A Novel Codebook Design Scheme for Sparse Code Multiple Access

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Abstract. The sparse code multiple access (SCMA) technology proposed by Huawei has become a very competitive one of many NOMA solutions due to its high overload capability. SCMA maps the user's binary bit data input by the system's sending end into a multi-dimensional complex number sequence through a pre-designed codebook, instead of the modulation and spreading in the orthogonal multiple access scheme. Based on the sparsity of codewords, the receiving end uses the Message Passing Algorithm (MPA) for multi-user detection. Among them, the excellent SCMA codebook can effectively improve the performance of the system under limited spectrum resources. Therefore, this paper conducts detailed research on the SCMA codebook design scheme. First, this article introduces the principle of the SCMA system in detail, and then proposes a SCMA codebook design method, including the design of the mapping matrix, the design of the mother constellation and the constellation operator. Among them, the mother constellation design adopts a reorganized multi-dimensional constellation scheme. The constellation operator obtains the user constellations of different users through phase rotation, and then maps the designed user constellations into a codebook through a mapping matrix. The simulation results show that the performance of this design is better than the sample codebook given by Huawei.

Keywords: Sparse code multiple access · Codebook design · Mother constellation

1 Introduction

The While the rise of the mobile Internet and the Internet of Things (IoT) has brought unprecedented experience to people, it has also brought massive terminal access to mobile communication systems. New applications and service industries have emerged in an endless stream, providing a broad range of communication systems. Prospects. By 2020, the capacity of mobile communication networks is expected to increase by about 1,000 times compared to 2018. Orthogonal multiple access schemes cannot theoretically reach the maximum capacity of the system, and cannot meet the needs of future mobile communication systems for massive user access. Therefore, the fifth generation mobile communication system (5G) uses non-orthogonal multiple access (NOMA) technology as its core technology. Among them, the SCMA technology proposed by

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Huawei has become a very competitive one among many NOMA solutions with its high overload capability. The SCMA system directly maps the user binary bit data input by the transmitter into a multi-dimensional complex number sequence through the pre-designed SCMA codebook, instead of modulation and spreading in the orthogonal multiple access scheme. The codewords sent by different users are at the same time frequency. The non-orthogonal superposition of resources allows the system to accommodate more users at the same time. The receiving end utilizes the sparsity of codewords and uses Message Passing Algorithm (MPA) for multi-user detection.

Excellent SCMA codebook can effectively improve the performance of the system under limited spectrum resources. Therefore, it is of practical significance to study the design of SCMA codebook. The design process of SCMA codebook is a non-convex quadratic programming secondary constraint problem. The optimal solution to this problem has not been found so far, so the design codebook has become an open optimization problem. This paper proposes a sub-optimal design scheme for SCMA codebook, which not only effectively reduces the design complexity, but also has good bit error rate performance.

2 SCMA Sender Principle

2.1 SCMA Encoding Process

The traditional CDMA modulation is to pass the input data stream through QAM modulation, and then generate the modulated spread spectrum signal through the spread spectrum sequence $\{s_1, s_2, s_3, s_4\}$, as shown in Fig. 1.

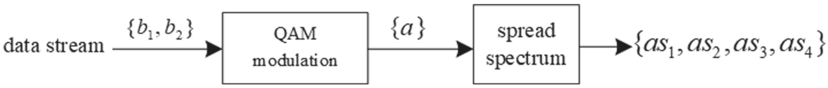

Fig. 1. CDMA modulation

The SCMA system is an improvement of CDMA and spread spectrum technology. The data stream is encoded by SCMA to generate the corresponding codeword. Since the spreading sequence of CDMA is composed of orthogonal codes, and the coding sequence of SCMA is composed of non-orthogonal codes, the SCMA system can accommodate more users (Fig. 2).

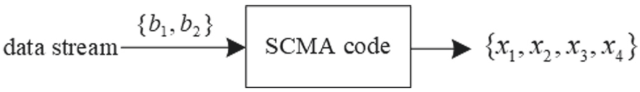

Fig. 2. SCMA modulation

As shown in Fig. 3, the SCMA encoding process can be understood as a process in which a data stream is mapped into a corresponding codeword in a codebook. For example, when data 10 enters the SCMA encoder, it will be mapped to a codeword numbered 10.

