



Vehicle Routing Problem for an Integrated Electric Vehicles and Drones System

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Abstract. This study explores a new research direction on the cooperation between electric vehicles (EVs) and drones in last-mile logistics in the form of *electric vehicle routing problem with drones* (E-VRPD). The primary objective of E-VRPD is to find an optimal vehicle tour to minimize the total completion time to deliver parcels to a set of customer nodes using a set of EVs, each equipped with a single drone. Due to the importance of such problems and the lack of existing techniques, in this work, we develop a sequential decomposition algorithm with an improvement phase to solve E-VRPD. This algorithm involves the development of a mathematical formulation for inserting drone sorties into an EV tour, leading to a *matheuristic* algorithm. The proposed method is evaluated on a set of instances involving up to 40 customers and 7 recharging nodes, with the experimental results showing the effectiveness and relevancy of E-VRPD.

Keywords: Electric vehicle routing problem · Last-mile logistics · Drones · Decomposition · Matheuristic

1 Introduction

The ease of getting desired items without having to go out of your house has led to the continuous growth of home deliveries in recent years. However, this rising number of customer home deliveries poses a hidden drawback for both firms and society. Although it sounds very convenient, direct deliveries to customer's houses may be costly, highly inefficient, and not environmental-friendly [1]. The inefficiency issue in last-mile logistics has been long known by researchers and practitioners. During these golden years of the E-commerce business, where the majority of parcels delivered are small-size packages [2], this issue has become more relevant than ever.

Several innovations have emerged as potential solutions in response to this particular challenge. These innovative ideas range from implementing novel delivery concepts, such as parcel lockers, to using new transportation technologies [1]. Accordingly, in this study, we aim to explore the cooperation of these innovations in last-mile logistics, particularly between electric vehicles (EVs) and drones.

Here, this cooperation is referred to as *electric vehicle routing problem with drones* (E-VRPD), where a firm must deliver parcels to a set of customer nodes using a set of EVs, each equipped with a single drone. Due to the range limitation of EVs, a set of recharging stations is available to recharge the battery of EVs, and the main purpose of E-VRPD is to find optimal vehicle tours to minimize the total completion time of delivery tasks.

E-VRPD can be seen as an extension of vehicle routing problem with drones (VRPD) [3] and electric vehicle routing problem (E-VRP) [4]. Compared to VRPD, E-VRPD considers the deployment of EVs instead of traditional internal combustion engine vehicles (ICEV). This corresponds to the pursuit of a more sustainable transportation mode amidst the realization of various negative externalities from freight transportation sectors to the environment [1]. As to E-VRP, E-VRPD practically extends E-VRP with the presence of drones as an additional transportation mode, leading to a larger solution space to be explored.

This work defines E-VRPD and develops a sequential decomposition algorithm with an improvement phase to solve E-VRPD. The algorithm involves the development of a mathematical formulation for inserting drone sorties into an EV tour, leading to the development of a *mathuristic* algorithm [5]. To this time, this is the first effort to propose an algorithm for E-VRPD, as we are not aware of previous works that deal with a similar problem. As such, we evaluate the performance of our proposed algorithm by comparing it with E-VRP solutions. Furthermore, with a set of numerical experiments, we show the relevancy of the E-VRPD as a promising research area in operations research and logistics.

The rest of this paper is structured in the following way. Section 2 presents a literature review on related studies. Section 3 describes E-VRPD. Section 4 delivers an explanation of our proposed algorithm. Then, Sect. 5 presents computational experiments and analyzes their results. Finally, in Sect. 6, we discuss some conclusions and future research directions.

2 Literature Review

The study of the cooperation between ground vehicles (e.g. trucks) and drones was introduced in [2], where the seminal mathematical formulation of coordination between a ground vehicle (GV) and a drone, namely the flying sidekick traveling salesman problem (FSTSP), was discussed. FSTSP considers a single truck assisted by a single drone to perform a parcel delivery task to a set of customer nodes in a graph. In this form of cooperation, a human operator can operate the attached drone by launching the drone from a launching node (a depot node or a customer node) to deliver a parcel to a single customer node. While the drone performs its *sortie* (drone-delivery operation), the truck can serve other customer nodes before recollecting the drone in a rendezvous node, as long as the endurance constraint of the drone is not violated. Another influential work from [6] presented an integer programming formulation for a similar model to FSTSP, namely the traveling salesman problem with drone (TSPD). By comparing their models to the classical traveling salesman problem (TSP)

model, these two seminal studies successfully illustrated how using drones in last-mile logistics with GV as its ‘moving hub’ can reduce the time required to complete delivery tasks.

Meanwhile, within the last decade, a handful of works have discussed the optimal way to implement EVs in logistics. Among them, the most popular direction is perhaps E-VRP, which deals with the optimization of EV routes by considering the constraints of battery and charging operations. For instance, a recent survey on E-VRP from [4] reported 136 published works related to E-VRP within the last ten years, which shows that the deployment of EVs in logistics is attracting the interest of researchers and practitioners.

