Robust Unscented Kalman Filter for Target Tracking Based on Mahalanobis Distance

Bingbing Gao[1], Wenmin Li[1](✉), Longqiang Ni[2], and Wei Wang[1]

[1] School of Automation, Northwestern Polytechnical University, Xi'an, China
cocogalaxy@163.com
[2] Northwest Institute of Mechanical and Engineering, Xianyang, China

Abstract. A robust unscented Kalman filtering (UKF) is presented on the basis of the theory of Mahalanobis distance to enhance radar target tracking's robustness against abnormal observation information. This method firstly uses the concept of Mahalanobis distance to identify the abnormal observation involved in the radar tracking; and then, a scaling factor with robust property is determined and used into the innovation covariance of classical UKF to weaken the negative effect of aberrant observations on system estimation. The designed robust UKF could effectively enhance the filter's robustness and improve the tracking accuracy of radar system. The simulation outcomes verify that the designed robust UKF has a better performance than the classical UKF and RUKF, leading to superior tracking performance for radar system.

Keywords: Radar target tracking · Unscented Kalman filter · Abnormal observations identification · Mahalanobis distance

1 Introduction

Target tracking technology has received an extensive application for military and civil fields like artillery radar, remote warning, civil aviation transportation, and so on [1–3]. It is constantly developing and updating in order to achieve the accurate tracking with the increasingly complex environment such as the human interference factors in the information war, Therefore, accurate target tracking problem has become the current research hotspot [3].

Filtering technology is the key for target tracking [2, 3]. At present, Kalman filtering (KF) is extensively applied in target tracking fields such as missile tracking, artillery radar system due to its advantages, for example strong real-time performance, small amount of calculation, high efficiency and stable prediction results. KF can obtain the optimal state estimation under the principle of the minimum mean square error for linear Gaussian system [4]. However, for the radar target tracking system, the state model and measurement model of a target cannot both be linear in the same coordinate system [3]. Under this condition, KF shows its shortcomings in dealing with nonlinear problems [4, 5]. In the field of target tracking, there are two commonly used methods to solve

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the above problem: one is extended Kalman filtering (EKF) [5, 6]. EKF transforms the nonlinear filtering problem into a linear KF one by using the first-order linearization of nonlinear function. However, the accuracy of state estimation for the EKF is poor when the system is highly nonlinear [7]; in addition, it greatly limits the application scope because EKF needs to compute complex Jacobian matrix in the process of implementation. The other is unscented Kalman filtering (UKF) [8]. Compared with the EKF, there is no need for UKF to calculate Jacobian matrix and its calculation amount is equivalent to EKF [9]. Furthermore, UKF can achieve higher approximation accuracy and effectively overcomes the shortcomings of EKF in estimation accuracy and filtering stability [10, 11]. However, UKF has poor robustness against abnormal observations. The estimation accuracy of the filter will be seriously deteriorated when the radar is subject to abnormal interference or equipment failure, which will has an impact on the system tracking performance [12, 13].

The existing researches mostly use H-infinity, noise statistics estimation and adjustment factor to enhance the robust performance of UKF for aberrant observation. Zhao et al. proposed a robust UKF by introducing H-infinity strategy into classical UKF. However, it is not suitable for the condition of random aberrant observation [14]. By using the maximum a posteriori estimation, Shi and Han established an adaptive UKF, which is actually integrated the classical UKF with the traditional Sage-Husa estimator [12]. However, in this method the determined forgetting factor is suboptimal since it is chosen by experience. Meng et al. proposed an adaptive UKF based on covariance matching method to estimate the observation noise covariance [15]. However, the steady-state estimation error limits the further improvement of UKF filtering accuracy. Soken and Hajiyev designed a robust UKF by embedding a robust factor to regulate the filter gain, thus the robustness of UKF against abnormal observation can be improved [16]. However, the robust factor in this method is also determined by experience, which leads to suboptimal filtering results.

In this paper, a robust UKF is proposed on the basis of Mahalanobis distance for enhancing the robust performance of radar single target tracking system against abnormal observation information. A novel method of abnormal observation identification is established based on the Mahalanobis distance. Next, a scaling factor with robust property is further determined and embedded into the innovation covariance matrix of classical UKF to adjust the filter gain for decreasing the negative impact of aberrant observation on state estimation. The stimulation on target tracking for a radar system has been conducted to comprehensively evaluate the proposed methodology.