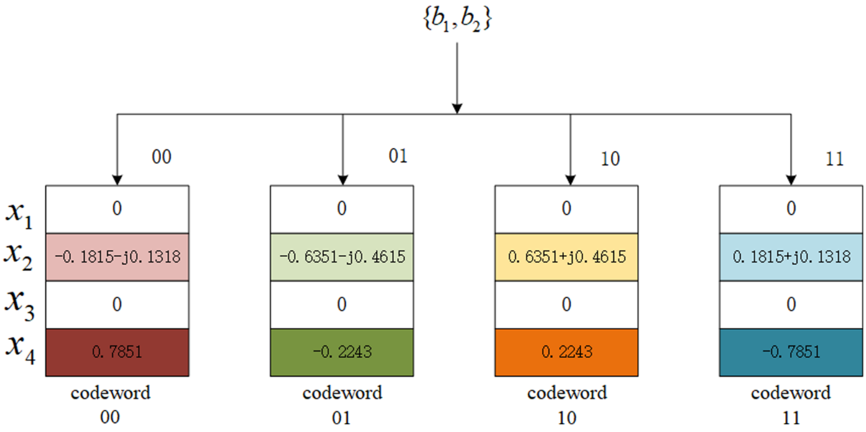

Fig. 3. SCMA codebook

Figure 3 shows the SCMA codebook with $M = 4$-point constellation used by each user. The data comes from the codebook provided by Huawei in the 5G competition [1]. Among them, all users share $K = 4$ orthogonal time-frequency resources. The codebook of each user consists of 4 code words, corresponding to a 4-point constellation. Each codeword consists of 4 blocks, and each block represents an orthogonal time-frequency resource. The squares are divided into white squares and colored squares. White squares represent zero elements, and colored squares represent non-zero elements. Therefore, it can be seen that each user only transmits data on orthogonal time-frequency resources representing non-zero elements. Assuming that the number of non-zero element blocks of each codeword is N, each user in the corresponding figure only transmits data on $N = 2$ orthogonal time-frequency resources.

After the data is mapped into corresponding codewords, the codewords of different users are transmitted after non-orthogonal superposition on the same orthogonal time-frequency resource. Since the codewords of different users interfere with each other when they are superimposed, in order to distinguish the data of each user, the positions of non-zero element blocks in the codebooks of different users cannot be completely the same, that is, N of any two user codebooks Orthogonal time-frequency resources cannot be completely the same. From this, we can get the expression of the maximum user access number $J_{\max}$ of the SCMA system as:

$$J_{\max} = \binom{K}{N} \tag{1}$$

Figure 4 shows the process of non-orthogonal superposition of codewords of $J = 6$ users on $K = 4$ orthogonal time-frequency resources.