Unfortunately, our literature review shows that only four research works are available on the integration between drones and EVs in logistics, as most of the previous works deal with the traditional delivery system with a single type of vehicle. To the best of our knowledge, [7] is the first study to consider the cooperation between EV and drone in a TSP-variant model. They presented an EV-based transportation model where an EV is equipped with a single drone. The model presented in [7] aims to minimize the total energy consumption of the ground vehicle, and they assumed that the drone could be launched multiple times within a tour. A recent study from [8] then developed a mathematical formulation for a similar problem and named the problem the electric traveling salesman problem with drone (E-TSPD). The model presented in [8] embodied two particular differences from the problem discussed in [7], in which [8] considered the presence of recharging stations in the graph and assumed that the drone battery could be recharged using the energy from the EV after finishing a sortie. That study was then continued in [9], where the authors extended E-TSPD to consider the possibility of a partial recharging policy. In this regard, [9] relaxed the classical assumption of E-VRP variants where the EVs must be fully recharged after entering a recharging station. Another recent study [10] discussed the first scaling-up extension of E-TSPD, with a new model named E-VRPD, with the objective function of minimizing total energy consumption. In [10], a mathematical formulation of E-VRPD was presented with a hybrid ant colony optimization algorithm introduced to solve E-VRPD instances with up to 50 customer nodes. Nevertheless, we note that the E-VRPD model presented in [10] assumed that the EVs are only used as a moving hub for launching and retrieving the drones, as only drones serve all of the demand from customer nodes. Borrowing the term of [11], the E-VRPD model presented by [10] can be classified as non-simultaneous coordination between ground vehicles and drones, where this class of model is highly-related to the 2-echelon routing problems [12]. Additionally, [10] also did not consider the presence of recharging stations as they assumed that the EVs would always have enough energy to finish their tour.

In summary, the contributions of this study are twofold. Firstly, we define the E-VRPD with simultaneous coordination between EVs and drones as a new form of optimization problem. We differ our contribution from [10] by discussing a simultaneous form of E-VRPD with recharging stations throughout the graph

and relaxing the assumption of unlimited energy of EVs. Secondly, a sequential decomposition algorithm with an improvement phase is proposed as the first solver for E-VRPD, as there is no existing approach for E-VRPD. This decomposition approach is executed by deconstructing E-VRPD into a set of smaller (and simpler) sub-problems that can be sequentially solved using a set of established tools. The idea of decomposition has been noted for a long time as an effective and practical way to solve any complex optimization problems, as shown by several classical decomposition techniques in operations research such as Dantzig-Wolfe, Lagrangian, and Benders decomposition [13]. In retrospect, the decomposition technique has also been noted as a traditional class of *matheuristic* approach [5], i.e. a practice of combining metaheuristic frameworks [14] with exact algorithms, which is known to be a very promising approach to solving variants of routing problems [15].

3 Problem Description of E-VRPD

This section briefly describes E-VRPD. The problem can be defined in an undirected graph $G = (V, A)$. The set $V = V_0 \cup V_S \cup V_C$ captures all the nodes within the graph, while set A comprises all arcs $(i, j) \in V, i \neq j$ between those nodes. In accordance, set $V_0 = \{0, r + n + 1\}$ is presented as the set of the depot node $\{0\}$ along with its dummy node $\{r + n + 1\}$, set $V_S = \{1, \dots, r\}$ contains all r recharging stations available on the graph, and $V_C = \{r + 1, \dots, r + n\}$ represents all the n customer nodes to be served.

The E-VRPD model aims to find a set of optimized routes to serve all the customer nodes using f homogeneous fleet of EVs and drones $F = \{1, \dots, f\}$. All the routes must be started from and finished at the depot node, and each customer node must be served only once, either by an EV or a drone. In the case where a certain node i is visited by both EV and drone, its demand q_i is served by the EV. Additionally, these routes are subject to the capacity constraint, where each EV and drone has a limited payload capacity, Q_t and Q_d , respectively. In some occasions, $q_i > Q_d$ is valid, therefore, we introduce a subset $V_D \subseteq V_C$ to list all the drone-eligible customer nodes.

Along the tour of EVs, the operator can launch the equipped drone from node $i \in V_L = \{0, \dots, r + n\}$ to serve the customer nodes in V_D . After getting launched, the drone can then deliver the parcel of the targeted customer node $j \in V_D$ with a service time s_j , then return back to its corresponding EV at a different node $k \in V_R = \{1, \dots, r + n + 1\}$ before its energy capacity, E_d , runs out. Borrowing the popular terminology from [2], we call such an operation a *sortie* and define it as a tuple $\langle i, j, k \rangle$, where $i \neq j \neq k$. Moreover, note that during a *sortie* of drone f , the corresponding EV can visit other customers and/or recharging nodes before visiting the rendezvous node k .

Similar to the drones, the EVs are also subject to the limitation of their battery E_t . The battery energy of EVs is drained to fuel its movement as well as recharge its drone's battery. The latter concept is borrowed from [8], as we assume that after finishing the *sortie* the drone batteries will be replaced with

a new one, and then the used batteries are recharged using the energy from the EV. If required, the EVs can stop at the available recharging stations $i \in V_S$ to recharge their battery with a recharging rate of R . These stations can also be used as a launching and retrieval point for a sortie. In accordance, we introduce h_t as the battery consumption rate of EVs and h_d as the battery consumption rate of drones.

