An Anti-jamming Game When None Player Knows Rival's Channel Gain

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Abstract. We consider a user's communication with a receiver in presence of a jammer, in the most competitive situation for user and jammer when they do not have access to complete information on channel gains of each other although they could have access to exact information on their own channel gains. The problem is modeled as a Bayesian power control game between user and jammer as players. Incomplete information is modeled as statistical data over possible channel gains (also referred as channel states). Since channel gain is a function on the distance to the receiver, this also covers scenarios where the user and jammer could know its own location via global positioning system (GPS), but none of them know the exact location of the other. A novel approach is suggested to derive equilibrium of such problems in closed form for two communication metrics: signal-to-interference-plus-noise ratio (SINR) metric, reflecting regular data transmission, and latency metric, reflecting emergency data transmission. In particular, it is shown that the user's equilibrium strategies corresponding to the latency metric is more sensitive to the a priori statistical information, as compared to the SINR metric. This reflects an advantage of implementing latency metric in case of availability of exact information on network parameters, and an advantage of implementing SINR metric in case of lack of such its availability.

Keywords: Jamming · SINR · Latency · Bayesian equilibrium

1 Introduction

The shared and open-access nature of the wireless medium renders wireless networks vulnerable to hostile interference or jamming. To design anti-jamming strategy in such a background, non-cooperative game theory has been widely employed due to in such problems each agent (say, a user who aims to communicate with the receiver and a jammer who aims to obstruct the user's communication via jamming) has its own objective [5,9,12,15,16,18–20]. While the above cited works about anti-jamming problems assume that each player, user and jammer, has complete information about own (network and communication) parameters as well as about such parameters of the rival, there are works where

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none of the players, user and jammer, could have access to the exact information about some own network parameters, or only one of them might have access to the exact information about own parameters [2,3,6,8,10,11,13,14,17]. For example, the players might not know exact values of some parameters involved in payoffs [2,3,6,10,11,14,17], the player might not know whether the other player is selfish or malicious [13], the player might not know whether the other player implements behavior's strategies or not [8]. Meanwhile in the real-world most competitive scenarios, the user as well as the jammer might have exact information about itself but do not have complete information about the rival. Such a scenario is the most competitive since the advantage that each player could gain via access to exact information about itself, could be used against the player by the rival due to lack of information for the player about the rival. To the best knowledge of the authors such anti-jamming problems have not been studied in literature.

Motivation of the research: All the above cited papers have been addressed to anti-jamming scenarios where either only one of the players (user or jammer) has access to exact information or none of them has such an access. Meanwhile, in the real-world scenarios related to anti-jamming problems under incomplete information on channel gains, both players, user and jammer, might have access to exact information about itself but do not have such access about the other. It is motivated, for example, by the fact that the channel gain is a function on the distance to the receiver, and each player, user and jammer, might have complete information about its own location via GPS, but none of them might not know another player's location. To model access of the player to exact information about its location, a player's type has to be assigned per location. That is why such a problem leads to an increase in its dimension and complexity in contrast to those studied in literature. Such an increase in complexity calls for development of a new approach to design anti-jamming strategies and methods to verify whether such a designed anti-jamming strategy could maintain stability in communication.

The main contributions of this paper are as follows: (i) A Bayesian game approach is employed to model a power control anti-jamming problem for different access of the players (user and jammer) to information (exact or only statistical) about channel states. (ii) A novel approach is developed allowing to find in closed form equilibrium strategies with such incomplete information for two communication metrics: SINR metric, reflecting regular data transmission, and latency metric, reflecting emergency data transmission. (iii) Stability of designed anti-jamming strategies are verified via proving uniqueness of equilibrium. (iv) An advantage of implementing latency metric in case of availability of exact information on network parameters as well as an advantage of implementing SINR metric in case of a lack of its availability are revealed via showing that user's equilibrium strategies corresponding to the latency metric is more sensitive to the a priori statistic information, as compared to the SINR metric.

2 The Communication Model

Let us consider a single carrier transmission scheme with two agents: a *user* and an *adversary*. The adversary is a jammer, who intends to degrade the user's communication by generating interference. User's resource is its transmission power P, $P \in [0, \overline{P}]$, and jammer's resource is its jamming power J, $J \in [0, \overline{J}]$. In contrast to [7,19], where communication channel and jammer's channel of each player are fixed and known to both players, we assume that: (a) communication channel can be in one of K states corresponding to fading gains $h_1, \ldots, h_K$, with probabilities $\alpha_1, \ldots, \alpha_K$, respectively, and (b) jammer's channel can be in one of M states corresponding to fading gains $g_1, \ldots, g_M$, with probabilities $\beta_1, \ldots, \beta_M$, respectively. Each of the players knows its own channel state, meanwhile regarding rival's channel state it only knows set of feasible states and corresponding a priori probabilities. Without loss of generality, let us assume that the gains of the user's channel and jammer's channel are arranged in increasing order, i.e.,

$$h_0 \triangleq 0 < h_1 < h_2 < \ldots < h_K < h_{K+1} \triangleq \infty, \tag{1}$$

$$g_0 \triangleq 0 < g_1 < g_2 \ldots < g_M < g_{M+1} \triangleq \infty. \tag{2}$$

In the following, by *type-k user* ($k \in \mathcal{K} \triangleq \{1, \ldots, K\}$) we refer to a user if its channel state k occurs, and let P_k be its strategy, and $\boldsymbol{P} \triangleq (P_1, \ldots, P_K)$.[1] By *type-m jammer* ($m \in \mathcal{M} \triangleq \{1, \ldots, M\}$) we refer to a jammer if its channel state m occurs, and let J_m be its strategy, and $\boldsymbol{J} \triangleq (J_1, \ldots, J_M)$.