2 System Model of Radar Target Tracking

Based on the two-dimensional Cartesian coordinate system, the system model for radar target tracking is established [17]. It is assumed that the target's motion obeys the CV model, its azimuth and slant range measured by a radar [18].

2.1 State Model

Supposed that a target P is moving in two-dimensional space x-y, and its position and velocity at time k can be expressed by

$$\boldsymbol{X}_k = \left[x_k, \dot{x}_k, y_k \dot{y}_k\right]^{\mathrm{T}} \tag{1}$$

It is assumed that P moves at approximately constant velocity in horizontal (x-axis) and vertical (y-axis) directions, and there is additive system noise $\boldsymbol{W}_k$ in both directions [19]. Then, the state model of the target motion in Cartesian coordinate system is described by

$$\boldsymbol{X}_k = \boldsymbol{F}_{k/k-1}\boldsymbol{X}_{k-1} + \boldsymbol{W}_k \tag{2}$$

where $\boldsymbol{W}_k$ is additive system noise; $\boldsymbol{F}_{k/k-1}$ is system transformation matrix, and its specific form is:

$$\boldsymbol{F}_{k/k-1} = \begin{bmatrix} 1\,1\,0\,0 \\ 0\,1\,0\,0 \\ 0\,0\,1\,1 \\ 0\,0\,0\,1 \end{bmatrix} \tag{3}$$

2.2 Observation Model

The slant range and azimuth of a target is measured by a radar, thus the observation model of radar target tracking system can be established as [20]

$$\boldsymbol{Z}_k = h(\boldsymbol{X}_k) + \boldsymbol{V}_k = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ ar\tan\left(\frac{y_k}{x_k}\right) \end{bmatrix} + \boldsymbol{V}_k \tag{4}$$

where $\boldsymbol{V}_k$ is the observation noise.

3 Robust Unscented Kalman Filter Based on Mahalanobis Distance

By utilizing the theory of Mahalanobis distance, we firstly develop a novel way of aberrant observation identification in this section. Then, we further propose a new robust UKF to improve the radar target tracking's robust performance against aberrant observations.

3.1 Classical UKF

By combining (2) and (4), the nonlinear Gaussian discrete model is considered as follows

$$\boldsymbol{X}_k = \boldsymbol{F}_{k/k-1}\boldsymbol{X}_{k-1} + \boldsymbol{W}_k \tag{5}$$

$$\boldsymbol{Z}_k = h(\boldsymbol{X}_k) + \boldsymbol{V}_k \tag{6}$$

where $\boldsymbol{X}_k \in \mathbf{R}^n$ is the system state variable, $\boldsymbol{F}_{k/k-1}$ is discrete state transform matrix, $\boldsymbol{Z}_k \in \mathbf{R}^m$ is the observation variable, $h(\cdot)$ is nonlinear observation function, $\boldsymbol{W}_k$ and $\boldsymbol{V}_k$ are the unrelated Gaussian noise processes with zero-mean, whose variances satisfy

$$E\left[\boldsymbol{W}_k\boldsymbol{W}_k^{\mathrm{T}}\right] = \boldsymbol{Q},\quad E\left[\boldsymbol{V}_k\boldsymbol{V}_k^{\mathrm{T}}\right] = \boldsymbol{R},\quad E\left[\boldsymbol{W}_k\boldsymbol{V}_k^{\mathrm{T}}\right] = \mathbf{0} \tag{7}$$

The procedures of classical UKF is described as:

Step 1: *Initialization.*
State estimate and its covariance are initialized as

$$\begin{cases} \hat{\boldsymbol{X}}_0 = E[\boldsymbol{X}_0] \\ \boldsymbol{P}_0 = E[(\boldsymbol{X}_0 - \hat{\boldsymbol{X}}_0)(\boldsymbol{X}_0 - \hat{\boldsymbol{X}}_0)^{\mathrm{T}}] \end{cases} \tag{8}$$

Step 2: *Prediction.*
The calculation of the state prediction and its error covariance matrix follow as the KF because of the system's linear state model

$$\hat{\boldsymbol{X}}_{k/k-1} = \boldsymbol{F}_{k/k-1}\hat{\boldsymbol{X}}_{k-1} \tag{9}$$

$$\boldsymbol{P}_{k/k-1} = \boldsymbol{F}_{k/k-1}\boldsymbol{P}_{k-1}\boldsymbol{F}_{k/k-1}^{\mathrm{T}} + \boldsymbol{Q} \tag{10}$$