The objective function considered here is to minimize the total completion time required, T , to perform all the delivery tasks. Let $F_a \subseteq F$ be the subset of active tours and T_f be the completion time of tour f , the value of T then can then be calculated as in Eq. (1). Borrowing the finding of [16], the completion time of a tour f can be decomposed into three components:

1. the total travel time of the GV. Here, it comprises the sum of travel time, service time and recharging time of EVs.
2. the total setup time required to launch (s_L) and recollect (s_R) the drone and
3. the total time spent by the GV to wait for the drone.

Accordingly, the value of T_f can be calculated as in Eq. (2) by introducing A^s as the set of all possible (i, j, k) sorties and the following parameters: $d_{i,j}^t$ as the distance through arc $(i, j) \in A$ for EVs, $d_{i,j}^d$ as the distance through arc $(i, j) \in A$ for drones, v_t and v_d to represent the speed of EVs and drones, s_i to represent the service time required to serve node i , $t_{i,j}^t = \frac{d_{i,j}^t}{v_t}$ as the time required for EVs to traverse through arc $(i, j) \in A$, and $t_{i,j}^d = \frac{d_{i,j}^d}{v_d}$ as the corresponding travel time parameter of drones.

$$C = \max_{f \in F_a}(T_f) \quad (1)$$

$$T_f = \sum_{i,j \in A} (t_{i,j}^t + s_j)X_{i,j,f} + \sum_{i,j,k \in A^s} (s_L + s_R)Y_{i,j,k,f} + \sum_{k \in V_S} R_{k,f} + \sum_{k \in V_R} W_{k,f} \quad (2)$$

In addition, the value of these decisions must be taken into account to calculate the completion time of each tour in Eq. (2). These are:

1. $X_{i,j,f} \in \{0, 1\}$ that sets whether EV f traverses through arc (i, j)
2. $Y_{i,j,k,f} \in \{0, 1\}$ to decide whether drone f performs sortie (i, j, k) ,
3. $R_{k,f} \geq 0$ as the recharging time spent by EV f in recharging station k ,
4. $W_{k,f} \geq 0$ to define the waiting time spent by EV f in node k to wait for the arrival of the drone,
5. $a_{i,f}^t, a_{i,f}^d \geq 0$ to respectively record the arrival time of the EV and drone f at node i , and
6. $e_{i,f}^t, e_{i,f}^d \geq 0$ to respectively record the battery state of EV and drone f when they arrive at node i

Then, the values of $R_{k,f}$ and $W_{k,f}$ can be defined as in Eqs. 3 and 4, while we also define several assumptions in E-VRPD as follows:

1. Drone can safely land at the rendezvous node before the arrival of the EV. In this regard, we define that if one vehicle arrives earlier, the early vehicle must wait for the latter.
2. Full recharging policy is applied for EVs, so they must be in a fully-charged state when departing a recharging station.
3. All customer nodes have a low-weight demand and are drone-eligible. Therefore, we can neglect the capacity constraint of EVs in this study.
4. Drones always start their sortie in a fully charged state, and the battery recharging time is negligible.
5. The battery energy of drones remains drained when the drone performs a service at drone node j , since it needs to be active during the whole service time. On the other hand, the engine of EVs is deactivated by the operator during a visit. In this regard, the battery consumption of drones within a sortie (i, j, k) can be defined as a time-based function $(t_{i,j}^d + s_j + t_{j,k}^d) h_d \leq E_d$, while the battery consumption of EVs in arc (i, j) can be calculated using a simple distance-based function $(d_{i,j}^t h_t)$ [4].
6. Every time the EV visits a node, the human operator must perform these tasks in the following order: retrieving the drone (if any), fulfilling the customer order (if any), recharging the EV (if any), and launching the drone (if any).

$$R_{k,f} = \frac{E_t - e_{k,f}^t - (E_d - e_{k,f}^d)}{R} \quad (3)$$

$$W_{k,f} = \max(0, a_{k,f}^d - a_{k,f}^t) \quad (4)$$

4 Sequential Decomposition Algorithm

This section demonstrates how to solve E-VRPD using a decomposition-based algorithm. This algorithm relies on the idea of sequential decomposition, where we deconstruct a complex optimization problem into a set of smaller (and simpler) sub-problems that can be solved sequentially [13]. The end product of these sequential decisions corresponds to the near-optimal solution of the original problem.

Accordingly, in order to deconstruct the relatively complex E-VRPD, we rely on a classic concept in developing a heuristic approach for vehicle routing problem (VRP), namely 'cluster-first, route second' [13]. The core idea is that, in order to find the near-optimal solution of VRP, one can divide the customer nodes into k clusters (where $k \leq f$), then optimize the route for each cluster (that comprises a smaller set of customer nodes). Additionally, in finding the near-optimal route for a truck-drone tandem, the authors of [2] derived the idea of 'truck-first, drone-second' that states that the route of a truck-drone tandem can be developed by finding the optimal truck-only TSP tour for all the corresponding nodes first, then inserting the drone sorties to re-optimize the tour.

