An Optimized Design of Golden Angle Modulation SCMA Codebook Based on Genetic Algorithms

Jingqiu Ren, Sen Li, Guanghua Zhang(✉), and Shang Liu

Northeast Petroleum University, DaQing, China
dazgh@nepu.com.cn

Abstract. Sparse Code Multiple Access (SCMA) is a novel non-orthogonal multiple access scheme, which maps information on different resources into mutidimensional codewords to improve system mapping diversity. However, the performance of the same codebook may vary greatly between different channels, so we propose a new SCMA codebook optimization design for the difference of the codebook, It can be used in the codebook generation process of multiple channels, and optimize the minimum Euclidean distance between code words by genetic algorithm to achieve the effect of improving the codebook performance. In this paper, based on the codebook design of Golden Angle Modulation (GAM), we optimize GAM with the help of genetic algorithm. The simulation results show that in the process of SCMA system simulation using the codebook generated by the optimized parameters in Gaussian channel and Rayleigh fading channel, the BER performance of the codebook is significantly improved.

Keywords: Sparse Code Multiple Access · Genetic algorithm · Minimum Euclidean distance · Codebook optimization · Rayleigh fading channel

1 Introduction

Non-orthogonal multiple access (NOMA) is a communication technology that allows multiple users to occupy the same resource for transmission in 5G and other networks. Recently, NOMA has attracted more and more researchers' attention because it meets various stringent requirements proposed in Massive machine type communication (mMTC). Its excellent characteristics such as very high spectrum efficiency, unscheduled delivery of small data packets at irregular times, large user capacity and ultra-low access delay make it considered as a very promising communication technology. NOMA requires specific technologies to realize, among which power domain non-orthogonal multiple access (PD-NOMA) distinguishes users by power, and code domain non-orthogonal multiple access (CD-NOMA) by code are two specific technical solutions for the implementation of NOMA. As a kind of CD-NOMA, SCMA is developed on the basis of Low density signature (LDS) [1, 2]. Different from LDS, SCMA directly maps user's bit information into a multidimensional complex sequence using a

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codebook, making it easier to distinguish at the receiver, and allocates the transmission of the user on multiple resource blocks to achieve higher spectral efficiency and improved BER performance. In SCMA, research mainly focuses on the codebook design, decoding algorithm and practical application of SCMA technology. The codebook design of SCMA mainly determines the BER performance of the system [3]. In recent years, many codebooks have been designed. Most of the codebooks generate the initial mother constellation by maximizing the minimum Euclidean distance (MED) index, and then perform a series of transformations on the mother constellation to generate the user's codebook. Since these codebooks are used in a specific channel, the same codebook in different channels may not show excellent performance, and may even increase the BER.

So the codebook design approach of GAM is adopted in this paper [4], Because in the two categories of codebooks design methods in GAM, codebooks with one and two optimization parameters, respectively, can construct codebooks for Rayleigh fading channels in the uplink and downlink. After testing, the performance of the optimized codebook in Gaussian channel is also improved compared with other methods. We first generate GAM codebooks for different channels, and then use Genetic Algorithm (GA) to optimize the parameters during the generation process, because GA is easy to optimize the number of parameters, it shows good optimization performance in this methods. Simulation results show that our codebooks optimized by GA significantly improves the performance over existing codebooks used in both Gaussian and Rayleigh fading channels.

The SCMA codebooks design is an important and challenging problem, Taherzadeh and Nikopour et al. first proposed SCMA method and basic codebook construction method [5]. Following this direction, many codebook design methods were proposed based on various theories came into being. In Downlink, Yu et al. proposed a design to generate the mother constellation based on the star-QAM signal constellation [6], and then generate the user codebook through some operators. However, although three operators were proposed in the paper, no specific design indicators were given. In Uplink, Tian, Zhong et al. formulated the design of the multidimensional mother constellation as a nonconvex optimization problem [7] to obtain a suboptimal solution, so as to obtain better coding gains. Gao et al. proposed an algorithm to construct a multidimensional mother constellation by using a two dimensional lattice constellation [8] and design of the uplink SCMA codebook in a multilevel optimization manner. However, most of the design work for uplink SCMA codebook needs to solve complex optimization problems, and there are few related works proposed for uplink. In the codebook design method under Gaussian channel, Zhou et al. proposed a codebook design method based on constellation rotation [9]. And Cai et al. further proposed a codebook generation method by adding interweaving after rotation. [10] However, in the above papers, the codebook was only generated for a certain channel, and it may not have excellent effects in other channel.

