Variational Quantum Eigensolver for Optimizing Network Scheduling Using QUBO Formulation

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Abstract. A wireless multihop network is characterized by nodes capable of direct communication or extending their reach beyond the transmission range by employing other nodes as relays. The significance of multihop networks is underscored by their advantages, including expanded coverage, reduced interference, increased spectrum reuse, and lower energy consumption. These networks find applications in wireless sensor networks, Internet of Things (IoT), and ad-hoc networks, among others. Effective operation of these networks relies on scheduling, where a subset of nodes is activated for specific durations while the rest remain inactive. This scheduling challenge falls within the domain of combinatorial optimization problems, which can be NP-hard for large-scale scenarios on classical computers. Quantum computers can offer a potential solution by transforming such problems into Quadratic Unconstrained Binary Optimization (QUBO) problems, presenting a possible speedup in certain instances or with scaling. This paper demonstrates how to formulate the multihop wireless network scheduling problem as a QUBO problem and solve it on a quantum computer using variational quantum eigensolver (VQE). The aim is to explore the practical application of quantum computing in the noisy intermediate-scale quantum (NISQ) era and assess potential benefits in real-world scenarios.

Keywords: Quantum Computing · Variational Quantum Eigensolver · Optimization · Wireless Networks

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1 Introduction

In the dynamic realm of wireless communication, the adoption of multihop networks has become a crucial strategy to meet the escalating demands for expansive coverage and energy-efficient communication. In wireless multihop networks, nodes establish communication through wireless channels, eschewing the need for a centralized control or shared infrastructure. Nodes collaborate by forwarding or relaying each other's packets, potentially involving numerous intermediary relay nodes. These networks play a foundational role in various applications, encompassing wireless sensor networks, the Internet of Things (IoT), ad-hoc networks, and more [14]. Recently, this technology has gained traction as a promising solution for the next generation of wireless communication systems. It is under consideration in the standardization processes of upcoming mobile broadband communication systems, including IEEE 802.11 s for WiFi [27], scatternet for bluetooth [17], and LTE D2D for 4G cellular networks [26].

Moreover, with the advent of the Internet of Things (IoT), an expanding network of interconnected devices, objects, and sensors has emerged, facilitating communication through the internet [13]. These devices encompass a broad spectrum, ranging from everyday objects like smart home appliances, wearable fitness trackers, and industrial machinery to more intricate systems such as self-driving cars and smart cities [2,8,10]. The implementation of a multihop wireless topology is crucial in the deployment of IoT, where a primary objective is to enhance the radio coverage of a base station. In this scenario, specific IoT devices function as relay nodes for other devices, allowing the connection of devices that may not be easily accessible via a single-hop link. [18].

However, one of the central challenges in multihop wireless networks revolves around network scheduling [15]. It is imperative to maintain the Signal-to-Interference-plus-Noise Ratio (SINR) above a certain threshold to ensure the successful decoding of signals. The primary complication in multihop wireless communication stems from the self-interference induced by the traffic relaying of intermediate nodes [31]. This interference manifests as packet reception interference from the upstream node and packet transmission interference to the downstream node. Such interferences are not confined solely to adjacent links but extend to all links within the interference range of each link. As a result, only a subset of edges within the network can be activated during the same time slot due to these interference constraints. Therefore, the operational efficiency of these networks depends on optimizing the scheduling of node activations [16]. These problems fall within the domain of combinatorial optimization problems, specifically Quadratic Unconstrained Binary Optimization (QUBO). It involves the task of determining the optimal solution from a finite set of possibilities, where each solution is a composite of discrete variables [21]. In the context of large-scale scenarios, tackling these problems on classical computers becomes NP-hard. The NP-hardness associated with solving QUBO problems on classical computers arises from the inherent complexity involving binary variables, the quadratic objective function, and the exponential expansion of the search space.