In this section we consider SINR as communication metric. Then, the type-k user's payoff reflects trade-off between its SINR and transmission cost and given as follows:

$$v_{U,k}(P_k, \boldsymbol{J}) \triangleq \sum_{m \in \mathcal{M}} \beta_m \frac{h_k P_k}{N + g_m J_m} - C_P P_k = (h_k T(\boldsymbol{J}) - C_P)\, P_k, \tag{3}$$

where N is the background noise variance and C_P is user's transmission cost per power unit and

$$T(\boldsymbol{J}) \triangleq \sum_{m \in \mathcal{M}} \beta_m / (N + g_m J_m). \tag{4}$$

The type-m jammer's payoff is given as follows:

$$v_{J,m}(\boldsymbol{P}, J_m) \triangleq -\sum_{k \in \mathcal{K}} \alpha_k \frac{h_k P_k}{N + g_m J_m} - C_J J_m = \frac{H(\boldsymbol{P})}{N + g_m J_m} - C_J J_m, \tag{5}$$

where C_J is jamming cost per power unit and

$$H(\boldsymbol{P}) \triangleq \sum_{k \in \mathcal{K}} \alpha_k h_k P_k. \tag{6}$$

[1] We use bold font in vector's notations.

Intuition behind $T(\boldsymbol{J})$ is that it is expected inverse total noise when all jammer's types apply strategies $\boldsymbol{J}$. Meanwhile $H(\boldsymbol{P})$ is that it is the expected power generated by all user's types implementing strategies $\boldsymbol{P}$.

Each of user's and jammer's types want to maximize its payoff. Thus, we look for Bayesian equilibrium [4]. In other words, we look for such vector of user's types strategies $\boldsymbol{P} = (P_1, \dots, P_K)$ and vector of jammer's types strategies $\boldsymbol{J} = (J_1, \dots, J_K)$ that each of these strategies is the best response to the others, i.e.,

$$P_k = \mathrm{BR}_{U,k}(\boldsymbol{J}) \triangleq \mathrm{argmax}\{v_{U,k}(\tilde{P}_k, \boldsymbol{J}) : \tilde{P}_k \in [0, \overline{P}]\},\ k \in \mathcal{K}, \tag{7}$$

$$J_m = \mathrm{BR}_{J,m}(\boldsymbol{P}) \triangleq \mathrm{argmax}\{v_{J,m}(\boldsymbol{P}, \tilde{J}_m) : \tilde{J}_m \in [0, \overline{J}]\},\ m \in \mathcal{M}. \tag{8}$$

These $K + M$ real-value best response equations can be rewritten as two vector-value equations:

$$\boldsymbol{P} = \mathbf{BR}_U(\boldsymbol{J}) \text{ and } \boldsymbol{J} = \mathbf{BR}_J(\boldsymbol{P}), \tag{9}$$

where

$$\mathbf{BR}_U(\boldsymbol{J}) \triangleq (\mathrm{BR}_{U,1}(\boldsymbol{J}), \dots, \mathrm{BR}_{U,K}(\boldsymbol{J})), \tag{10}$$

$$\mathbf{BR}_J(\boldsymbol{P}) \triangleq (\mathrm{BR}_{J,1}(\boldsymbol{P}), \dots, \mathrm{BR}_{J,M}(\boldsymbol{P})). \tag{11}$$

Let us denote by Γ this game.

Proposition 1. *In game Γ there is at least one equilibrium.*

Please find the proof in Appendix.

3 Equilibrium

In this section solving best response equation (9) we establish the condition when equilibrium is unique or multiple equilibria arise.

3.1 The Players' Best Responses

In the following proposition we provide the players' best responses in closed form.

Proposition 2. *For a fixed $\boldsymbol{J}$, type-k user's best response is given as follows*

$$BR_{U,k}(\boldsymbol{J}) \begin{cases} = 0, & T(\boldsymbol{J}) < C_P/h_k \\ \in [0, \overline{P}], & T(\boldsymbol{J}) = C_P/h_k, \\ = \overline{P} & T(\boldsymbol{J}) > C_P/h_k. \end{cases} \tag{12}$$

For a fixed $\boldsymbol{P}$, type-m jammer's best response is given as follows

$$BR_{J,m}(\boldsymbol{P}) = \begin{cases} 0, & H(\boldsymbol{P}) \le A_m, \\ \sqrt{\dfrac{H(\boldsymbol{P})}{C_J g_m}} - \dfrac{N}{g_m}, & A_m < H(\boldsymbol{P}) < B_m, \\ \overline{J} & B_m \le H(\boldsymbol{P}), \end{cases} \tag{13}$$

where

$$A_m \triangleq C_J N^2/g_m \text{ and } B_m \triangleq C_J(N + g_m\overline{J})^2/g_m. \quad (14)$$

Please find the proof in Appendix.