In this paper we propose the design method can use a simple algorithm to generate suitable codebooks between channels with different performance, so as to reduce the bit error rate during transmission. The rest of this paper is organized as follows: Sect. 2 introduces the traditional system model of SCMA, Sect. 3 describes the method of using GA to optimize the GAM codebook, Sect. 4 describes the comparison of different codebooks and BER simulation and analysis in different channels, and Sect. 5 concludes the paper.

2 SCMA System Model

The SCMA encoder is defined as a mapping from $\log_2(M)$ bits to a K-dimensional complex codebook of size M. In the downlink SCMA system, each SCMA encoder consists of J Internet of Things (IOT) devices (users) occupying K orthogonal resource nodes. Among them, orthogonal resources can be subcarriers, time slots, etc. Each user has a multidimensional SCMA codebook with M codewords $\boldsymbol{x}_j$ of length K. The SCMA encoder for user j is defined as the mapping:

$$f_j : \mathbb{B}^{\log_2(M)} \rightarrow \chi_j \tag{1}$$

where χ_j is the set of codebooks, M is the modulation order, $\mathbb{B}^{\log_2(M)}$ is the binary bit at the encoder side, and $\boldsymbol{x}_j$ is extracted from $\chi_j \in \mathbb{C}^{K\times M}$. Set of codebooks:

$$\chi_j = \left\{x_{j,1}, x_{j,2}, \ldots, x_{j,M}\right\} \in \mathbb{C}^{K\times M}, i.e. x_j = f_j b_j \tag{2}$$

$$b_j = \left[b_{j,1}, b_{j,2}, \ldots, b_{j,\log_2(M)}\right]^\top \in \mathbb{B}^{\log_2(M)} \tag{3}$$

where $\boldsymbol{b}_j$ is the incoming binary message vector from the j^{th} user.

A K-dimensional complex codeword of a codebook is a sparse vector with $N < K$ nonzero entries. All codewords in the codebook contain zeros in the same $K - N$ dimension. The nonzero dimensions of the mapping matrix V are mapped to the K-dimensional complex domain. Let C_j be the N-dimensional compound constellation obtained by removing all zero elements in χ_j. Define the mapping from $\mathbb{B}^{\log_2(M)}$ to C_j:

$$g_j : \mathbb{B}^{\log_2(M)} \rightarrow C_j, i.e. c_j = g_j b_j. \tag{4}$$

$$C_j \subset \mathbb{C}^{N\times M} \tag{5}$$

Then the SCMA encoder in (1) can be redefined as

$$f_j : V_j g_j, i.e. x_j = V_j g_j b_j \tag{6}$$

where $\boldsymbol{V}_j \in \mathbb{B}^{K\times N}$ is a binary mapping matrix that maps N-dimensional constellation points to K-dimensional SCMA codewords. The mapping matrix $\boldsymbol{V}_j$ contains $K - N$ all zero rows. The inclusion of a row $r \in \{1, \ldots, K\}$ with value 1 in $\boldsymbol{V}_j$ means that user j is using the r^{th} resource. Thus, the set of resources occupied by user j depends on $\boldsymbol{V}_j$. Define the factor matrix $\boldsymbol{F}$ as follows:

$$F_{K\times J} = [f_1, \ldots, f_2] \subset \mathbb{B}^{K\times J}, f_j = diag\left(V_j V_j^\top\right) \tag{7}$$

User node j is connected to resource node k if and only if $f_{k,j} = 1$. An example of a factor graph representation of $\boldsymbol{F}$ is shown in Fig. 1. There are 6 symbol nodes and 4 resource nodes (Fig. 2).