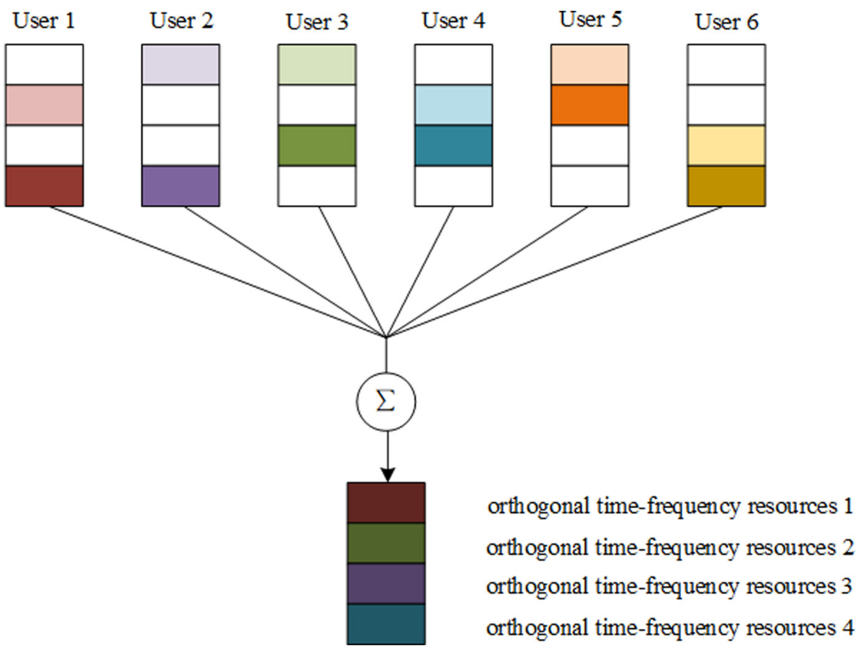

Fig. 4. Overlay of codewords of different users

2.2 Sparse Matrix

It can be seen from Fig. 4 that user 3 transmits data on orthogonal time-frequency resource 1 and orthogonal time-frequency resource 3. The column vector $(1 \quad 0 \quad 1 \quad 0)^T$ can be used to represent its codeword structure, and 1 indicates that data is transmitted on the orthogonal time-frequency resource at the corresponding position. 0 means that no data is transmitted on the corresponding orthogonal time-frequency resource. The matrix $\boldsymbol{F}_{K \times J}$ composed of column vectors of the codeword structure of all users is called a sparse matrix. The sparse matrix corresponding to Fig. 4 is:

$$\boldsymbol{F}_{4\times 6} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

From the sparse matrix of Eq. 2, it can be seen that any two column vectors are mainly non-orthogonal, which reflects the non-orthogonality of SCMA. The sparsity is reflected by the low proportion of 1 in each column. Each user occupies 2 orthogonal time-frequency resources, and the proportion of 1 in each column is 50%. The number of 1 s in each row is 3, which means that each orthogonal time-frequency resource has 3 users' data for non-orthogonal superposition. The number of users d_f superimposed on each orthogonal time-frequency resource can be calculated by Eq. 3:

$$d_f = \frac{JN}{K} \tag{3}$$

The system in Fig. 4 shows that the SCMA system needs only 4 orthogonal time-frequency resources to transmit data of 6 users at the same time, and the overload rate λ of users is defined as the ratio of the number of users to the number of orthogonal time-frequency resources:

$$\lambda = \frac{J}{K} \tag{4}$$

The overload rate of this system is 150%. Compared with the 100% overload rate of the OFDMA system, it can be clearly seen that the SCMA system can accommodate more users in limited resources.

3 SCMA Codebook Design Scheme

Suppose the number of users connected to the SCMA system is J. When transmitting data, if the user adopts an M-point constellation, the SCMA codebook of each user is composed of M codewords, and each codeword contains K orthogonal time-frequency resources, the actual number of orthogonal time-frequency resources occupied by each user is N. SCMA encoding can be regarded as a process of selecting corresponding codewords for $\log_2 M$ bits of data from M codewords containing K orthogonal time-frequency resources. Therefore, the process of SCMA codebook design can be expressed by Equation 5 and Equation 6 [5]:

$$\boldsymbol{C}_j = g_j(\boldsymbol{b}_j) \tag{5}$$

$$\boldsymbol{X}_j = \boldsymbol{V}_j\boldsymbol{C}_j \tag{6}$$

First, the data stream $\boldsymbol{b}_j$ sent by the user j is substituted into the constellation generation function g_j to obtain an N-dimensional user constellation $\boldsymbol{C}_j$ containing M constellation points, and then the mapping matrix $\boldsymbol{V}_j$ is applied to the user constellation $\boldsymbol{C}_j$, and the SCMA codebook $\boldsymbol{X}_j$ is mapped.

Use V to represent the set of mapping matrices and G to represent the set of constellation generating functions, then there are $V = \{\boldsymbol{V}_j \widehat{u} j \in [1, J]\}$, $G = \{g_j \widehat{u} j \in [1, J]\}$. Then the SCMA codebook design problem can be simplified to find the optimal solution set of V and G under the given conditions of J, M, K, and N. However, the optimal solution for this problem has not been found yet. When designing SCMA codebooks, we usually adopt a sub-optimal solution, which is mainly divided into three links: SCMA mapping matrix design, mother constellation design, and constellation operator design. It is a common scheme for SCMA codebook design.