Algorithm 1: A sequential decomposition algorithm for E-VRPD

Input: E-VRPD instance
Result: S^*, T^*

- 1 $S \leftarrow \emptyset, T \leftarrow 0, m \leftarrow 1$
- 2 **Phase I - Decomposition**
- 3 $V_{CL} \leftarrow kMeans(V_C, t_{i,j}^t, f)$
- 4 **while** $m \leq CL$ **do**
- 5 $S_m^{TSP} \leftarrow MinTimeTSP(V_{C,m}, t_{i,j}^t)$
- 6 $S_m^{E-TSP} \leftarrow RechargingInsertion(S_m^{TSP}, V_S, t_{i,j}^t)$
- 7 $S_m^{E-TSPD}, T_m \leftarrow DroneInsertion(S_m^{E-TSP}, V_{C,m}, t_{i,j}^t, t_{i,j}^d)$
- 8 **if** $T_m > T$ **then**
- 9 $T \leftarrow T_m$
- 10 **end**
- 11 $S \leftarrow Merge(S_m^{E-TSPD})$
- 12 $m = m + 1$
- 13 **end**
- 14 **Phase II - Improvement**
- 15 $S^*, T^* \leftarrow RandomizedVND(S, T)$

These two ideas are utilized to build the first phase of our proposed algorithm, which comprises two phases: decomposition and improvement. Algorithm 1 presents the pseudocode of our algorithm. We denote S as a solution array and T as the objective value (of total completion time). Accordingly, S^* and T^* are the final solution and the objective value obtained from the algorithm. In the following subsections, each of these phases will be explained.

4.1 Decomposition Phase

The first phase of the algorithm decomposes an E-VRPD instance into four sub-problems: clustering with k-Means, TSP with a time minimization objective function, recharging insertion problem, and drone insertion problem. These sub-problems are visually presented in Fig. 1, where the output of each sub-problem will be used as an input for the next one.

Clustering: First, all the customer nodes, V_C , are clustered into $m \leq f$ clusters. Here, we deploy the classic k-Means clustering algorithm to create clusters using $t_{i,j}^t \in A$ as the distance function.

Notably, one major task in the k-Means algorithm is finding the appropriate value of m . From a practical standpoint, one important observation here is that deploying all available vehicles ($m = f$) tends to minimize the total completion time, as this balances the workload of each vehicle and reduces the chance of having an overloaded resource (that corresponds to an increasing value of maximum completion time). On the other hand, for cost minimization variants, limiting the value of k to be as small as possible is a logical choice, as it corresponds to the reduction of fixed vehicle cost [17].

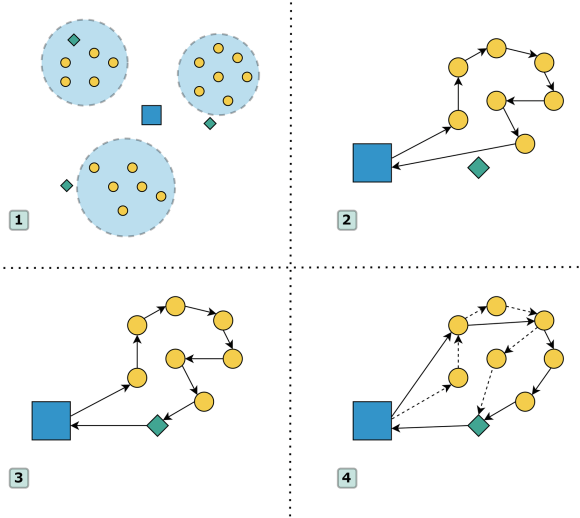


Fig. 1. Decomposing E-VRPD into four stages: (1) clustering, (2) traveling salesman problem, (3) recharging insertion, and (4) drone insertion

From the deployment of k-Means, we derive V_{CL} as the set of customer nodes that has been divided into CL clusters. In this regard, we denote $V_{C,m} \subset V_{CL}$ as the subset of customer nodes included in cluster m .

Traveling Salesman Problem: the second step of this algorithm is to find the optimal single EV tour (started and ended at the depot node) that captures all the customer nodes in each $V_{C,m}$. For each m , we define a TSP to minimize the completion time that can be defined as in Eq. (5). In this step, we simply deploy the Concorde solver [18], widely known as a state-of-the-art solver for TSP.

$$T_m^{TSP} = \sum_{i \in \{V_0 \cup V_{C,m}\} \setminus \{j\}} \sum_{j \in \{V_0 \cup V_{C,m}\} \setminus \{i\}} (t_{i,j}^t + s_j) X_{i,j,m} \quad (5)$$

Recharging Insertion: from the TSP tour of EV (S_m^{TSP}), we now aim to see whether it is required to insert any recharging stations within the tour. In this step, the battery state of EV m on its arrival at node $i \in S_m^{TSP}$ is examined. Then, a recharging station is inserted into the tour before the point where the EV’s battery runs out. At this point, one might argue that inserting the drone sorties before inserting recharging stations could perhaps be more beneficial, as it could reduce the need to use any recharging stations for the EVs. Nevertheless, our pilot study during the development phase of this algorithm showed that the