Step 3: *Sigma point selection.*
The Sigma points are selected according to the state prediction $\hat{\boldsymbol{X}}_{k/k-1}$ and associated error covariance matrix $\boldsymbol{P}_{k/k-1}$

$$\begin{cases} \boldsymbol{\xi}_{i,k/k-1} = \hat{\boldsymbol{X}}_{k/k-1} & i = 0 \\ \boldsymbol{\xi}_{i,k/k-1} = \hat{\boldsymbol{X}}_{k/k-1} + a\left(\sqrt{nP_{k/k-1}}\right)_i & i = 1, 2, \cdots, n \\ \boldsymbol{\xi}_{i,k/k-1} = \hat{\boldsymbol{X}}_{k/k-1} - a\left(\sqrt{nP_{k/k-1}}\right)_{i-n} & i = n+1, n+2, \cdots, 2n \end{cases} \tag{11}$$

where $\left(\sqrt{n\boldsymbol{P}_{k/k-1}}\right)_i$ represents the ith column of $\sqrt{n\boldsymbol{P}_{k/k-1}}$; $a \in \mathbf{R}$ is adjustment parameter, which is determined by the trial and error method, and it controls Sigma points' distribution around $\hat{\boldsymbol{X}}_{k/k-1}$.

Step 4: *Update.*
The selected Sigma points in (11) are transformed by observation function

$$\gamma_{i,k/k-1} = h\left(\boldsymbol{\xi}_{i,k/k-1}\right)(i = 0, 1, \cdots, 2n) \tag{12}$$

The observation prediction and its error covariance matrix were calculated by

$$\hat{\boldsymbol{Z}}_{k/k-1} = \sum_{i=0}^{2n} \omega_i \gamma_{i,k/k-1} \tag{13}$$

$$\boldsymbol{P}_{\hat{\boldsymbol{Z}}_{k/k-1}} = \sum_{i=0}^{2n} \omega_i \left(\gamma_{i,k/k-1} - \hat{\boldsymbol{Z}}_{k/k-1}\right)\left(\gamma_{i,k/k-1} - \hat{\boldsymbol{Z}}_{k/k-1}\right)^{\mathrm{T}} + \boldsymbol{R} \tag{14}$$

The cross-covariance matrix is calculated as

$$\boldsymbol{P}_{\hat{\boldsymbol{X}}_{k/k-1}\hat{\boldsymbol{Z}}_{k/k-1}} = \sum_{i=0}^{2n} \omega_i \left(\xi_{i,k/k-1} - \hat{\boldsymbol{X}}_{k/k-1} \right) \left(\gamma_{i,k/k-1} - \hat{\boldsymbol{Z}}_{k/k-1} \right)^{\mathrm{T}} \tag{15}$$

where $\begin{cases} \omega_i = 1 - \dfrac{1}{a^2} & i = 0 \\ \omega_i = \dfrac{1}{2na^2} & i = 1, 2, \cdots, n \end{cases}$

Determine the gain matrix

$$\boldsymbol{K}_k = \boldsymbol{P}_{\hat{\boldsymbol{X}}_{k/k-1}\hat{\boldsymbol{Z}}_{k/k-1}} \boldsymbol{P}^{-1}_{\hat{\boldsymbol{Z}}_{k/k-1}} \tag{16}$$

Update the state estimation and its error covariance

$$\hat{\boldsymbol{X}}_k = \hat{\boldsymbol{X}}_{k/k-1} + \boldsymbol{K}_k \left(\boldsymbol{Z}_k - \hat{\boldsymbol{Z}}_{k/k-1} \right) \tag{17}$$

$$\boldsymbol{P}_k = \boldsymbol{P}_{k/k-1} - \boldsymbol{K}_k \boldsymbol{P}_{\hat{\boldsymbol{Z}}_{k/k-1}} \boldsymbol{K}_k^{\mathrm{T}} \tag{18}$$

Step 5: Return to Step 2, and perform the following time's filtering solution until all data is conducted.