3.1 Design Mapping Matrix

The purpose of constructing the mapping matrix is to allocate orthogonal time-frequency resources to users. A good mapping matrix can reduce the complexity of MPA decoding at the receiving end, which is equivalent to the spread spectrum function in CDMA. The construction rules of the mapping matrix are as follows:

1.
$$\boldsymbol{V}_j \in \boldsymbol{B}^{K \times N}$$

2.
$$\boldsymbol{V}_i \neq \boldsymbol{V}_j, \forall i \neq j$$

3.
$$\boldsymbol{V}_j^{[\emptyset]} = \boldsymbol{I}_N$$

Among them, $\boldsymbol{B}^{K \times N}$ is a matrix of order $K \times N$ consisting of only 0 and 1, $\boldsymbol{I}_N$ is a unit matrix of order N, and $\boldsymbol{V}_j^{[\emptyset]}$ is the matrix obtained by removing all 0 rows from V_j.

Define L as the number of orthogonal time-frequency resources occupied by two users. Since the orthogonal time-frequency resources occupied by two users are not completely the same, $L \leqslant N - 1$. When $2N > K$, the number of the same orthogonal time-frequency resources occupied by two users is at least $2N - K$, otherwise it is 0. Therefore, the value range of L is:

$$\max(0, 2N - K) \leqslant L \leqslant N - 1 \tag{7}$$

Take the SCMA system with $J = 6$, $K = 4$, $N = 2$ as an example, at this time $0 \leqslant L \leqslant 1$. The matrix obtained by removing all 0 rows from V_j is:

$$\boldsymbol{V}_j^{[\emptyset]} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} j \in [1, 6] \tag{8}$$

The six mapping matrices expanded by $\boldsymbol{V}_j \in \boldsymbol{B}^{4 \times 2}$ are as follows:

$$\begin{array}{lll} \boldsymbol{V}_1 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} & \boldsymbol{V}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} & \boldsymbol{V}_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \boldsymbol{V}_4 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} & \boldsymbol{V}_5 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} & \boldsymbol{V}_6 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \end{array} \tag{9}$$

3.2 Design Mother Constellation

After the mapping matrix is designed, the SCMA codebook design problem can be simplified to find the optimal solution set of G under the given conditions of V, J, M, K, and N. At this time, it is necessary to design J N-dimensional constellations containing M constellation points. We can design a mother constellation first, and then obtain the constellations of other users through operations such as phase rotation. Define the matrix operation that converts the mother constellation to other user constellations as a constellation operator. The user constellation can be expressed as:

$$\boldsymbol{C}_j = (\Delta_j)\boldsymbol{C}, \forall j \tag{10}$$

Among them, $\boldsymbol{C}$ represents the mother constellation, and Δ_j represents the constellation operator.

Use $\boldsymbol{C}^{(n)}$ to represent the nth dimension of the mother constellation, and its length is M, that is, each dimension contains M constellation points:

$$\boldsymbol{C}^{(n)} = \begin{pmatrix} c_{n1} & c_{n2} & \cdots & c_{nN} \end{pmatrix} \forall n \in [1, N] \tag{11}$$

The mother constellation is composed of vectors of N dimensions:

$$\boldsymbol{C} = (\boldsymbol{C}^{(1)}, \boldsymbol{C}^{(2)}, \cdots, \boldsymbol{C}^{(N)})^T \tag{12}$$

Substitute Equation 11 into Equation 12 to get:

$$\boldsymbol{C} = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1M} \\ c_{21} & c_{22} & \cdots & c_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N1} & c_{N2} & \cdots & c_{NM} \end{pmatrix} \tag{13}$$

We use the method of reorganizing the real and imaginary axes of the N-dimensional constellation to design the mother constellation. As shown in Fig. 5, we first rotate the A and B constellations, and project the constellation points vertically to the dimensional axis of the constellation. Then, the k-th dimension axis of the A constellation is taken as the real axis, and the k-th dimension axis of the B constellation is taken as the imaginary axis to form the k-th dimension recombined constellation of the mother constellation. The N reorganized complex constellations correspond to N orthogonal time-frequency resources respectively.