It is clear that, by (14),

$$A_m < B_m \quad (15)$$

and

$$0 < \sqrt{\frac{H(\boldsymbol{P})}{C_J g_m}} - \frac{N}{g_m} < \overline{J} \text{ for } A_m < H(\boldsymbol{P}) < B_m. \quad (16)$$

Thus, (13) defines jammer's types best responses correctly.

3.2 Structure of User Types' Equilibrium Strategies

In the following proposition we derive in the parameterized form, given by (18) below, all plausible candidates to be user types' equilibrium strategies. In particular, it means that there are no user types' equilibrium strategies which are not given by (18) for a specific value of its parameter x. Intuition behind this parameter x in such form is that it allows to identify threshold user's type which separates user's types implementing the maximal and minimal (zero) transmission power as well as to specify the transmission power implemented by this threshold user's type. Finally, we derive the K-dimension vector equation to obtain the value of this parameter. Such an approach (method) to design equilibrium and investigate its uniqueness via parameterizing all plausible candidates to be equilibrium we call a parameterization method.

Proposition 3. *$(\boldsymbol{P}, \boldsymbol{J})$ is an equilibrium in game Γ if and only if there is an $x \in [0, K\overline{P}]$ such that*

$$\boldsymbol{P} = \boldsymbol{P}(x) \triangleq (P_1(x), \dots P_K(x)) \text{ and } \boldsymbol{J} = \mathbf{BR}_J(\boldsymbol{P}(x)), \quad (17)$$

where for $x \in [0, K\overline{P}]$ vector $(P_1(x), \dots P_K(x))$ is given as follows:

$$P_k(x) \triangleq \begin{cases} 0, & k < \lfloor x/\overline{P} \rfloor + 1, \\ (\lfloor x/\overline{P} \rfloor + 1)\overline{P} - x, & k = \lfloor x/\overline{P} \rfloor + 1, \text{ for } k \in \mathcal{K}, \\ \overline{P}, & k > \lfloor x/\overline{P} \rfloor + 1 \end{cases} \quad (18)$$

where $\lfloor \xi \rfloor$ is the greatest integer less than or equal to ξ. Moreover, x is the root in $[0, K\overline{P}]$ of the following K-dimension vector equation:

$$P_k(x) = \begin{cases} 0, & \mathcal{T}(x) < C_P/h_k, \\ \in [0, \overline{P}], & \mathcal{T}(x) = C_P/h_k, \; k \in \mathcal{K}, \\ \overline{P}, & \mathcal{T}(x) > C_P/h_k, \end{cases} \quad (19)$$

where

$$\mathcal{T}(x) = \sum_{m \in I_0(x)} \frac{\beta_m}{N} + \sum_{m \in I(x)} \frac{\beta_m}{\sqrt{g_m}} \sqrt{\frac{C_J}{\mathcal{H}(x)}} + \sum_{m \in \overline{I}(x)} \frac{\beta_m}{N + g_m \overline{J}} \tag{20}$$

with

$$\mathcal{H}(x) \triangleq H(\boldsymbol{P}(x)), \tag{21}$$

$$I_0(x) \triangleq \{m \in \mathcal{M} : a_m \le x\}, \tag{22}$$

$$I(x) \triangleq \{m \in \mathcal{M} : b_m < x < a_m\}, \tag{23}$$

$$\overline{I}(x) \triangleq \{m \in \mathcal{M} : x \le b_m\}, \tag{24}$$

where

$$a_m \triangleq \begin{cases} K\overline{P}, & \mathcal{H}(0) \le A_m \\ \mathcal{H}^{-1}(A_m), & \mathcal{H}(0) > A_m \end{cases} \text{ and } b_m \triangleq \begin{cases} 0, & \mathcal{H}(0) \le B_m, \\ \mathcal{H}^{-1}(B_m), & \mathcal{H}(0) > B_m. \end{cases} \tag{25}$$

Please find the proof in Appendix.

Let us explain intuition behind functions $\mathcal{H}(x)$ and $\mathcal{T}(x)$ implemented in Proposition 3. By (6) and (21), $\mathcal{H}(x)$ is the expected power generated by all user's types implementing strategies $\boldsymbol{P}(x)$, and, by (6), (18) and (21),

$$\mathcal{H}(x) \text{ is strictly decreasing from } \mathcal{H}(0) = \overline{P} \sum_{k \in \mathcal{K}} \alpha_k h_k \text{ to } \mathcal{H}(K\overline{P}) = 0. \tag{26}$$

Thus, by (26), a_m and b_m are correctly defined by (25).