3.2 Mahalanobis Distance Based Robust Unscented Kalman Filter

3.2.1 Identification of Aberrant Observation

In statistics, we usually use a criterion named Mahalanobis distance to detect outliers of multivariate data [21, 22]. Assume that a multi-dimensional vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_p)^{\mathrm{T}}$, whose mean is $\boldsymbol{\mu} = (\mu_1, \mu_2, \cdots, \mu_p)^{\mathrm{T}}$ and the covariance matrix is $\boldsymbol{\Sigma}$, we can define the Mahalanobis distance as follow [22].

$$D(\boldsymbol{x}) = \sqrt{(\boldsymbol{x} - \boldsymbol{\mu})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\boldsymbol{x} - \boldsymbol{\mu})} \tag{19}$$

We can identify aberrant observations for radar target tracking system via the concept of Mahalanobis distance, thus the innovation of filter is defined as

$$\tilde{\boldsymbol{Z}}_k = \boldsymbol{Z}_k - \hat{\boldsymbol{Z}}_{k/k-1} \tag{20}$$

When a Gaussian system does not have any aberrant observations, the innovation $\tilde{\boldsymbol{Z}}_k$ will be a multivariate Gaussian distribution $N\left(0, \boldsymbol{P}_{\tilde{\boldsymbol{Z}}_{k/k-1}}\right)$, where $\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_{k/k-1}}$ is innovation covariance and its specific form is [4, 15].

$$\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_{k/k-1}} = \boldsymbol{P}_{\hat{\boldsymbol{Z}}_{k/k-1}} = \sum_{i=0}^{2n} \omega_i \left(\gamma_{i,k/k-1} - \hat{\boldsymbol{Z}}_{k/k-1} \right) \left(\gamma_{i,k/k-1} - \hat{\boldsymbol{Z}}_{k/k-1} \right)^{\mathrm{T}} + \boldsymbol{R} \tag{21}$$

We denote following formula through the definition of Mahalanobis distance's in (19)

$$\beta_k = D^2(\tilde{\boldsymbol{Z}}_k) = \tilde{\boldsymbol{Z}}_k^{\mathrm{T}} \boldsymbol{P}_{\tilde{\boldsymbol{Z}}_{k/k-1}}^{-1} \tilde{\boldsymbol{Z}}_k \tag{22}$$

It is known that $D^2(\tilde{\boldsymbol{Z}}_k)$ obeys the χ^2 distribution according to the statistical knowledge [4].

$$\beta_k = D^2(\tilde{\boldsymbol{Z}}_k) \sim \chi_m^2 \tag{23}$$

When a significance level $\alpha(0 < \alpha < 1)$ is given, there is a threshold $\chi_{m,\alpha}^2$ to generate the relationship as below in the light of χ^2 test theory

$$P\{\beta_k > \chi_{m,\alpha}^2\} = \alpha \tag{24}$$

Therefore, the following criteria can be constructed to identify the aberrant observation of the system

$$\begin{cases} H_0 : \beta_k \le \chi_{m,\alpha}^2 \text{ without aberrant observation} \\ H_1 : \beta_k > \chi_{m,\alpha}^2 \text{ with aberrant observation} \end{cases} \tag{25}$$

where $\chi_{m,\alpha}^2$ is the pre-set test threshold.

After completing aberrant observation identification of system model, a robust UKF is proposed on the basis of Mahalanobis distance in the following to resist the aberrant observations for improving the robustness of tracking system.

3.2.2 Calculation of the Robust Factor

Robust UKF is to modify observation noise covariance matrix by a robust factor, which can affect the $\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_{k/k-1}}$ and reduce gain matrix, leading to weakened influence of abnormal observation on system solution. Therefore, its key is how to determine the robust factor.

The core of the proposed method is to embed the robust factor κ_k into the classical UKF's innovation covariance, so the proposed method's innovation covariance can be formulated as

$$\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_{k/k-1}}^{*} = \sum_{i=0}^{2n} \omega_i \left(\gamma_{i,k/k-1} - \hat{\boldsymbol{Z}}_{k/k-1}\right)\left(\gamma_{i,k/k-1} - \hat{\boldsymbol{Z}}_{k/k-1}\right)^{\mathrm{T}} + \kappa_k \boldsymbol{R} \tag{26}$$

Substituting (26) into (22), the following nonlinear function can be achieved:

$$g(\kappa_k) = \beta_k^* - \chi_{m,\alpha}^2 = \tilde{\boldsymbol{Z}}_k^{\mathrm{T}} \left(\boldsymbol{P}_{\tilde{\boldsymbol{Z}}_{k/k-1}}^{*}\right)^{-1} \tilde{\boldsymbol{Z}}_k - \chi_{m,\alpha}^2 \tag{27}$$

The robust factor can be calculated by solving the equation when we let $g(\kappa_k) = 0$. The Newton iterative method [9, 21] with good convergence is adopted in this paper to solve the above nonlinear equation, and the following formula is obtained

$$\kappa_k(i+1) = \kappa_k(i) - \frac{g[\kappa_k(i)]}{g'[\kappa_k(i)]} \tag{28}$$

where i is the iteration number, and $g'[\cdot]$ represents the derivative of $g[\cdot]$.