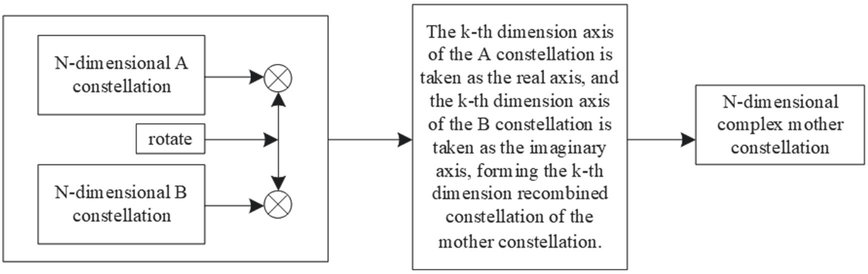

Fig. 5. Mother constellation design process

Next, follow the above design method to design an SCMA system with $J = 6$, $K = 4$, $N = 2$, and $M = 4$.

In the first step, we use two BPSK constellations as A constellation and B constellation (Fig. 6):

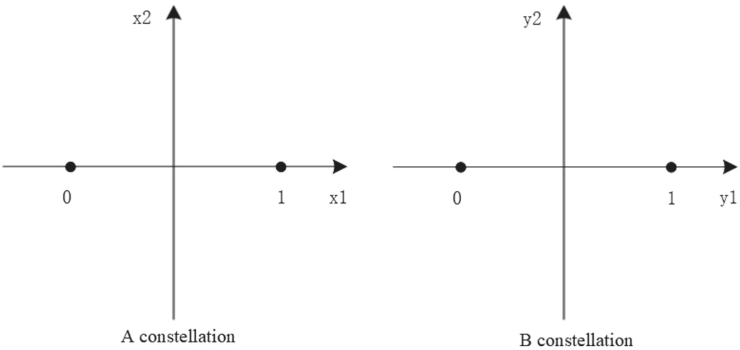

Fig. 6. Original constellation

The second step is to rotate the A and B constellations by the same angle θ (Fig. 7):

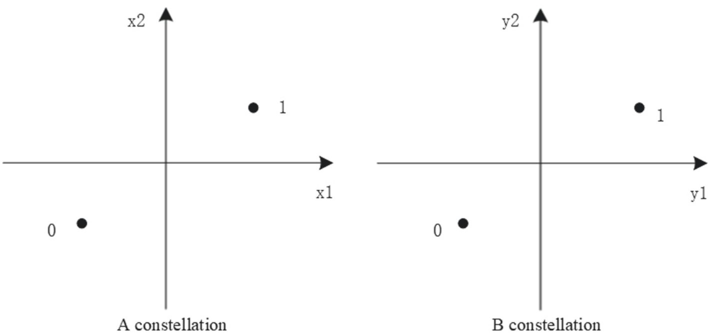

Fig. 7. Constellation after rotation

The third step is to project the rotated constellation points of the A and B constellations on the coordinate axes to obtain new coordinate points. The x1 axis of the A constellation is used as the real axis, and the y1 axis of the B constellation is used as the imaginary axis to form the first of the mother constellation. The x2 axis of the constellation A is used as the real axis, and the y2 axis of the constellation B is used as the imaginary axis to form the second dimension of the mother constellation to generate a reorganized 2-dimensional mother constellation, as shown in Fig. 8.

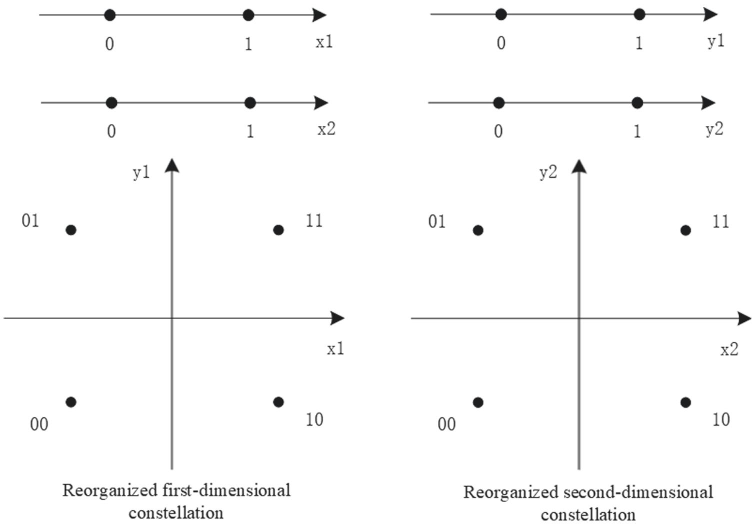

Fig. 8. Reorganized two-dimensional mother constellation

In the second step, the criterion of maximizing the minimum product distance is used to find the optimal rotation angle θ [2]. Define the product distance d_p as:

$$d_{P,i} = \prod_{x_i \neq y_i}^{N} |x_i - y_i| \tag{14}$$

Among them, x_i and y_i are two different points in the reorganized same constellation diagram.