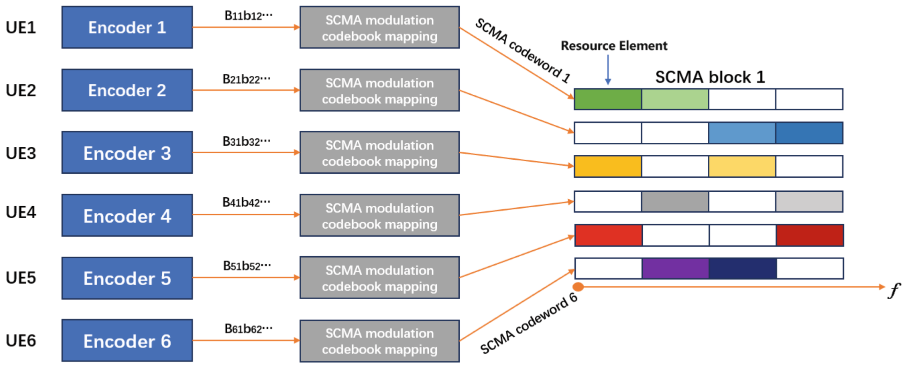

Fig. 1. SCMA system model

$$F_{4\times 6} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Fig. 2. Factor graph $\boldsymbol{F}$

For a known channel $\boldsymbol{h}$ and a received signal $\boldsymbol{y}$, a near optimal detection of layers J can be performed iteratively by applying a message passing algorithm (MPA). The complexity of MPA is proportional to Md_f where d_f is the number of branches reaching the resource node. The sparse codebook can effectively control the number of branches at each resource node, thus limiting the complexity of the MPA receiver. The data of J users are first superimposed at the base station and then transmitted over K resource blocks, with a total of M^J superimposed codewords. The signal received by the user can be written as

$$y = diag(h_j) \sum_{j=1}^{J} x_j + n \tag{8}$$

where $\boldsymbol{x}_j = \boldsymbol{V}_j g_j \boldsymbol{b}_j$ is the vector of SCMA codewords of user J, h_j is the channel vector of layer J, and $\boldsymbol{n}$ is the ambient noise. In the case where all layers transmit from the same transmitting point, all channels to the destination receiver are the same, *i.e.* $h_j = \boldsymbol{h}, \forall j$. The overload factor of the code is defined as $\lambda = J/K$ by multiplexing J layers on K resources.

3 The Optimized GAM Codebook

3.1 Mathematical Model

The GAM structure proposed in [1] is adopted, which can design the downlink and uplink SCMA codebooks based on golden Angle modulation [11]. One optimization parameter (θ) and two optimization parameters (θ, ρ) are used respectively to optimize the codebook design for the downlink and uplink. The optimization parameter is independent of the codebook and system parameters, so a simple optimization method can be used to optimize the parameters to design a better codebook. By adjusting the value of θ and ρ, the BER of the proposed codebook can be reduced and the minimum normalized Euclidean distance can be improved.

The specific codebook design method of GAM is provided in [1], no further details here. GAM point x_n is generated as follows.

$$x_n = r_{n+\rho} e^{i2(\pi+\theta)n}, n = 1, \ldots, N_p \tag{9-a}$$

$$r_{n+\rho} = c_{norm}\sqrt{n+\rho} \tag{9-b}$$

$$c_{norm} = \sqrt{\frac{2P}{N_P + 1}} \tag{9-c}$$

where $N_p = N \cdot \frac{M}{2}$, θ and ρ are the free parameters proposed in θ-GAM, which can be optimized to improve the codebook performance, x_n are used to generate the mother constellation to design SCMA codebook (Fig. 3).