Quantum computers can tackle these challenges of solving QUBO problems by potentially providing accelerated solutions in specific scenarios or with scaling [6,29]. Operating on the principles of quantum physics, quantum computers function in a distinct way from classical computers. Within the quantum realm, qubits can exist in a superposition of different states, allowing them to explore the solution space exponentially faster than their classical counterparts [3,9].

In this paper, we present a comprehensive demonstration of formulating the multihop wireless network scheduling problem as a QUBO problem. Within the domain of quantum computing, the conventional approach for solving QUBO problems involves the annealing method, where the system incrementally progresses toward the solution [11]. However, this paper adopts a different strategy by addressing the QUBO problem through a gate-based quantum computer by utilizing Variational Quantum Eigensolvers (VQEs). In the VQE framework, the problem is encoded as a Hamiltonian. Employing a classical optimization algorithm, we iteratively update the parametrized quantum circuit to minimize the expectation value of the system with respect to the observable [28]. This algorithm has proven to be valuable in quantum machine learning and has played a pivotal role in the development of hybrid algorithms that integrate both quantum and classical computing approaches [24,25]. Furthermore, the variational nature of VQE makes it well-suited for implementation on Noisy Intermediate-Scale Quantum (NISQ) devices, which are characterized by a limited number of qubits and some level of noise [23].

The paper is structured as follows: Sect. 2 encompasses the introductory discussion on quantum computing, Hamiltonian dynamics, VQE, and QUBO. Section 3 delineates the methodologies employed for formulating QUBO in the context of optimizing scheduling in multihop wireless networks. Section 4 presents the framework's results and analysis, and Sect. 5 serves as the conclusion of the paper.

2 Preliminary

2.1 Quantum Computing

Quantum computing is a field within computer science that harnesses quantum theory's principles. Traditional computers encode information in bits that are strictly binary, represented as 0 s or 1 s. In contrast, quantum computers utilize qubits, which embody information within a quantum state that encapsulates both 0 and 1 in a complex, multidimensional manner [4,19,20].

The elementary unit of quantum information is the qubit, denoted by $|\psi\rangle$, which can exist in a superposition of states $|0\rangle$ and $|1\rangle$, expressed as

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{1}$$

where α and β are complex probability amplitudes that satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. In this computational framework, quantum gates manipulate qubits by applying unitary transformations, which are reversible and preserve the quantum state's norm. Among single qubit quantum gates, few of the widely used are the Hadamard gate and the Pauli gates (X, Y, and Z). Pauli gates are given as

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{2}$$

Another category of single qubit quantum gates is quantum rotation gates, employed to rotate the state of a qubit. They are generally expressed through unitary matrices and are characterized by a rotation angle, influencing the qubit state rotation around the X, Y, and Z axes. There are three common type of rotaion gates namely $R_x(\theta)$, $R_y(\theta)$, and $R_z(\theta)$ that are given in computational basis by

$$R_x(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}, R_y(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}, R_z(\theta) = \begin{bmatrix} e^{-i\frac{\theta}{2}} & 0 \\ 0 & e^{i\frac{\theta}{2}} \end{bmatrix}. \tag{3}$$

Moreover, in order to have a complete set of gates for quantum computation, the inclusion of two-qubit gates becomes essential. Among these, one widely utilized two-qubit gate is the Controlled NOT (CNOT) gate. This gate flips the state of the target qubit when the control qubit is in the state $|1\rangle$ and leaves it unaffected when the control qubit is in the state $|0\rangle$.

Measurement is another important part in quantum computing. It involves a set of measurement operators $\{M_m\}$. These operators are defined to act on the state space of the quantum system, where m indexes the possible measurement outcomes. For a two-level system (qubit), the measurement is typically performed in the computational basis, which consists of the basis states $|0\rangle$ and $|1\rangle$. In this case, the measurement operators M_m become $M_0 = |0\rangle\langle 0|$ and $M_1 = |1\rangle\langle 1|$. The likelihood of obtaining a particular outcome m is determined using the Born rule. The probability $p(m)$ of the outcome m is given by

$$p(m) = \langle\psi| M_m^\dagger M_m |\psi\rangle \tag{4}$$

where $|\psi\rangle$ is the state vector of the quantum system before measurement and the operator $M_m^\dagger$ is the Hermitian adjoint (or conjugate transpose) of the measurement operator M_m. This formula calculates the probability of observing the outcome m when the system is in state $|\psi\rangle$ [20,30].