The minimum product distance diagram under different rotation angles obtained by simulation is as follows (Fig. 9):

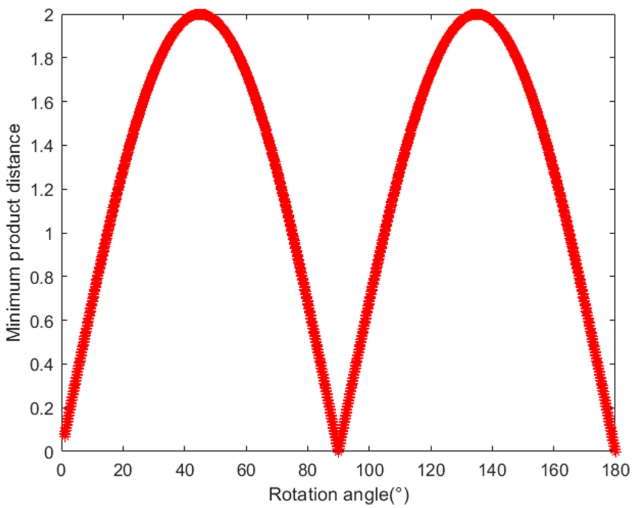

Fig. 9. The minimum product distance of different rotation angles

Through simulation, the optimal rotation angles of the SCMA system with $J = 6$, $K = 4$, $N = 2$, and $M = 4$ are 45° and 135°. When the value of θ is 45°, We choose 45° as the optimal rotation angle.

3.3 Design Constellation Operator

After the mother constellation design is completed, the SCMA codebook design problem can be further simplified as finding the operation matrix that converts the mother constellation into each user constellation. This operation matrix is the constellation operator.

In the SCMA system with $J = 6$, $K = 4$, $N = 2$, $M = 4$, we can get that the number of users superimposed on each orthogonal time-frequency resource is 3 from Eq. 3. Let C^1 and C^2 respectively denote the two mother constellations obtained in the previous section, and suppose that the phase rotation angle of the mother constellation of user j on the k-th orthogonal time-frequency resource is θ_{kj}, then a mother constellation allocation scheme can be designed as follows [4]:

$$\begin{pmatrix} 0 & \boldsymbol{C}^1 \cdot e^{j\theta_{12}} & \boldsymbol{C}^1 \cdot e^{j\theta_{13}} & 0 & \boldsymbol{C}^2 \cdot e^{j\theta_{15}} & 0 \\ \boldsymbol{C}^2 \cdot e^{j\theta_{21}} & 0 & \boldsymbol{C}^2 \cdot e^{j\theta_{23}} & 0 & 0 & \boldsymbol{C}^1 \cdot e^{j\theta_{26}} \\ 0 & \boldsymbol{C}^2 \cdot e^{j\theta_{32}} & 0 & \boldsymbol{C}^1 \cdot e^{j\theta_{34}} & 0 & \boldsymbol{C}^2 \cdot e^{j\theta_{36}} \\ \boldsymbol{C}^1 \cdot e^{j\theta_{41}} & 0 & 0 & \boldsymbol{C}^2 \cdot e^{j\theta_{44}} & \boldsymbol{C}^1 \cdot e^{j\theta_{45}} & 0 \end{pmatrix} \tag{15}$$

Since the rotation angle of the constellation diagram of different users has different effects on the decoding complexity of the MPA algorithm, we use the criterion of maximizing the minimum Euclidean distance to find the optimal rotation angle of the user constellation diagram [3].