By (4), (13) and (20)–(25), we have that $T(\mathbf{BR}_J(\boldsymbol{P}(x))) = \mathcal{T}(x)$. Thus, $\mathcal{T}(x)$ is the expected inverse total noise when the jammer's types apply best response to user's types strategies $\boldsymbol{P}(x)$. Moreover, $\mathcal{T}(x)$ is non-decreasing continuous function. Since $\mathcal{T}(x)$ might be constant within some sub-intervals it might lead to multiple solutions of K-dimension vector equation (19) if and only if there is a k such that $x = k\overline{P}$ is solution of (19). In this case multiple equilibrium strategies arise for the only type-k user.

3.3 Equilibrium in Closed Form

By Proposition 1, in game Γ, there is at least one equilibrium. In the following theorem we find equilibrium as well as establish condition when equilibrium is unique. Moreover, we prove that the jammer types' equilibrium strategies are always unique while multiple user types' equilibrium strategies might arise for the only user's type.

Theorem 1. *In game Γ let $(\boldsymbol{P}, \boldsymbol{J})$ be an equilibrium. Then $\boldsymbol{P} = \boldsymbol{P}(x)$ and $\boldsymbol{J} = \mathbf{BR}_J(\boldsymbol{P}(x))$ with $x \in [0, K\overline{P}]$, where $\boldsymbol{P}(x)$ given by (18). Moreover, such x is defined as follows:*

(a) If

$$C_P/h_1 < \mathcal{T}(0) \tag{27}$$

then $x = 0$;

(b) If

$$\mathcal{T}(K\overline{P}) < C_P/h_K \tag{28}$$

then $x = K\overline{P}$;

(c) If (27) and (28) do not hold then there is an unique $\tilde{k}$ *such that either (29) or (30) given below hold and*

(c-i) if

$$C_P/h_{\tilde{k}} < \mathcal{T}(\tilde{k}\overline{P}) < C_P/h_{\tilde{k}-1} \tag{29}$$

then $x = \tilde{k}\overline{P}$;

(c-ii) if

$$\mathcal{T}(\tilde{k}\overline{P}) \le C_P/h_{\tilde{k}} \le \mathcal{T}((\tilde{k}+1)\overline{P}) \tag{30}$$

then x *is the root in* $[\tilde{k}\overline{P}, (\tilde{k}+1)\overline{P}]$ *of the equation*

$$\mathcal{T}(x) = C_P/h_{\tilde{k}}. \tag{31}$$

This x *can be found via the bisection method.*

Please find the proof in Appendix.

Finally note that $\mathcal{T}(x)$ is non-decreasing continuous function. This and Theorem 1 imply that if x is given by this theorem and there is no $[a, b]$ such that $x \in [a, b]$ and $\mathcal{T}(t)$ is constant in $[a, b]$ then user's types equilibrium strategies are unique. Otherwise, $\boldsymbol{P}(t)$ with $t \in [a, b]$ is also equilibrium.

4 Latency Metric

For further illustration of the suggested parameterization method, let us consider latency communication metric modeled by the inverse SINR [9]. It might be interesting to note that SINR and negative latency might be included into a uniform scale of user's communication utilities via α-fairness utility with $\alpha = 0$ and $\alpha = 2$, respectively [1]. The user wishes to find the trade-off between a reduction in latency of the signal at the receiver and transmission power cost. Then, the k-type user's payoff is given as follows:

$$v_{U,k}^L(P_k, \boldsymbol{J}) \triangleq -\sum_{m=1}^{M} \beta_m \frac{N + g_m J_m}{h_k P_k} - C_P P_k = -\frac{W(\boldsymbol{J})}{h_k P_k} - C_P P_k, \tag{32}$$

where

$$W(\boldsymbol{J}) \triangleq N + \sum_{m \in \mathcal{M}} \beta_m g_m J_m. \tag{33}$$

The type-m jammer's payoff is given as follows

$$v^L_{J,m}(\boldsymbol{P}, J_m) \triangleq \sum_{k=1}^{K} \alpha_k \frac{N + g_m J_m}{h_k P_k} - C_J J_m = NR(\boldsymbol{P}) + (g_m R(\boldsymbol{P}) - C_J) J_m, \tag{34}$$

where

$$R(\boldsymbol{P}) \triangleq \sum_{k \in \mathcal{K}} \alpha_k / (h_k P_k). \tag{35}$$

Intuition behind $W(\boldsymbol{J})$ is that it is the expected total noise when the jammer's types apply jamming strategies $\boldsymbol{J}$. Meanwhile, $R(\boldsymbol{P})$ is that it is the expected inverse power generated by all user's types implementing strategies $\boldsymbol{P}$.

Let us denote by Γ^L such Bayesian game with user's and jammer's payoffs given by (32) and (34), respectively.

Proposition 4. *In game Γ^L there is at least one equilibrium.*

Please find the proof in Appendix.

4.1 The Best Responses in Game $\boldsymbol{\Gamma^L}$

In this section we provide the players' best responses in closed form.