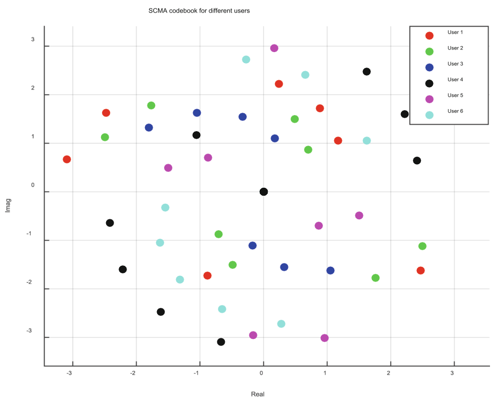

Fig. 3. The user codebook constellation points generated using GAM in the Rayleigh channel downlink, $\theta = -2.8329$, $\rho = 0$.

3.2 Optimization Algorithm

GA optimizes the θ and ρ parameters of GAM in a simple optimization manner [12]. For convenience, in this paper, the normalized minimum Euclidean distance $\tilde{d}_{\min}$ is used as the fitness to calculate the θ and ρ that are optimal for the GAM points.

$$\tilde{d}_{\min} = \min\left\{ \frac{|x_i - x_j|}{\sqrt{E}}, \forall x_i x_j \in x_n, x_i \neq x_j \right\} \tag{10}$$

where x_n denotes the nth codeword in the GAM constellation point, E is the average power of the codeword. Before obtaining the codebook, we propose Algorithm 1 to optimize the parameters, which is used to build the codebook of each user later. In GA, we try to increase the initial population size, and appropriately adjust the crossover probability, mutation probability and generation gap value to prevent the generation of local optimal solution in the iterative process. The algorithm steps are as follows:

1) Population initialization. It is Set that *NIND* individuals (chromosomes) need to be initialized as the population number, so n numbers are randomly generated at first, and each number is tested whether it satisfies the range constraint of θ. According to this method, all *NIND* individuals are initialized, and the initialization of *Chrom* of the parent population is completed after the completion.
2) Fitness function. A proper evaluation of each chromosome is needed to find the optimal value in the GA optimization process. The design criterion of the optimal θ value is to maximize the minimum Euclidean distance between codewords [13]. Therefore, $\tilde{d}_{\min}$ is used as the evaluation index, and the Fitness function is calculated as follows:

$$Fitness = \max\left(\tilde{d}_{\min}|\theta_i\right)\ i = 1, \ldots, NIND \tag{11}$$

3) Selection. From the biological point of view, although there are *NIND* individuals in the population, not all of them can give birth to offspring, so instead of selecting out *NIND* individuals, but $Nsel = NIND \times GGAP$ individuals. *GGAP* is the generation gap, and the selection method uses the roulette wheel selection strategy. Specifically, the probability of being selected is assigned to each different individual, and which one is selected is decided by rotating the roulette wheel. in conclusion, the selected individuals need to be rotated Nsel times to form the Sel$C$h(select Chrom).
4) Crossover and mutation. The genes of the offspring population individuals generated by the selection operation have not changed, and the individual genes need to be changed to make the population evolve. The first operation is crossover, which usually requires two objects, but there is only one parameter for the optimization of θ, so the crossover adopts a simple parameter perturbation. The second operation is mutation, which takes a small value in code because mutation is less common in nature.
5) Reorganization and update. Nsel individuals are obtained through the above series of operations, and the individuals with the top $NIND - Nsel$ fitness rank are found out from the parent population *Chrom* and added to the child population *SelCh*. Then the new parent population *Chrom* is updated and used as the population in the next selection operation. The reproduction algebra *MAXGEN* was set, which was the number of iterations. Finally, the optimal individual generated in each iteration was recorded and updated to generate the optimal individual value.

