| \bar{\tau}^l - \tau^l\|^2$, where L is the total number of independent simulations for each value of σ^2. Furthermore $\boldsymbol{p}^l$ and τ^l are denoted as actual values at the l^{th} simulation of L, while $\bar{\boldsymbol{p}}^l$ and $\bar{\tau}^l$ denote the estimation values of $\boldsymbol{p}^l$ and τ^l, respectively. Specifically, in our simulation, $L = 10000$. The simulation results are presented in Fig. 3 and Fig. 4 respectively.

In Fig. 3, it can be seen the proposed GTRS based algorithm outperforms ULLS and nearly coincides with the CRLB. Similar phenomenon can be observed in Fig. 4, the estimation accuracy of clock bias of our proposed algorithm outperforms ULLS and nearly coincides with CRLB.

5 Conclusion

In this paper, we investigated the joint time synchronization and localization in wireless sensor networks. We proposed an optimal GTRS based joint time synchronization and localization algorithm by using TOA measurements, which can outperform traditional ULLS algorithm and nearly coincide with the CRLB.

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