Proposition 5. *For a fixed $\boldsymbol{J}$, the type-k user best responses is*

$$BR^L_{U,k}(\boldsymbol{J}) = \min\{\sqrt{W(\boldsymbol{J})/(h_k C_P)}, \overline{P}\}. \tag{36}$$

For a fixed $\boldsymbol{P}$, the type-m jammer best response is:

$$BR^L_{J,m}(\boldsymbol{P}) = \begin{cases} 0, & R(\boldsymbol{P}) < C_J/g_m, \\ \in [0, \overline{J}], & R(\boldsymbol{P}) = C_J/g_m, \\ \overline{J}, & R(\boldsymbol{P}) > C_J/g_m. \end{cases} \tag{37}$$

Please find the proof in Appendix.

4.2 Structure of Jammer Types' Equilibrium Strategies

In the following proposition we derive in the parameterized form, given by (39) below, all plausible candidates to be jammer types' equilibrium strategies. In particular, it means that there are no jammer types' equilibrium strategies which are not given by (39) for a specific value of its parameter y. Intuition behind this parameter y in such form is that it allows to identify threshold jammer's type which separates jammer's types implementing the maximal and minimal (zero) jamming power as well as to specify the jamming power implemented by this threshold jammer's type. Finally, we derive M-dimension vector equation to obtain the value of this parameter.

Proposition 6. *$(\boldsymbol{P}, \boldsymbol{J})$ is an equilibrium in game Γ^L if and only if there is a $y \in [0, M\overline{J}]$ such that*

$$\boldsymbol{J} = \boldsymbol{J}(y) \triangleq (J_1(y), \dots J_M(y)) \text{ and } \boldsymbol{P} = \mathbf{BR}_U^L(\boldsymbol{J}(y)), \tag{38}$$

where for a fixed $y \in [0, M\overline{J}]$ vector $(J_1(y), \dots J_M(y))$ is given as follows:

$$J_m(y) \triangleq \begin{cases} 0, & m < \lfloor y/\overline{J} \rfloor + 1, \\ (\lfloor y/\overline{J} \rfloor \overline{J} + 1)\overline{J} - y, & m = \lfloor y/\overline{J} \rfloor + 1, \\ \overline{J}, & m > \lfloor y/\overline{J} \rfloor + 1 \end{cases} \text{ for } m \in \mathcal{M}. \tag{39}$$

Moreover, such y is solution in $[0, M\overline{J}]$ of the following M-dimension vector equation:

$$J_m(y) = \begin{cases} 0, & \mathcal{R}(y) < C_J/g_m, \\ \in [0, \overline{J}], & \mathcal{R}(y) = C_J/g_m, m \in \mathcal{M}, \\ \overline{P}, & \mathcal{R}(y) > C_J/g_m, \end{cases} \tag{40}$$

where

$$\mathcal{R}(y) \triangleq \sum_{k \in I^L(y)} \alpha_k \sqrt{\frac{h_k C_P}{W(\boldsymbol{J}(y))}} + \sum_{k \in \overline{I}^L(y)} \frac{\alpha_k}{h_k \overline{P}} \tag{41}$$

with

$$I^L(y) \triangleq \left\{ k \in \mathcal{K} : W(\boldsymbol{J}(y)) \le h_k C_P \overline{P}^2 \right\}, \tag{42}$$

$$\overline{I}^L(y) \triangleq \left\{ k \in \mathcal{K} : h_k C_P \overline{P}^2 < W(\boldsymbol{J}(y)) \right\}. \tag{43}$$

Please find the proof in Appendix.

Let us explain intuition behind functions $W(\boldsymbol{J}(y))$ and $\mathcal{R}(y)$ implemented in Proposition 6. By (33), $W(\boldsymbol{J}(y))$ is total noise when the jammer's types apply jamming strategies $\boldsymbol{J}(y)$. By (35), (36) and (41)–(43), we have that $R(\mathbf{BR}_U(\boldsymbol{J}(y))) = \mathcal{R}(y)$. Thus, $\mathcal{R}(y)$ the expected inverse power generated by all user's types when the user's types apply best response to jammer's types strategies $\boldsymbol{J}(y)$.

4.3 Equilibrium in Closed Form

By Proposition 4, in game Γ^L there is at least one equilibrium. In the following theorem we find equilibrium as well as establish its uniqueness.