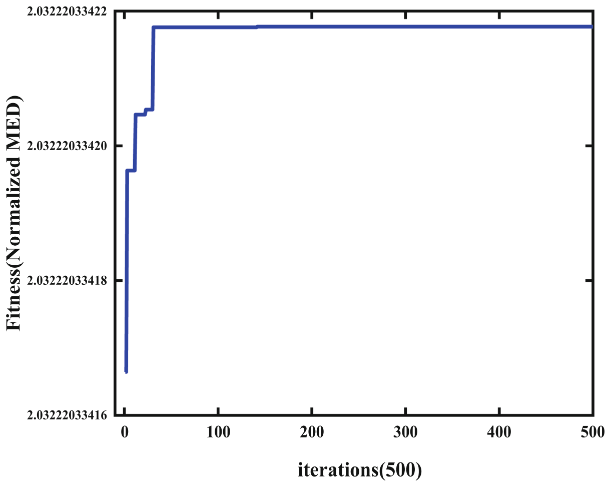

Fig. 4. Diagram of iterative process of GA minimum Euclidean distance iterations = 500

4 Simulation and Numerical Analysis

In this section, the performance of codebooks generated using GA optimized parameters θ, ρ is evaluated under different channels. We take the classic case of 6 users, 4 resource blocks, and the number of resources used by each user N = 2 to evaluate the average BER of the codebook. Figure 1 represents the factor matrix. According to GA in Gauss and downlink, the minimum Euclidean distance is optimized to generate parameters as $\theta_1 = -2.8329$, $\rho = 0$.The optimized parameter in the uplink is $\theta_2 = 3.9549$, where is the value of θ selected for the best performance, because the optimization of θ does not depend on the system. Then we fix the θ_2 and optimize ρ separately to produce $\rho = 1.054$. Using the above optimized parameters in the GAM method, three different codebooks are generated in the Gaussian channel and Rayleigh fading channel up/downlink, and their performance is compared with other codebooks. Figure 4, Fig. 5, and Fig. 6 show the BER performance of the GAM codebook optimized using GA, tentatively referred to as GA-GAM, compared with the original GAM and the proposed codebook in [6, 14, 15] in different channels. We observe that The $\tilde{d}_{\min}$ between the GA-GAM codewords is larger under Gaussian and Rayleigh channels, and this codebook has a lower BER than other codebooks (Fig. 7).

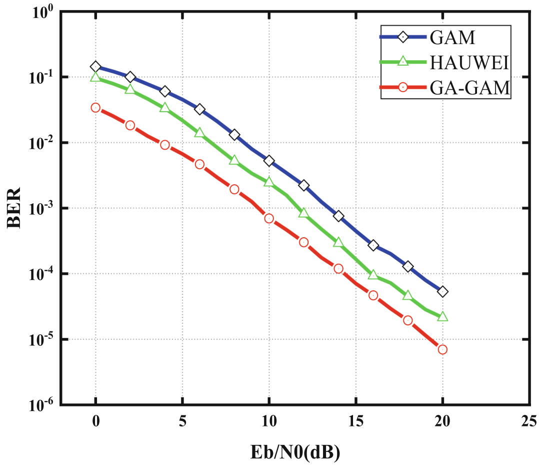

Fig. 5. Performance comparison of GAM codebook in [4], Huawei codebook in [15] and GA-GAM in Rayleigh downlink channel with diversity, $\theta = -2.8329$ and $\rho = 0$ after optimization.

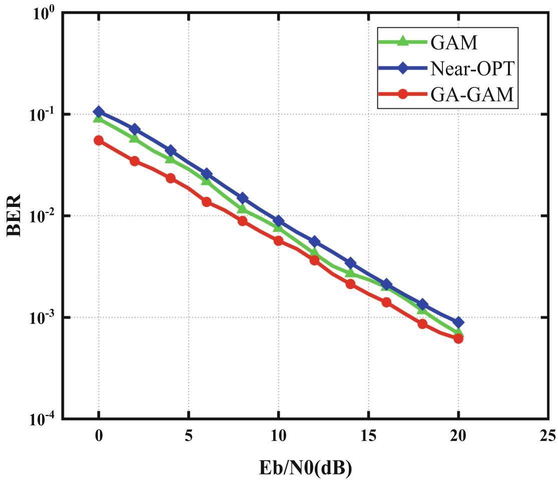

Fig. 6. Performance comparison of GAM codebook in [4], Near-OPT codebook in [14] and GA GAM in Rayleigh fading uplink channel with diversity, $\theta = 3.9549$ and $\rho = 1.054$ after optimization