Algorithm 2: A constructive algorithm for the recharging insertion problem

Input: $M_m, N, d_{i,j}^t, E_t, H_t$
Result: S_m^{E-TSP}

- 1 $S \leftarrow [0], e_0^t \leftarrow E_t, i \leftarrow 1, j \leftarrow 1$
- 2 **while** $i \leq |M_m|$ **do**
- 3 $C_r \leftarrow \left(d_{(M_m, j-1, M_m, i)}^t + d_{(M_m, i, N(M_m, i))}^t \right) H_t$
- 4 **if** $e_{j-1}^t - C_r > 0$ **then**
- 5 $S \leftarrow \text{Merge}(M_m, i)$
- 6 $e_j^t \leftarrow (e_{j-1}^t - d_{(M_m, j-1, M_m, i)}^t) H_t$
- 7 $j = j + 1$
- 8 **else**
- 9 $S \leftarrow \text{Merge}(N(M_m, j-1), M_m, i)$
- 10 $e_j^t \leftarrow (e_{j-1}^t - d_{(M_m, j-1, N(M_m, j-1))}^t) H_t$
- 11 $e_{j+1}^t \leftarrow (E_t - d_{(N(M_m, j-1), M_m, i)}^t) H_t$
- 12 $j = j + 2$
- 13 **end**
- 14 $i = i + 1$
- 15 **end**
- 16 $S_m^{E-TSP} \leftarrow S$

presence of a recharging station within an E-VRPD tour could potentially be beneficial objective-wise due to the possibility of launching and/or retrieving drones there (see Fig. 1). This corresponds to the finding of [19] that showed how the performance of a truck-drone logistics system could be improved by including several parking lots for launching-retrieving purposes.

Moving on, this recharging insertion task can be easily solved with a constructive algorithm. The core concept of this constructive algorithm is as follows: “if one cannot reach the nearest recharging station from node $i+1$ (the next location in the tour), one should detour to the nearest recharging station from node i (current location) before visiting node $i+1$ ”. For this task, we define $M_{m,i}$ as the i -th node in the EV tour m and N_i as the nearest recharging station from node $i \in \{V_0 \cup V_C\}$. For a case where there are two recharging stations with the same distance from i , we simply take one of them randomly. The constructive algorithm is presented in Algorithm 2.

Drone Insertion: After obtaining the full EV tour of cluster m (S_m^{E-TSP}), which corresponds to the electric traveling salesman problem (E-TSP), the next step is to insert drone sortie(s) with the aim of reducing the completion time of tour m . In this regard, we define the drone insertion task as a mixed-integer linear program (MILP), adapting the approach of [20]. In order to build the MILP formulation, we first define L as the length of solution array and $L_m = |S_m^{E-TSP}|$,

then, we label the nodes in S_m^{E-TSP} as $[1, \dots, L]$ according to the order of the EV tour m , where both nodes ‘1’ and ‘L’ stand for the depot node (start and finish). By defining the following notations and sets:

- A_m as the set of all (i, j) arcs in S_m^{E-TSP} where $i < j$,
- $V_{L,m} = \{1, \dots, L - 1\}$, $V_{D,m} = V_{C,m} \cap V_D$, and $V_{R,m} = \{2, \dots, L\}$ as the required sets to define possible sorties in cluster m ,
- A_m^s as the set of all feasible sortie arcs (i, j, k) from $V_{L,m}$, $V_{D,m}$ and $V_{R,m}$ where $i < j < k$.

we can present the MILP of the drone insertion problem for each value of m , as in Eqs. (6)–(15).

$$\begin{aligned} \min \quad & \sum_{i,j \in A_m} \left(t_{(M_m,i,M_m,j)}^t + s_{(M_m,i)} \right) X_{i,j,m} \\ & + \sum_{i,j,k \in A_m^s} (s_L + s_R) Y_{i,j,k,m} + \sum_{k \in V_{R,m}} W_{k,m} \end{aligned} \tag{6}$$

subject to:

$$\sum_{\substack{i \in \{1, \dots, j-1\} \\ i < j}} X_{i,j,m} + \sum_{i \in \{1, \dots, j-1\}} \sum_{k \in \{j+1, \dots, L\}} Y_{i,j,k,m} = 1 \quad \forall j \in \{2, \dots, L-1\} \tag{7}$$

$$\sum_{\substack{i \in V_{L,m} \\ i < j}} X_{i,j,m} \leq 1 \quad \forall j \in V_{R,m} \tag{8}$$

$$\sum_{j,k \in A_m^s} Y_{i,j,k,m} \leq \sum_{i \in \{1, \dots, L\}} X_{i,j,m} \quad \forall i \in V_{L,m} \tag{9}$$

$$\sum_{i,j \in A_m^s} Y_{i,j,k,m} \leq \sum_{\substack{i \in \{1, \dots, L\} \\ j < k}} X_{j,k,m} \quad \forall k \in V_{R,m} \tag{10}$$

$$\left(t_{(M_m,i,M_m,j)}^d + s_{(M_m,j)} + t_{(M_m,j,M_m,k)}^d \right) Y_{i,j,k,m} \leq E_d \quad \forall (i, j, k) \in A_m^s \tag{11}$$

$$\sum_{q,r \in A_m^s} Y_{p,q,r,m} \leq \sum_{\substack{i,j,k \in A_m^s \\ i < p, k \leq p}} Y_{i,j,k,m} \quad \forall p > 0, p \in V_{L,m} \tag{12}$$