Taking orthogonal time-frequency resource 1 as an example, a total of 3 users are superimposed on this orthogonal time-frequency resource, they are user 2, user 3, and user 5. The rotation angle of their constellation diagram is represented by θ_{12}, θ_{13}, and θ_{15} respectively. Assuming that the rotation angle of user 2 is 0°, the contours of the minimum Euclidean distance corresponding to different phase rotation angles obtained through simulation are shown in Fig. 10:

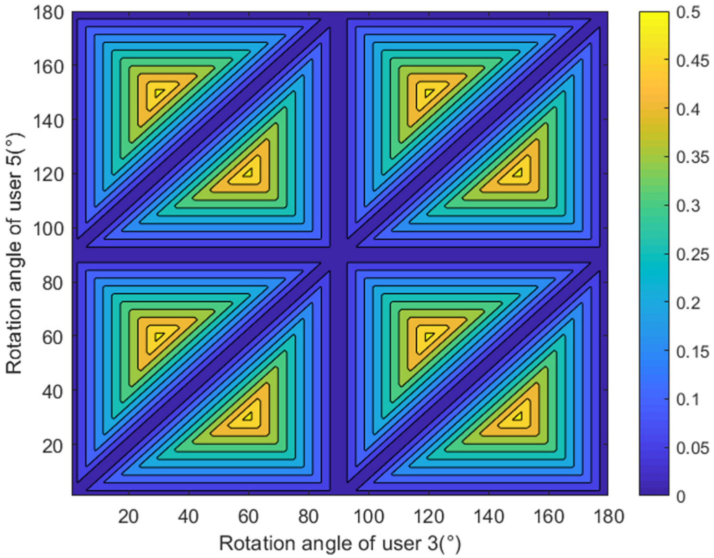

Fig. 10. Contours of the minimum Euclidean distance corresponding to different phase rotation angles (Color figure online)

The blue area in Fig. 10 indicates that the minimum Euclidean distance of the corresponding rotation angle is small, and the yellow area indicates that the minimum Euclidean distance of the corresponding rotation angle is large. It can be seen from the figure that there are eight groups of optimal solutions for rotation angles. The specific optimal rotation angle is obtained by accurate solution as follows:

- $$\theta_{12} = 0^{\circ},\ \theta_{13} = 30^{\circ},\ \theta_{15} = 60^{\circ}$$
- $$\theta_{12} = 0^{\circ},\ \theta_{13} = 30^{\circ},\ \theta_{15} = 150^{\circ}$$
- $$\theta_{12} = 0^{\circ},\ \theta_{13} = 60^{\circ},\ \theta_{15} = 30^{\circ}$$

- $$\theta_{12} = 0°,\ \theta_{13} = 60°,\ \theta_{15} = 120°$$
- $$\theta_{12} = 0°,\ \theta_{13} = 120°,\ \theta_{15} = 60°$$
- $$\theta_{12} = 0°,\ \theta_{13} = 120°,\ \theta_{15} = 150°$$
- $$\theta_{12} = 0°,\ \theta_{13} = 150°,\ \theta_{15} = 30°$$
- $$\theta_{12} = 0°,\ \theta_{13} = 150°,\ \theta_{15} = 120°$$

4 Performance Simulation

A total of eight schemes of user constellation rotation angles are obtained by maximizing the minimum Euclidean distance criterion. The following firstly compares the performance of these eight schemes. From the analysis in Fig. 10, we can use symmetry to select three asymmetric solutions, and reduce the eight solutions to three as follows:

1. $$\theta_{12} = 0°,\ \theta_{13} = 30°,\ \theta_{15} = 60°$$
2. $$\theta_{12} = 0°,\ \theta_{13} = 30°,\ \theta_{15} = 150°$$
3. $$\theta_{12} = 0°,\ \theta_{13} = 60°,\ \theta_{15} = 120°$$

The codebook calculated from the above three phase rotation angles is simulated for bit error rate performance and the result is shown in Fig. 11. The specific simulation parameters are shown in Table 1:

Table 1. Simulation parameters

Parameter	Value
M	4
J	6
K	4
d_f	3
λ	150%
Number of iterations	6
Codebook 1 phase rotation angle	0°, 30°, 60°
Codebook 2 phase rotation angle	0°, 30°, 150°
Codebook 3 phase rotation angle	0°, 60°, 120°
Channel model	Rayleigh channel

It can be seen from Fig. 11 that when the signal-to-noise ratio is low, the simulation curves under the three phase rotation angles are not much different, but with the increase of the signal-to-noise ratio, the three schemes have some subtle differences. When the signal-to-noise ratio reaches 8 dB, the error rate of codebook 1 and codebook 3 is almost the same, and both are lower than codebook 2. When the signal-to-noise ratio reaches 10 dB, the bit error rate decreases sequentially from codebook 1 to codebook 2 to codebook 3.