Taking (27) into (28), it will be

$$\boldsymbol{\kappa}_k(i+1) = \boldsymbol{\kappa}_k(i) - \frac{\tilde{\boldsymbol{Z}}_k^{\mathrm{T}}\left(\boldsymbol{P}^*_{\tilde{\boldsymbol{Z}}_{k/k-1}}\right)^{-1}\tilde{\boldsymbol{Z}}_k^{\mathrm{T}} - \chi^2_{m,\alpha}}{\tilde{\boldsymbol{Z}}_k^{\mathrm{T}}\left[\left(\boldsymbol{P}^*_{\tilde{\boldsymbol{Z}}_{k/k-1}}(i)\right)^{-1}\right]\tilde{\boldsymbol{Z}}_k^{\mathrm{T}}} \tag{29}$$

By using the matrix formula [9]

$$\frac{d}{dt}(\boldsymbol{M}^{-1}) = -\boldsymbol{M}^{-1}\frac{d\boldsymbol{M}}{dt}\boldsymbol{M}^{-1} \tag{30}$$

it is easy to get

$$\kappa_k(i+1) = \kappa_k(i) + \frac{\tilde{\boldsymbol{Z}}_k^{\mathrm{T}}\left(\boldsymbol{P}^*_{\tilde{\boldsymbol{Z}}_{k/k-1}}(i)\right)^{-1}\tilde{\boldsymbol{Z}}_k - \chi^2_{m,\alpha}}{\tilde{\boldsymbol{Z}}_k^{\mathrm{T}}\left[\left(\boldsymbol{P}^*_{\tilde{\boldsymbol{Z}}_{k/k-1}}(i)\right)^{-1}\boldsymbol{R}\left(\boldsymbol{P}^*_{\tilde{\boldsymbol{Z}}_{k/k-1}}(i)\right)^{-1}\right]\tilde{\boldsymbol{Z}}_k} \quad (i = 0, 1, 2, \cdots) \tag{31}$$

where $\boldsymbol{M}$ represents a matrix, which is invertible about t.

The above iteration's initial value is set as $\kappa_k(0) = 1$ when using (31) for iterative calculation, and the iterative result of each step is brought into (22) for calculating β_k^*. The iteration ends at the time of $\beta_k^* \leq \chi^2_{m,\alpha}$, and the last iterative result is the determined robust factor.

3.2.3 Proposed Methodology

The proposed methodology is summarized as the Fig. 1, which has the following procedures:

Step 1: *Initialization.*
The filter is initiated as (8).

Step 2: *Prediction.*
Trough (9) and (10), the state prediction $\hat{\boldsymbol{X}}_{k/k-1}$ and error covariance $\boldsymbol{P}_{k/k-1}$ is obtained.

Step 3: *Update.*

i) The innovation $\tilde{\boldsymbol{Z}}_k$ and error covariance $\boldsymbol{P}_{\hat{z}_{k/k-1}}$ of the filter are calculated by (20) and (21).
ii) Set $\kappa_k(0) = 1$ and calculate β_k^*;
iii) If $\beta_k^* \leq \chi^2_{m,\alpha}$,
- Perform the classical UKF steps (15)–(18) to achieve the $\hat{\boldsymbol{X}}_k$.

Else,

- κ_k is solved by (31) until it meets $\beta_k^*(i) \le \chi_{m,\alpha}^2$.
- Update innovation covariance $\boldsymbol{P}^*_{\tilde{\boldsymbol{Z}}_{k/k-1}}$
- Execute the steps (15)–(18) in classical UKF to renewal the $\hat{\boldsymbol{X}}_k$.

Step 4: Return to Step 2, and perform the following time's filtering solution until all data is conducted.