The expectation value of the operator $\hat{M}$ for the state $|\psi\rangle$ is a measure of the average outcome that would be obtained if the measurement corresponding to $\hat{M}$ were performed many times on identically prepared systems in state $|\psi\rangle$ and is given by,

$$\langle\hat{M}\rangle = \langle\psi| \hat{M} |\psi\rangle \tag{5}$$

2.2 The Hamiltonian

The Hamiltonian H in quantum computing is an Hermitian operator that embodies the total energy of the quantum system and dictates its temporal evolution. The Hamiltonian can have its spectral decomposition written as

$$H = \sum_{E} E \left| E \right\rangle \left\langle E \right| \tag{6}$$

where the E are the eigenvalues corresponding to the eigenvector $|E\rangle$. These states, denoted as $|E\rangle$, are commonly recognized as energy eigenstates, with E representing the energy of each state. The minimum energy value is referred to as the ground state energy of the system, and the associated energy eigenstate is termed the ground state [20].

Quantum algorithms often involve the preparation and manipulation of quantum states based on the system's Hamiltonian. In quantum computing, variational quantum algorithms focus on finding the optimal parameters of a parametrized quantum circuit to minimize the expectation value of the Hamiltonian. This approach allows for the approximation of solutions to problems in various domains, including chemistry, optimization, and machine learning [7].

2.3 Variational Quantum Eigensolver (VQE)

In the realm of variational quantum algorithms and hybrid quantum-classical approaches, VQE stands out as a promising algorithm, particularly when taking into account the limitations posed by the circuit depth of NISQ devices [7]. Initially developed by Peruzzo et al. [22], VQE utilizes the variational principle to compute the ground state energy of a Hamiltonian, representing the stable, lowest-energy configuration of a quantum system. In general, the algorithm encodes problems into a Hamiltonian, describing the total energy of the system, and employs a trial wavefunction through a parameterized quantum circuit known as an ansatz. This ansatz, comprised of adjustable parameters, enables the expression of the quantum state in various configurations, this can be represented as

$$\left| \psi(\theta) \right\rangle = U(\theta) \left| 0 \right\rangle \tag{7}$$

where θ is the parameters to be optimized and $U(\theta)$ is the unitary operator representing the quantum circuit. Classical optimization algorithms, such as gradient descent, refine the ansatz parameters to minimize the Hamiltonian's expectation value E, which is given as

$$E(\theta) = \left\langle \psi(\theta) | H | \psi(\theta) \right\rangle . \tag{8}$$

The goal of optimization can be represented as

$$\theta_{\min} = \arg\min_{\theta} E(\theta). \tag{9}$$

By iteratively updating the parametrized quantum circuit, the algorithm seeks to minimize the expectation value concerning the observable.

2.4 Quadratic Unconstrained Binary Optimization (QUBO) Problem

Combinatorial optimization is a branch of optimization that deals with problems involving discrete or combinatorial structures. These problems aim to determine the optimal combination of variable values to maximize or minimize an objective function. Examples of combinatorial optimization problems include the traveling salesman problem, knapsack problem, and graph coloring problem [12]. QUBO is a distinct form of combinatorial optimization, which deals with binary decision variables and a quadratic objective function. In the realm of QUBO problems, the objective is to discover the binary value assignment (0 or 1) for decision variables that minimizes or maximizes the quadratic objective function. The transformation of combinatorial optimization problems to QUBO typically involves defining binary variables to signify the presence or absence of edges connecting nodes. The general form of QUBO is given as

$$f(x_1, \ldots, x_n) = -\sum_{m=1}^{N} c_m x_m + \sum_{1 \leq m < n}^{N} J_{mn} x_m x_n \tag{10}$$

where $x_m \in \{0, 1\}$, and c_m, J_{mn} represent QUBO parameters.