Theorem 2. *Let $(\boldsymbol{P}^L, \boldsymbol{J}^L)$ be an equilibrium in game Γ^L. Then $\boldsymbol{J}^L = \boldsymbol{J}(y)$ and $\boldsymbol{P}^L = \mathbf{BR}_U(\boldsymbol{J}^L(y))$ with $y \in [0, M\overline{J}]$, where $\boldsymbol{J}(y)$ given by (39). Moreover, such y is defined as follows:*

(a) If

$$C_J/g_1 < \mathcal{R}(0) \tag{44}$$

then $y = 0$;

(b) If

$$\mathcal{R}(M\overline{J}) < C_J/g_M \tag{45}$$

then $y = M\overline{J}$;

(c) If (44) and (45) do not hold then there is an unique $\tilde{m}$ *such that either (46) or (47) given below hold and*

(c-i) if

$$C_J/g_{\tilde{m}} < \mathcal{R}(\tilde{m}\overline{J}) < C_J/g_{\tilde{m}-1} \tag{46}$$

then $y = \tilde{m}\overline{J}$*;*

(c-ii) if

$$\mathcal{R}(\tilde{m}\overline{J}) \le C_J/h_{\tilde{m}} \le \mathcal{R}((\tilde{m}+1)\overline{J}) \tag{47}$$

then y is the root in $[\tilde{m}\overline{J}, (\tilde{m}+1)\overline{J}]$ *of the equation*

$$\mathcal{R}(y) = C_J/g_{\tilde{m}}. \tag{48}$$

This y can be found via the bisection method.

Please find the proof in Appendix.

Since $\mathcal{R}(y)$ is either strictly increasing or constant, equilibrium is unique except the only case if $\mathcal{R}(y)$ is constant in $[0, M\overline{J}]$.

5 Discussion of the Results

The complexity to design an equilibrium of game Γ, by Theorem 1, is $\kappa = \log_2(\overline{P}/\epsilon)$, where ϵ is the tolerance of the bisection method. The complexity to design an equilibrium of game Γ^L is similar, and, by Theorem 2, it equals to $\kappa^L = \log_2(\overline{J}/\epsilon)$. Thus, in game Γ, the impact of the maximal transmission power on the complexity to design an equilibrium is larger compare with the impact of the maximal jamming power. While, in game Γ^L, we can observe the inverse influence. To illustrate the obtained equilibrium strategies in Theorem 1 and Theorem 2, let us consider the network with background noise $N = 0.2$, and transmission and jamming costs $C_P = C_J = 1$. We compare the equilibrium strategies for scenario where the channels can be in two states: good and bad, i.e., $K = M = 2$, and let fading gains are $(g_1, g_2) = (0.3, 1) = (h_1, h_2)$. Also let the total powers resource of the players be $\overline{P} = \overline{J} = 5$, and the a priori probabilities are parameterized by probability for the first state to occur, i.e., $(\alpha_1, \alpha_2) = (\alpha, 1-\alpha)$ and $(\beta_1, \beta_2) = (\beta, 1-\beta)$. Figure 1 illustrates that

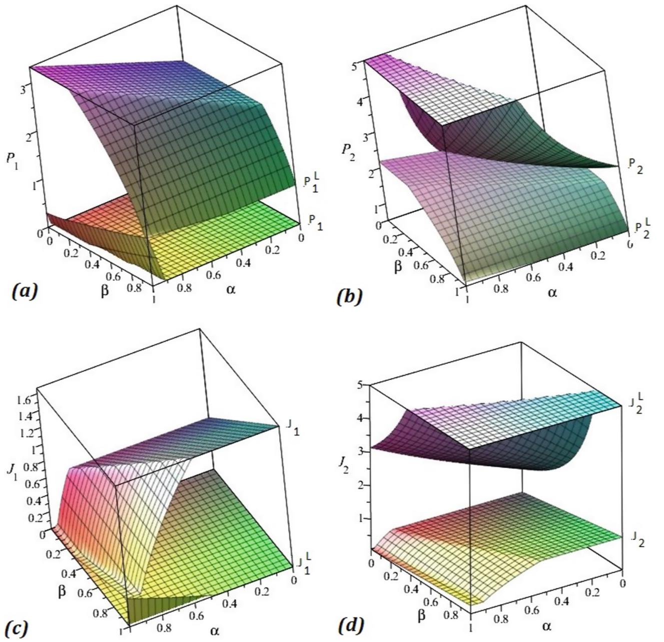

Fig. 1. (a) P_1 and P_1^L, (b) P_2 and P_2^L, (c) J_1 and J_1^L and (d) J_2 and J_2^L.

the user with latency as communication utility is more sensitive to the a priori probabilities compare with the user with latency as communication utility. This is reflected by an increase in zone where the corresponding strategies are constant, and, thus do not react on small variation of the a priori probabilities. The jammer's sensitiveness on the a priori probabilities is higher for SINR as communication utility. Thus, SINR metric assumes less information requirements for the user, and, so, communication with such metric might be easier to maintain for the user. An increase in probability that the user's channel is in good state leads to a decrease in applied transmission power for all user's types in both metrics. An increase in probability that the jammer's channel is in good state leads to a decrease in applied jamming power for all jammer's types in both metrics. The user with SINR metric applies higher transmission power if the user's channel is in good state compare with bad state. Moreover, if the user's channel is in bad state the user might prefer not to communicate with the receiver (Fig. 1(a), Fig. 1(b) and (12)). The user with latency metric, independent on in which state the channel is, keeps on communicating with the receiver. Moreover, when the channel is in good state, the user can transmit at lower power, while the user need to increase their power when the channel worsens to keep on communicating (Fig. 1(a), Fig. 1(b) and (36)).