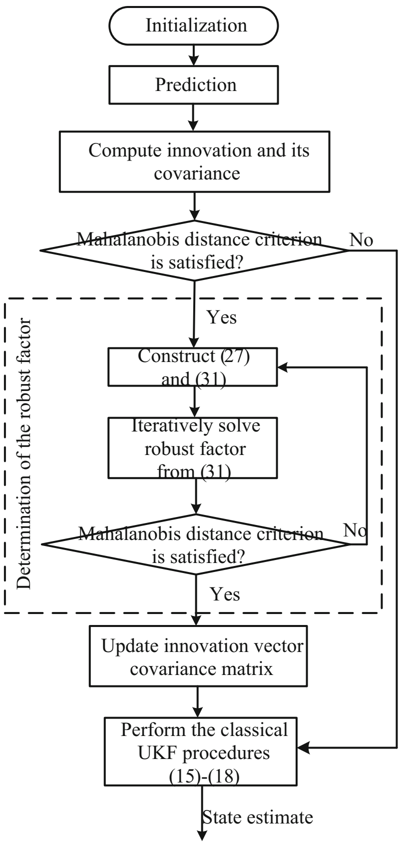

Fig. 1. The calculation process of the MRUKF

4 Simulation Validation and Analysis

The Mahalanobis distance based robust UKF (MRUKF) presented in this paper is applied to radar target tracking system. Compare the results of Monte Carlo simulations of the

MRUKF with the classical UKF and the robust UKF (RUKF) in [16]. Monte Carlo runs $M = 10$.

It is assumed that the covariance of $\boldsymbol{V}_k$ is set to

$$\boldsymbol{R} = diag\left(5^2, 0.005^2\right) \tag{32}$$

We try to evaluate the MRUKF when there exists abnormal observations, and supposed that the observation noise covariance suddenly increases to 50 times of its real value in [400s, 600s]. Therefore, the real covariance matrix of observation noise can be described as

$$\boldsymbol{R}_k = \begin{cases} \boldsymbol{R} & k < 400 \\ 50 \cdot \boldsymbol{R} & 400 \le k \le 600 \\ \boldsymbol{R} & 600 < k < 1000 \end{cases} \tag{33}$$

Figures 2, 3, 4, and 5 show the target's position and velocity errors for the x-axis and y-axis calculated from the classical UKF, RUKF and MRUKF. We can conclude from Figs. 2, 3, 4, and 5 that:

(i) For (0s, 400s) and (600s, 1000s), since the abnormal observation is not introduced into the target tracking system, the classical UKF, RUKF and the proposed MRUKF algorithms all can quickly and accurately estimate the target's position and velocity, and the three algorithms' tracking precision is very close.

(ii) During the time of [400s, 600s], the classical UKF's estimation precision is obviously decreased because of the aberrant observation for the target tracking system. RUKF can reduce the negative impact of aberrant observation on state estimation and improve the estimation precision of classical UKF. But, there is always a violate fluctuation in the estimation error curve of RUKF. Compared with the above two methods, the proposed MRUKF significantly have a more accurate estimation for the radar target tracking system by embedding a robust factor to regulate the filter gain. Its estimation error is significantly smaller than both the mentioned classical UKF and RUKF. Different from RUKF, the robust factor in MRUKF is directly derived from the concept of Mahalanobis distance, which is not determined by artificial experience, thus it has better regulation effect.

Figures 6, 7, 8, and 9 depicts the intuitive comparison of the position and velocity's average absolute errors for the target in x-axis and y-axis directions achieved from the classical UKF, RUKF and MRUKF during [400s, 600s] and other time interval respectively. Further, the maximum velocity and position errors achieved from the classical UKF, RUKF and MRUKF during [400s, 600s] are also calculated and intuitively compared in Figs. 10 and 11. The calculated data in Figs. 6, 7, 8, and 9 as well as Figs. 10 and 11 also fully confirm the above conclusion.

The simulation results show that when the radar target tracking system involves abnormal observation, the estimation accuracy of proposed MRUKF is better than the mentioned classical UKF and RUKF. At the same time, it significantly enhances the robustness of filter, thus improving the target tracking performance of the radar.