$$Y_{i,j,k,m} + Y_{p,q,r,m} \leq 1 \quad \forall (i, j, k) \in A_m^s, (p, q, r) \in A_m^s, \substack{i \leq p, k > p \\ q \neq j} \tag{13}$$

$$Y_{i,j,k,m} + Y_{p,q,r,m} \leq 1 \quad \forall (i, j, k) \in A_m^s, (p, q, r) \in A_m^s, \begin{matrix} i < r, k \geq r \\ q \neq j \end{matrix} \quad (14)$$

$$0 \leq \left(t_{(M_m,i,M_m,j)}^d + s_{(M_m,j)} + t_{(M_m,j,M_m,k)}^d \right) Y_{i,j,k,m} - \sum_{\substack{p \in V_{L,m} \\ p < q}} \sum_{\substack{q \in V_{R,m} \\ i \leq p, q \leq k}} \left(t_{(M_m,p,M_m,q)}^t + s_{(M_m,q)} \right) X_{p,q,m} \leq W_{k,m} \quad \forall (i, j, k) \in A_m^s \quad (15)$$

The objective function (6) aims to minimize the total completion time of tour m , by reducing the sum of travel time, setup time, and waiting time of the EV. Equation (7) ensures that each customer node is visited at least once, either by the EV or drone. Equation (8) sets the direction of an EV tour. Equations (9) and (10) guarantee that the launching and rendezvous nodes of a sortie of a certain drone are also visited by the corresponding EV. Equation (11) is the endurance constraint of drone sorties. Equations (12)–(14) guarantee the continuity of a drone sortie, such that the drone could not be launched before it is retrieved back from the previous sortie. Lastly, Eq. (15) defines the value of $W_{k,m}$.

4.2 Improvement Phase

The previous decomposition phase is executed iteratively for each cluster m . The solution of each cluster then corresponds to an E-TSPD solution, and obviously, those m E-TSPD solutions are together a feasible solution for an E-VRPD instance. This solution set can be used as an input for any improvement-based metaheuristics, such as variable neighborhood search, tabu search, or genetic algorithm [14]. Here, we are interested to examine the suitability of this two-stage concept for E-VRPD. Thus, we append a simple local-search improvement phase to further improve the quality of the solution produced by the decomposition phase. In this regard, we employ a *randomized variable neighborhood descent* (VND) with eight neighborhood moves: (1) swap node, (2) swap whole, (3) insertion node, (4) insertion whole, (5) reverse node, (6) reverse whole, (7) remove sortie node and (8) add sortie node. This VND procedure and its neighborhood moves are adopted from [21]. Additionally, interested readers can check the work of [22] to read the complete explanation of the VND procedure.

5 Numerical Experiments and Analysis

This section describes the numerical experiments executed in this study, alongside the results from these experiments.

Table 1. Summary of problem parameters

Parameter	Value	References
EV	Renault Kangoo Z.E	[24]
Drones	DJI Matrice 300 RTK	[25]
Charger	Alpitronic Hypercharger HYC 300 kW	[26]
$d_{i,j}^t$	Calculated by Manhattan distance	[2]
$d_{i,j}^d$	Calculated by Euclidean distance	[2]
s_i	1 min	[2]
E_t	33 kWh	[24]
E_d	55 min	[25]
v_t	35 mph	[3]
v_d	50 mph	[3]
h_t	0.159 kWh/km	[24]
h_d	0.01 kWh/min	[25]
R	300 kWh	[26]
s_L, s_R	1 min	[2]

5.1 Experiments Settings

All experiments were implemented on a personal computer with AMD Ryzen 5 1600 Six-core Processor 3.2 GHz, 16 GB DDR4 memory, NVIDIA Quadro K-1200 GPU and Windows 11 operating system. The algorithm is coded in Julia language and executed as a single-thread code. Both k-Means and Concorde solver were implemented using available wrapper packages in the Julia environment, while whenever a MILP formulation needs to be solved, we deploy Gurobi as a solver using the JuMP environment [23]. The runtime limit of Gurobi is set as 10 min (600s), while the improvement phase of our algorithm is executed for 3 min (180s). In addition, constraint checking for each E-VRPD solution is performed with a penalty function technique. In this regard, whenever a solution violates the E-VRPD constraints (i.e. drone endurance, continuity of tours and/or sorties), a large penalty value is incurred to the objective value.

5.2 Benchmark Problems

In order to evaluate the performance of our algorithm, we derived a set of test instances for E-VRPD. These instances are adapted from the popular E-VRP instances from [17], in which we take the location demography of the nodes (depot, recharging stations, and customer nodes). We generate 30 instances in total with various combination values of customer nodes $n \in \{5, 6, 7, 8, 10, 15, 20, 25, 30, 40\}$, recharging stations $r \in \{3, 4, 5, 7\}$, and available vehicles $f \in \{2, 3, 4\}$. Then, to develop a more realistic dataset for E-VRPD, we modify the instances with several parameters presented in Table 1, where several of them are based on real-life data (i.e. E_t , E_d , h_t , h_d , and R).