6 Conclusions

The optimal power control anti-jamming strategy has been designed in a game-theoretical framework for the most competitive scenario where the user and the jammer could have access to exact information about own channel state and only statistical information about rival's channel states. A novel approach, called parameterization method, has been developed to design equilibrium strategies in such problems with incomplete information. Based on this parameterization method, equilibrium strategies have been found in closed form and compared for two communication metrics: SINR and latency metrics reflecting regular and emergency data transmission, respectively. In particular, it has been established that user's equilibrium strategies corresponding to latency metric is more sensitive to varying a priori probabilities, as compared to the SINR. This reflects an advantage of implementing latency metric in case of availability of exact information on network parameters, and an advantage of implementing SINR metric in case of lack of such its availability.

Appendix

Proof of Proposition 1: Since $v_{J,m}(\boldsymbol{P}, J_m)$ is concave in J_m, and $v_{U,k}(P_k, \boldsymbol{J})$ is linear in P_k, and set of feasible strategies of each player is compact, the result follows from Nash theorem [4]. ■

Proof of Proposition 2: By (3), $v_{U,k}(P_k, \boldsymbol{J})$ is linear in P_k. Then, (12) follows from (3) and (7). By (5), we have that

$$\frac{\partial v_{J,m}(\boldsymbol{P}, J_m)}{\partial J_m} = \frac{g_m H(\boldsymbol{P})}{(N + g_m J_m)^2} - C_J. \tag{49}$$

Since $v_{J,m}(\boldsymbol{P}, J_m)$ is concave in J_m, by (8) and (49), we have that

$$\mathrm{BR}_{J,m}(\boldsymbol{P}) = \begin{cases} 0, & \dfrac{\partial v_{J,m}(\boldsymbol{P}, 0)}{\partial J_m} \le 0, \\ \dfrac{\partial v_{J,m}(\boldsymbol{P}, J_m)}{\partial J_m} = 0, & \dfrac{\partial v_{J,m}(\boldsymbol{P}, \overline{J})}{\partial J_m} < 0 < \dfrac{\partial v_{J,m}(\boldsymbol{P}, 0)}{\partial J_m}, \\ \overline{J}, & 0 \le \dfrac{\partial v_{J,m}(\boldsymbol{P}, \overline{J})}{\partial J_m}. \end{cases} \tag{50}$$

Substituting (49) into (50) and taking into account (6) and (14) imply (13). ■

Proof of Proposition 3: By (1) and (12), for a fixed $\boldsymbol{J}$ there exists an unique $\tilde{k}(\boldsymbol{J}) \in \{0, 1, \dots, K, K+1\}$ such that

$$\mathrm{BR}_{U,k}(\boldsymbol{J}) = \begin{cases} 0, & k < \tilde{k}(\boldsymbol{J}), \\ \in [0, \overline{P}], & k = \tilde{k}(\boldsymbol{J}), \\ \overline{P} & k > \tilde{k}(\boldsymbol{J}) \end{cases} \text{ or } \mathrm{BR}_{U,k}(\boldsymbol{J}) = \begin{cases} 0, & k < \tilde{k}(\boldsymbol{J}), \\ \overline{P} & k \ge \tilde{k}(\boldsymbol{J}). \end{cases} \tag{51}$$

Thus, user's types equilibrium strategies have to have the form $\boldsymbol{P}(x)$ given by (18) with an $x \in [\tilde{k}(\boldsymbol{J})\overline{P}, (\tilde{k}(\boldsymbol{J})+1)\overline{P})$. To derive an equation to find such x, note that, by (9), $\boldsymbol{P}(x)$ is user's types equilibrium strategies if and only if

$$\boldsymbol{P}(x) = \mathbf{BR}_U(\mathbf{BR}_J(\boldsymbol{P}(x))). \tag{52}$$

By (4), (13) and (20), we have that

$$T\left(\mathbf{BR}_J(\boldsymbol{P}(x))\right) = \mathcal{T}(x), \tag{53}$$

with $I_0(x) \triangleq \{m \in \mathcal{M} : \mathcal{H}(x) \le A_m\}$, $I(x) \triangleq \{m \in \mathcal{M} : A_m < \mathcal{H}(x) < B_m\}$ and $\overline{I}(x) \triangleq \{m \in \mathcal{M} : B_m \le \mathcal{H}(x)\}$. By (25) and (26) these sets $I_0(x)$, $I(x)$ and $\overline{I}(x)$ can be present in equivalent form given by (22)–(24). Then substituting (12) and (53) into right side of fixed point equation (52), and (18) into its right side imply (19), and the result follows. ■