Quantum computers are positioned to be pivotal in addressing NP-Hard problems, particularly those in the class of QUBO. VQE can be applied to solve QUBO problems when encoded as a Hamiltonian using a quantum computer. In order to map a QUBO problem to a quantum computer, we need to change the binary variable $x_i \in \{0, 1\}$ to $z_i \in \{-1, 1\}$ respectively, which can be done using

$$x_i = \frac{1 - z_i}{2}. \tag{11}$$

3 Methods

In this paper, we take a simple example network comprising 10 nodes, denoted as x_i where i ranges from 1 to 10, as depicted in Fig. 1. In the NISQ era, hardware limitations compel us to showcase a modest example . However, it is envisioned that as quantum computers with a substantial number of qubits become available in the future, the demonstrated problem can be scaled up to address large-scale networks. Among these nodes, one is specified as the gateway node, tasked with gathering packets from the remaining nodes. The communication between nodes and the gateway node is considered bidirectional. As a result, the network topology forms a tree structure, with the gateway node serving as the root and all packets directed towards it. If the node is set to transmit, then $x_i = 0$ else $x_i = 1$ when receiving data. The scenario posits that interference occurs exclusively between nodes connected by a shared link (edge), particularly during simultaneous transmission. We can write this using a function that assigns lower values when adjacent nodes (connected by link) are in same states. The problem is formulated as an optimization challenge, where the goal is to maximize the

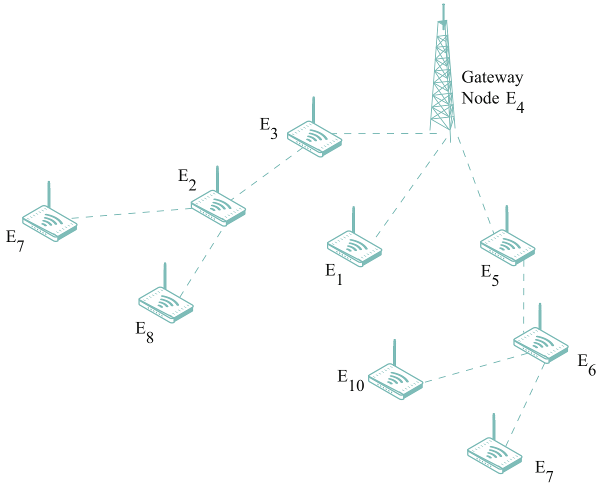

Fig. 1. Mulithop network consisting of 10 nodes.

objective function's value, such that more nodes can transmit at once. We can write the function that acts as our QUBO formulation of the given multihop scheduling problem as

$$f(x_1, x_2, ..., x_{10}) = 9 + x_1 + 3x_2 + 2x_3 + 3x_4 + 2x_5 + 3x_6 + x_7 + x_8 + x_9 + x_{10} \\ -2x_1x_4 - 2x_2x_3 - 2x_4x_5 - 2x_3x_4 - 2x_2x_8 - 2x_2x_7 - 2x_5x_6 - 2x_6x_{10} - 2x_6x_9. \tag{12}$$

In order to run this problem on a quantum computer, we need to convert it using (11). Our new equation can then be written as

$$f(z_1, z_2, ..., z_{10}) = 0.5(27 - z_1z_4 - z_2z_3 - z_3z_4 - z_2z_7 - z_4z_5 - z_2z_8 - z_5z_6 \\ -z_6z_9 - z_6z_{10}). \tag{13}$$