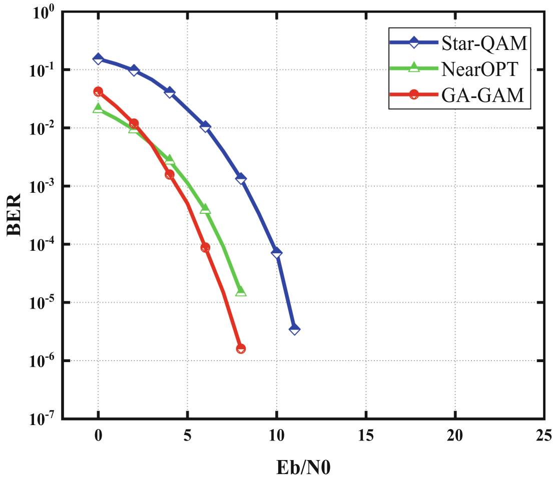

Fig. 7. Performance comparison of the Star-QAM codebook in [6], Near-OPT codebook in [14] and GA-GAM in Gaussian channel codebook, θ = –2.8329 and ρ = 0 after optimization

Table 1. Comparison of BER and $\tilde{d}_{\min}$ for different SNR in uplink/downlink of Rayleigh channel

DOWNLINK

BER \ Eb/No	5dB	15 dB	$\tilde{d}_{min}$
[4]	0.06051	0.00045	1.2886
[15]	0.03298	0.00017	1.4142
This work	0.0092	0.00007	1.8068

UPLINK

BER \ Eb/No	5dB	15dB	$\tilde{d}_{min}$
[4]	0.03335	0.00266	1.4120
[14]	0.02873	0.00235	1.8354
This work	0.01858	0.0017	2.0322

Table 2. Comparison of BER and $\tilde{d}_{\min}$ for different SNR in Gaussian channel

AWGN

BER \ Eb/No	5dB	10dB	$\tilde{d}_{min}$
[6]	0.02078	0.0000712	1.4426
[14]	0.00113	0.00000046	1.6135
This work	0.0005	0	1.6348

5 Conclusion

Based on the method of GAM codebook design, we use GA to optimize its proposed θ, ρ, and thus propose an optimal SCMA codebook design method for different channels. Our proposed method optimizes one parameter (θ) or two parameters (θ, ρ) respectively by maximizing the minimum Euclidean distance. The results of numerical experiments on codebooks show that the GA method can easily find better optimization parameters. Table 1 and Table 2 show that the optimized codebook is used to compare the Huawei generated codebook, the approximate optimal algorithm [14], star-QAM and the original GAM method under the Gaussian channel and Rayleigh channel down/uplink. It is proved that the proposed codebook optimization method has larger $\tilde{d}_{\min}$ value and better BER performance. However, the codebook proposed in this scheme is still limited to the three commonly used channels, and whether it can maintain good flexibility and robustness when the channel changes remains to be tested and studied (Table 3).

Table 3. A θ is optimized by the GA algorithm.

```
Set Chrom = InitPop (NIND, θ − range)
Set gen = 1
bestIndividual = Chrom (1, : )
while gen <= MAXGEN do
    for i = 1: NIND do
        CB = GenerateCB(Chrom(i))
        Obj = CalculateMED (Chrom(i))
        FitnV = Obj
    end for
    SelCh = Select (Chrom, FitnV, GGAP)
    SelCh = Mutate (SelCh, θ − range)
    Chrom = Reins (Chrom, SelCh, Obj)
    BestObj(gen, 1) = bestObj
    gen = gen + 1
end while
Output the best individual
```

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