Proof of Theorem 1: Let $\boldsymbol{P}(x)$ be an equilibrium. Then, by Proposition 3, there are $\tilde{k}$ and $x \in [\tilde{k}\overline{P}, (\tilde{k}+1)\overline{P})$ such that

$$P_k(x) = \begin{cases} 0, & k < \tilde{k}, \\ \in [0, \overline{P}], & k = \tilde{k}, \\ \overline{P} & k > \tilde{k} \end{cases} \text{ or } P_k(x) = \begin{cases} 0, & k < \tilde{k}, \\ \overline{P} & k \ge \tilde{k} \end{cases} \tag{54}$$

with

$$P_{\tilde{k}}(x) = (\tilde{k}+1)\overline{P} - x \tag{55}$$

and

$$P_k(x) = \begin{cases} 0, & \mathcal{T}(x) < C_P/h_k, \\ \in [0, \overline{P}], & \mathcal{T}(x) = C_P/h_k, \\ \overline{P}, & \mathcal{T}(x) > C_P/h_k, \end{cases} \text{ for } k \in \mathcal{K}. \tag{56}$$

Note that, by (20),

$$\mathcal{T}(x) \text{ is non-decreasing continuous function.} \tag{57}$$

Thus, by (56), if (27) holds then $x = 0$, and (a) follows. If (28) holds then $x = K\overline{P}$, and (b) follows. Let (27) and (28) do not hold. Let user's types equilibrium strategies are given by the right formula in (54). Then, by (55), $x = \tilde{k}\overline{P}$, and, thus, by (56), (29) holds, and (c-i) follows.

Let user's types equilibrium strategies are given by the left formula in (54). Then, by (55), $\tilde{k}\overline{P} < x < (\tilde{k}+1)\overline{P}$, and by (56), $\mathcal{T}(x) = C_P/h_{\tilde{k}}$. This equation has a root in $[\tilde{k}\overline{P}, (\tilde{k}+1)\overline{P}]$ if and only if (30) holds, and (c-ii) follows. ■

Proof of Proposition 4: It is clear that $v^L_{U,k}(P_k, \boldsymbol{J})$ is concave in P_k and $v^L_{J,m}(\boldsymbol{P}, J_m)$ is linear in J_m. Thus, by Nash theorem [4], equilibrium exists. ■

Proof of Proposition 5: By (34), $v^L_{J,m}(\boldsymbol{P}, J_m)$ is linear in J_m. Then, (37) follows from (34). Further, by (34), we have that

$$\frac{\partial v^L_{U,k}(P_k, \boldsymbol{J})}{\partial P_k} = \frac{W(\boldsymbol{J})}{h_k P_k^2} - C_P. \tag{58}$$

Since $v^L_{U,k}(P_k, \boldsymbol{J})$ is concave in P_k and $v^L_{U,k}(0, \boldsymbol{J}) = -\infty$ we have that

$$\mathrm{BR}^L_{U,k}(\boldsymbol{J}) = \begin{cases} \dfrac{\partial v^L_{U,k}(P_k, \boldsymbol{J})}{\partial P_k} = 0, & \dfrac{\partial v^L_{U,k}(\overline{P}, \boldsymbol{J})}{\partial P_k} < 0, \\ \overline{P}, & 0 \le \dfrac{\partial v^L_{U,k}(\overline{P}, \boldsymbol{J})}{\partial P_k}. \end{cases} \tag{59}$$

Substituting (58) into (59) implies (36). By (34), $v^L_{J,m}(\boldsymbol{P}, J_m)$ is linear in J_m, and, then, (37) immediately follows from (34). ■

Proof of Proposition 6: By (2) and (37), for a fixed $\boldsymbol{P}$ there exists an unique $\tilde{m}(\boldsymbol{P}) \in \{0, 1, \dots, M, M+1\}$ such that

$$\mathrm{BR}^L_{J,m}(\boldsymbol{P}) = \begin{cases} 0, & m < \tilde{m}(\boldsymbol{P}), \\ \in [0, \overline{J}], & m = \tilde{m}(\boldsymbol{P}), \\ \overline{J} & m > \tilde{m}(\boldsymbol{P}) \end{cases} \text{ or } \mathrm{BR}^L_{J,m}(\boldsymbol{P}) = \begin{cases} 0, & m < \tilde{m}(\boldsymbol{P}), \\ \overline{J} & m \ge \tilde{m}(\boldsymbol{P}). \end{cases} \tag{60}$$

Thus, jammer's types equilibrium strategies have to have the form $\boldsymbol{J}(y)$ given by (39) with $y \in [\tilde{m}(\boldsymbol{P})\overline{J}, (\tilde{m}(\boldsymbol{P})+1)\overline{J})$. To derive an equation to find such y, note that, $\boldsymbol{J}(y)$ is jammer's types equilibrium strategies if and only if

$$\boldsymbol{J}(y) = \mathbf{BR}^L_J(\mathbf{BR}^L_U(\boldsymbol{J}(y))). \tag{61}$$

By (35), (36) and (41)–(43), we have that $R\left(\mathbf{BR}_U(\boldsymbol{J}(y))\right) = \mathcal{R}(y)$. Then this, (37), (39) and (61) and (39) imply (40), and the result follows. ■

Proof of Theorem 2: By (33) and (39), $W(\boldsymbol{J}(y))$ is strictly decreasing in $[0, M\overline{J}]$, by (41)–(43), $\mathcal{R}(y)$ is either constant or strictly increasing in $[0, M\overline{J}]$. Then the result follows from Proposition 6. ■

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