The provided function can be transformed into a Hamiltonian. This can be done by substituting the z_i terms with Pauli Z, identity matrix I for any scalar, and the dot product by a tensor product in the expression. This transformation can be expressed as

$$H = 0.5(27I - [Z_1 \otimes Z_4] - [Z_2 \otimes Z_3] - [Z_3 \otimes Z_4] - [Z_2 \otimes Z_7] - [Z_4 \otimes Z_5] \\ -[Z_2 \otimes Z_8] - [Z_5 \otimes Z_6] - [Z_6 \otimes Z_9] - [Z_6 \otimes Z_{10}]). \tag{14}$$

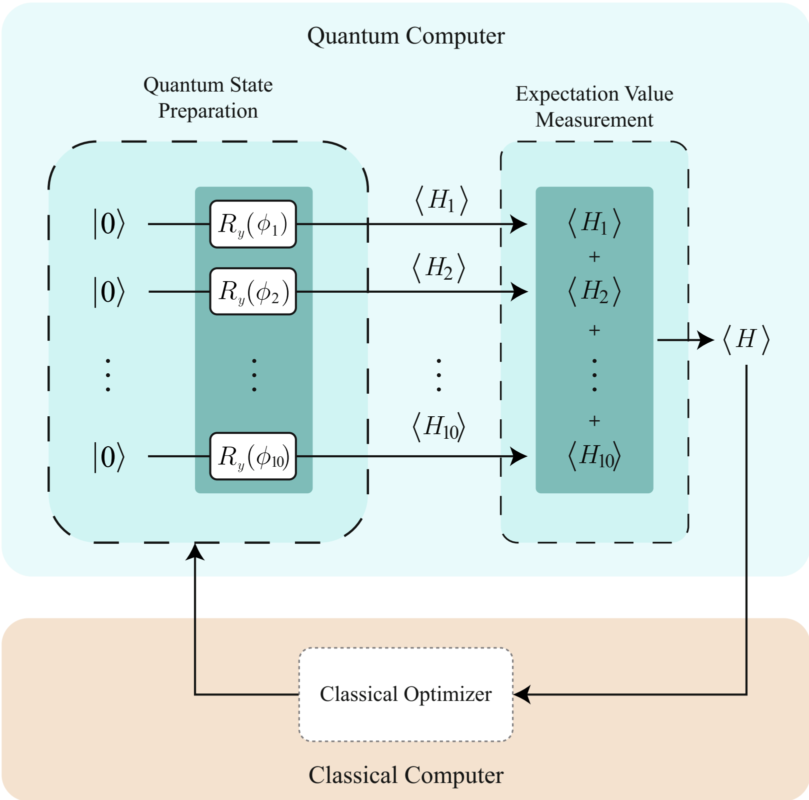

Fig. 2. VQE circuit for our optimization problem.

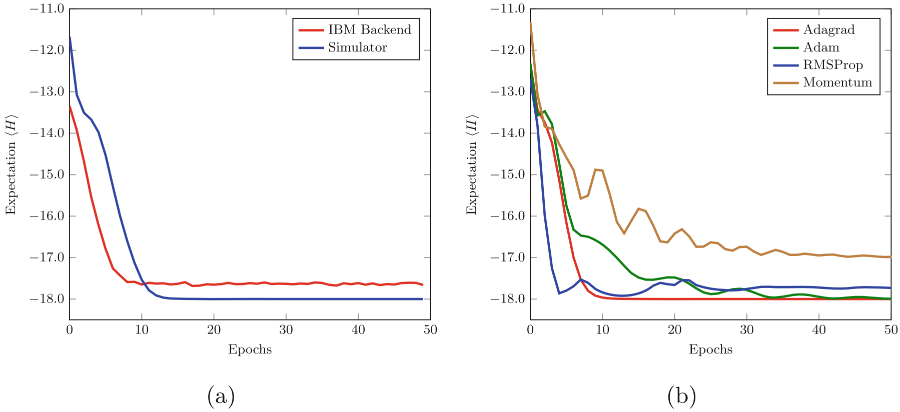

Fig. 3. Expected value of the Hamiltonian H plotted as a function of epochs for (a) IBM backend and (b) different classical optimizers on simulator.