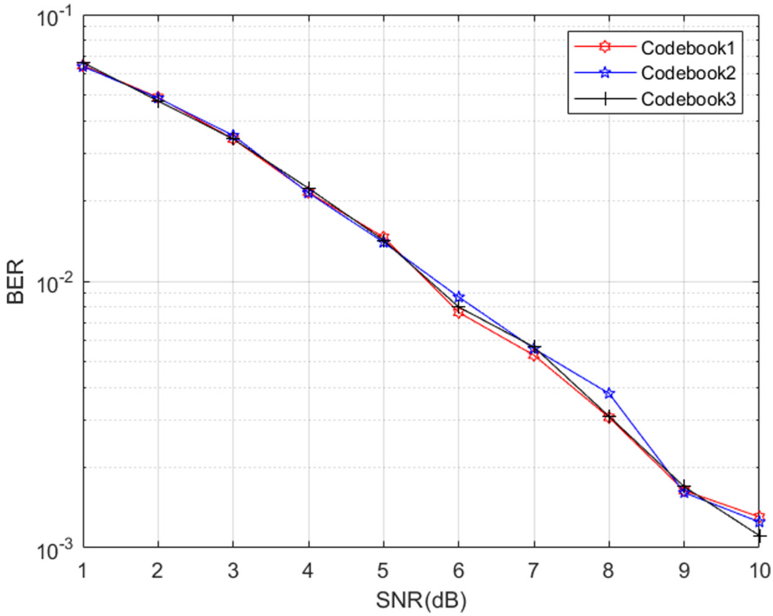

Fig. 11. BER comparison chart for different phase rotation angles

Compare and analyze the codebook design proposed in this section with the codebook disclosed by Huawei in the 5G competition. We use the codebook 1 above for comparison, and the simulation results are shown in Fig. 12:

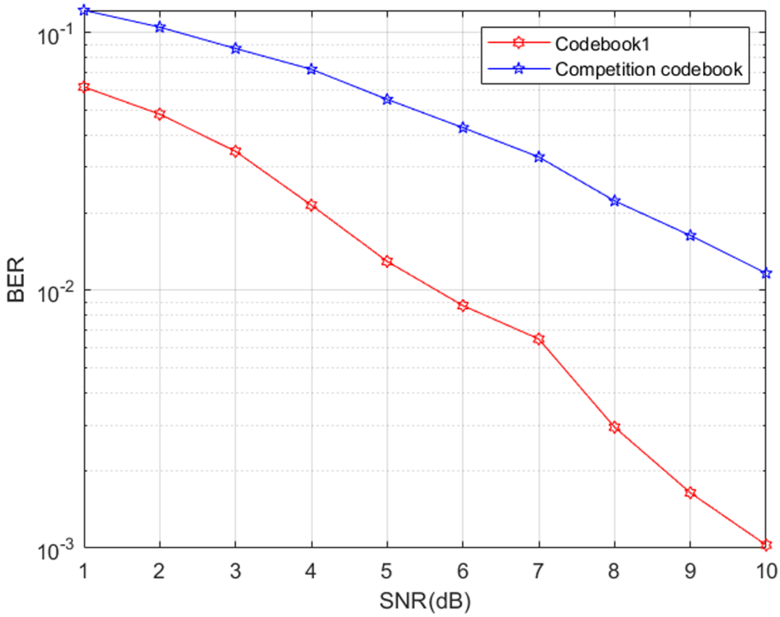

Fig. 12. BER comparison chart of different design schemes

It can be seen from Fig. 12 that as the signal-to-noise ratio increases, the bit error rate of codebook 1 will be significantly reduced compared to the competition codebook, and the performance will be significantly improved.

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