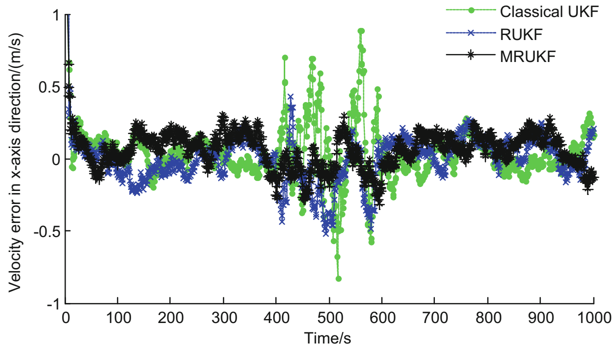

Fig. 2. Velocity errors in *x*-axis by the classical UKF, RUKF and MRUKF

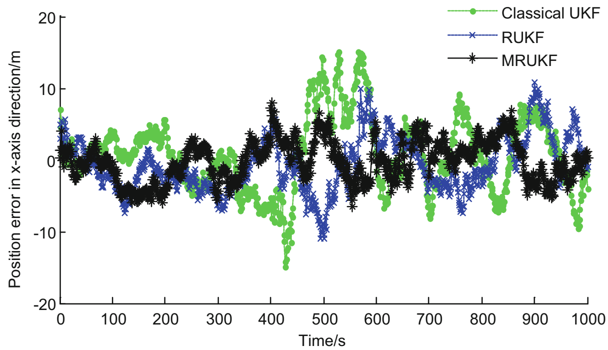

Fig. 3. Position errors in *x*-axis by the classical UKF, RUKF and MRUKF

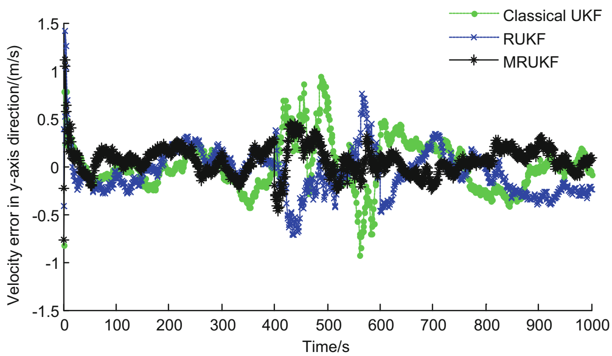

Fig. 4. Velocity errors in *y*-axis by the classical UKF, RUKF and MRUKF

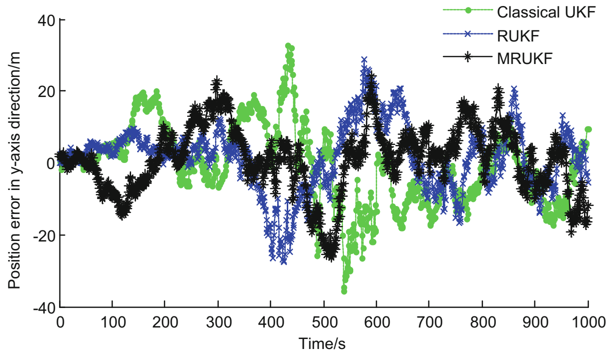

Fig. 5. Position errors in y-axis by the classical UKF, RUKF and MRUKF

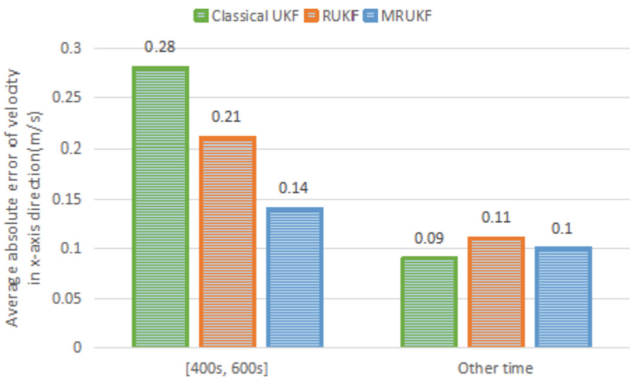

Fig. 6. Average absolute error of velocity in x-axis by the classical UKF, RUKF and MRUKF

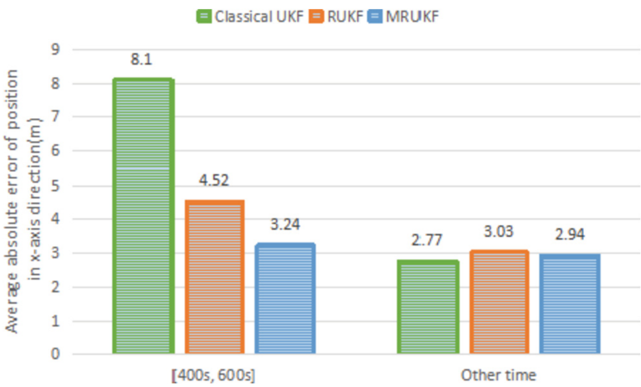

Fig. 7. Average absolute error of position in x-axis by the classical UKF, RUKF and MRUKF