Algorithm 1. Variational Quantum Eigensolver (VQE)

procedure VQE(H, ansatz, optimizer, max_iter)
  Initialize parameters θ for the quantum circuit (ansatz) randomly
  for $iter = 1$ to max_iter **do**
    Prepare the quantum state $|\psi(\theta)\rangle$ using the ansatz
    Measure the expectation value $E(\theta) = \langle\psi(\theta)|H|\psi(\theta)\rangle$
    $E_{min} \leftarrow E(\theta)$, $\theta_{min} \leftarrow \theta$
    Update θ using the optimizer based on $E(\theta)$
  end for
  return Optimal parameters θ_{min} and minimum eigenvalue E_{min}
end procedure

It is often more practical to minimize a cost function. Consequently, we can convert our problem into a minimization scenario by altering the signs of the terms in equation (14). The flow of solving the problem using VQE is demonstrated in Algorithm 1.

4 Results

We conducted the experiment using actual quantum hardware and compared the results with those obtained from a quantum simulator. The physical device employed was IBM's *ibmq_kolkata*, boasting 27 qubits, with each run utilizing 1024 shots [1]. The quantum simulator utilized in this study was Pennylane's simulator [5]. In our experimental setup, a parameterized quantum circuit consisting of 10 qubits was employed. Each qubit underwent a R_y gate rotation, with the rotation angle θ being iteratively optimized. The circuit configuration is depicted in Fig. 2, where initial parameter values were selected randomly. For both the real quantum backend and the simulator, we utilized Adagrad as the classical optimizer, conducting 50 iterations with an optimization step size of 0.5.

In Fig. 3a, the real quantum backends converge to a minimum expectation value of -17.6 around the 10th epoch, while the quantum simulator achieves a minimum of -17.9 after the 13th epoch. The slight disparity is attributed to noise in the real quantum backend. The notable advantage of employing variational algorithms lies in their robustness on noisy hardware, as evidenced by the minimal difference between the real backend and simulator results.

The ground state of Hamiltonian corresponds to the optimal solution of the QUBO problem. Therefore we sampled the optimized configuration of the quantum circuit, thereby providing a solution to the node activation scheduling problem. From both the experiments, we obtained the same solution of [0 1 0 1 0 1 0 0 0 0]. This resulting configuration indicates that nodes 1, 3, 5, 7, 8, 9, and 10 are designated for transmitting, while nodes 2, 4, and 6 are designated for receiving to mitigate interference among them. Subsequently, the roles of transmitting and receiving nodes are interchanged which optimizes the overall performance of the network.

Moreover, we conducted an analysis of different classical optimizers using the quantum simulator for the same problem. Specifically, we employed four classical optimizers-Adagrad, Adam, RMSProp, and Momentum. Their respective performances are depicted in Fig. 3b. Notably, Adagrad demonstrated superior efficacy by converging in the minimum number of epochs, indicating adept adaptation to the optimization landscape. RMSProp closely followed Adagrad in performance, displaying reliable convergence behavior. In contrast, both Adam and Momentum optimizers either failed to reach a minimum within the allotted maximum epochs or were very slow, suggesting challenges in adapting learning rates or efficiently navigating the optimization landscape.

5 Conclusion

This study explores the realm of quantum computing, focusing specifically on leveraging the QUBO framework to tackle the intricate challenges of scheduling multihop wireless networks. Through the application of the VQE algorithm on a quantum computer, we demonstrated the potential applications of quantum computing, particularly within the constraints of the noisy NISQ era. In this era, NISQ computers demonstrate the capability to effectively address small to medium-scale QUBO problems, limited by the current hardware's available qubit count. As quantum computing advances toward fault tolerance and an expanded qubit count, the hybrid approach demonstrated in this paper anticipates the possibility of more efficiently solving larger and more complex problems for optimization problems in many areas including wireless communication.

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