5.3 Comparison to Existing Techniques

Then, in order to show the relevancy of E-VRPD, we compare the E-VRPD solutions from our proposed algorithm with E-VRP solutions. This comparison is selected to gauge the gap of quality that can be attained from implementing drones into an E-VRP system. In this regard, the hybrid variable neighborhood search/tabu search (VNS/TS) algorithm [17] was selected as a solver for E-VRP.

We implemented VNS/TS by carefully following the design and parameter settings presented in [17]. Nevertheless, our pilot study showed that the quality of E-VRP solutions produced by VNS/TS is largely influenced by its initial solution. In the original design of [17], the initial solution of VNS/TS was produced with the classic nearest-neighbor algorithm. However, we note that [17] aimed to minimize total transportation cost in their study, while we consider to minimize total completion time instead. Due to the difference in these objective functions, in this study, we feed VNS/TS with S^{E-TSP} solutions to ensure the quality of the produced E-VRP solutions.

5.4 Experimental Results and Analysis

Here, we present and discuss the experiment results. These discussions are driven by the two research questions (RQs) listed below:

RQ 1: “Is the proposed algorithm effective for solving E-VRPD instances?”

RQ 2: “What are the benefits to obtain from implementing drones into an electric vehicles-based transportation system?”

First, Fig. 2 presents the full numerical experiment results. Each sub-graph in Fig. 2 corresponds to each instance, which is presented as a combination of $n-r-f$. From Fig. 2, we can observe the effectiveness of the proposed algorithm in finding quasi-optimal solutions for E-VRPD, as the final solution values from our algorithm are always superior to the E-VRP solution from VNS/TS for all instances. In addition, it is also observed that the solutions produced by the first phase (decomposition phase) of the proposed method are comparable to the E-VRP solutions. Nevertheless, from Fig. 2, it is clear that the effectiveness of our algorithm is enhanced by the addition of a local-search procedure in the second stage. By comparing the quality of the solutions produced by the first stage to those of the second stage, our experiments show that the presence of the VND procedure reduces the objective value by around 22% on average. This corresponds to the classic adage that optimizing each sub-system (E-TSPD tour of each vehicle) does not always correspond to the optimal solution of a complex system (full E-VRPD instances) [27].

Then, addressing the second RQ, Fig. 2 also shows that the inclusion of drones into an EV-based delivery system could improve the performance of the system. This is indicated by the reduction in the total completion time, which can be seen from the comparison between the quality of E-VRPD final solutions with the final solutions for E-VRP. In this regard, the total completion time of E-VRPD final solutions is consistently lower than E-VRP, as E-VRPD is able to exploit the faster travel time of drones to provide services to a set of drone-eligible customer