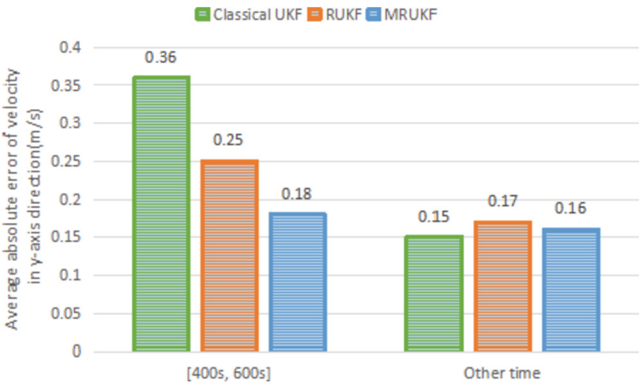

Fig. 8. Average absolute error of velocity in *y*-axis by the classical UKF, RUKF and MRUKF

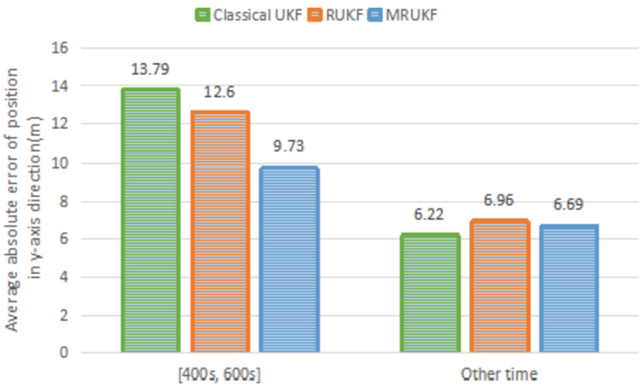

Fig. 9. Average absolute error of position in *y*-axis by the classical UKF, RUKF and MRUKF

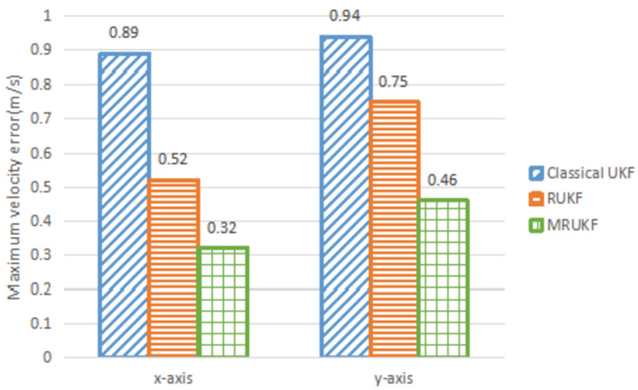

Fig. 10. Maximum velocity error by the classical UKF, RUKF and MRUKF during the time interval [400s, 600s]

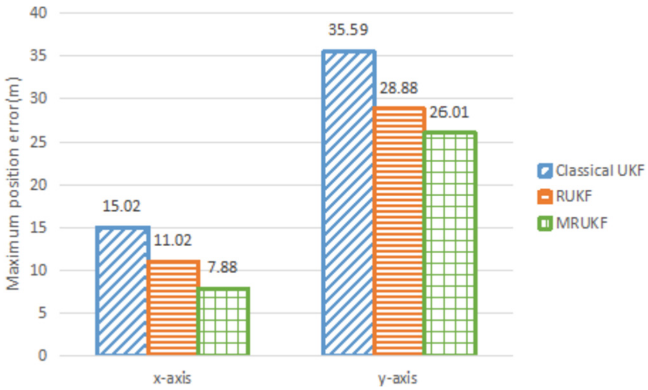

Fig. 11. Maximum position error by the classical UKF, RUKF and MRUKF during the time interval [400s, 600s]

5 Conclusions

This paper describes a MRUKF algorithm to enhance the robustness of radar target tracking system against aberrant observation information. A method of the identification of abnormal observation information for target tracking system is established based on the method of Mahalanobis distance; According to this, a scaling factor with robust property is further determined and embed into the classical UKF's innovation covariance matrix to adjust the filter gain and restrain the negative effect of abnormal observation on system estimation. The simulation outcomes verify that the designed MRUKF has better estimation performance compared to the classical UKF and RUKF when the system involves abnormal observations, which effectively improves the robustness of the filter and significantly enhances the target tracking performance for the radar system.

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