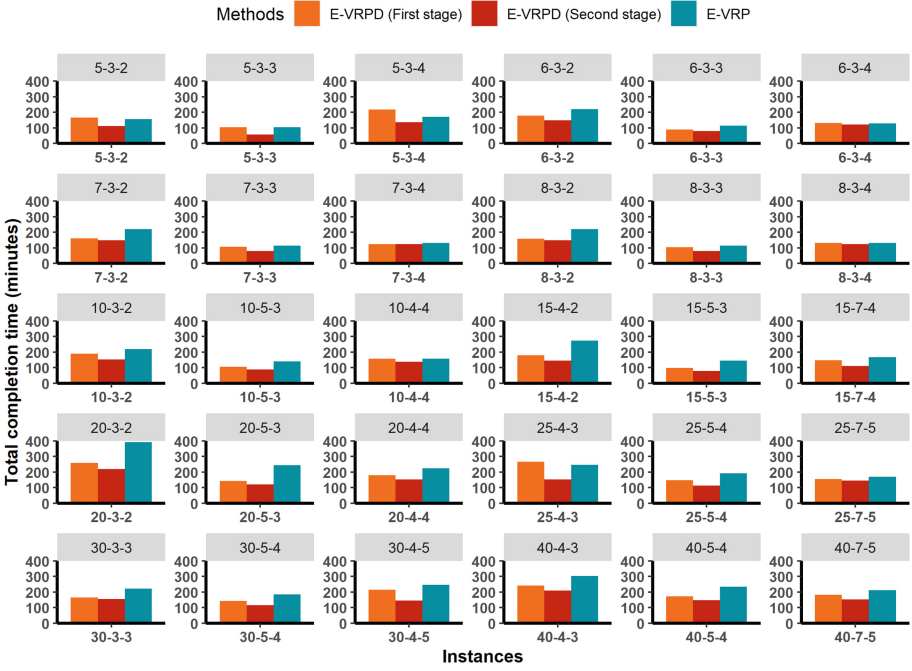


Fig. 2. Mean of total completion time for each method

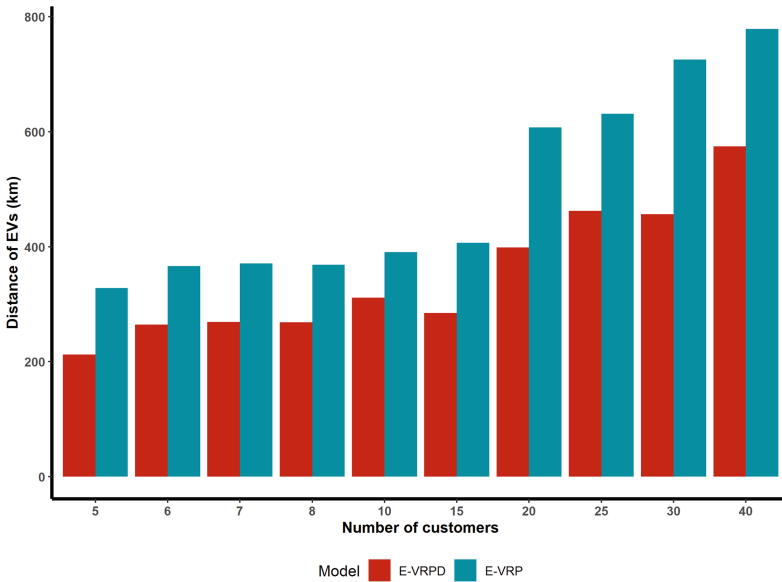


Fig. 3. Comparison of total distance traveled by EVs

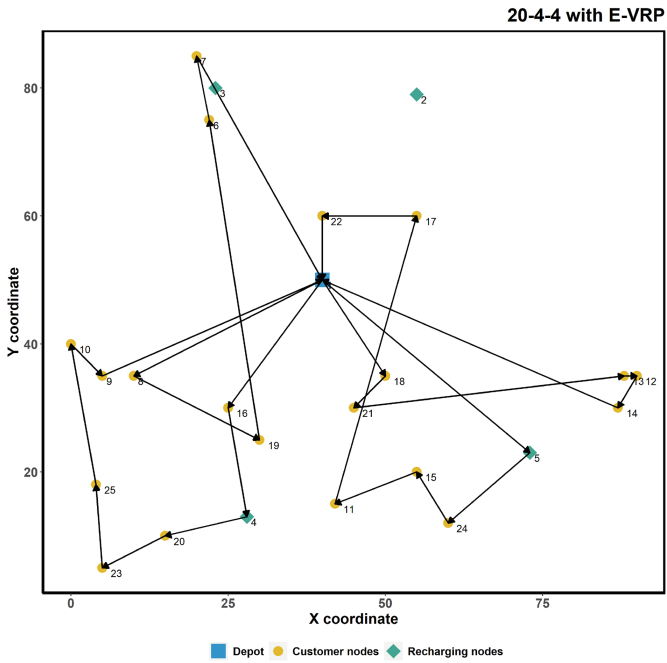
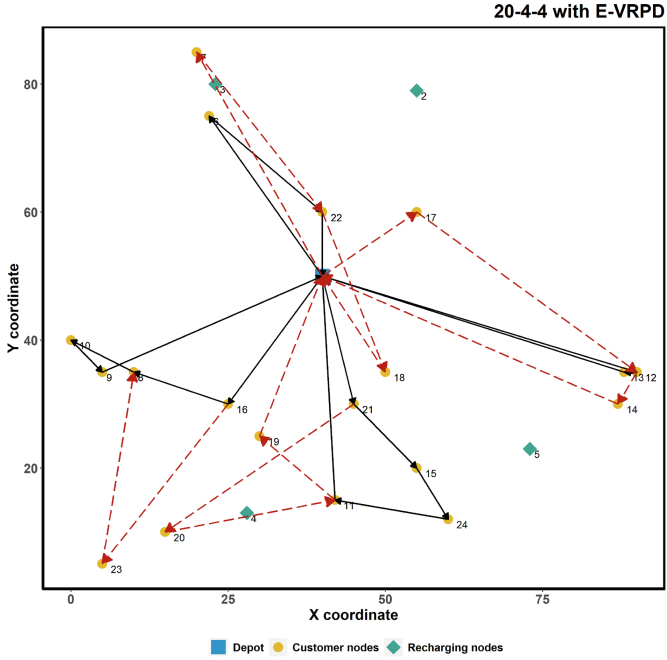


Fig. 4. Visualization of solutions

nodes. Furthermore, we also derive a bar plot to compare the travel distance of EVs from E-VRPD and E-VRP solutions in Fig. 3. From Fig. 3, it can be seen that the EVs always travel less distance in the E-VRPD solutions. This indicates that the presence of drones can reduce the distance that must be traveled by EVs, confirming the findings of previous works in VRPD [21]. In this regard, it is evident that the reduction in EVs' travel distance corresponds to the reduction of transportation costs, as the cost per distance of drones is generally still smaller than EVs (and obviously, ICEVs) [2]. Accordingly, this distance reduction may bring several benefits, such as the potential reduction in total energy usage and the number of charging visitations needed. Furthermore, Fig. 4 presents a visualization of E-VRPD and E-VRP solutions for instance '20-4-4', where the straight lines refer to a tour of an EV while the dashed lines refer to drone sorties. From Fig. 4, one can observe how drones reduce the working load of EVs by simultaneously delivering goods to drone-eligible nodes. A simple indication of this phenomenon is that the E-VRPD solution for instance '20-4-4' does not require any visitation to recharging stations. Meanwhile, the corresponding E-VRP solution for the given instance involves two recharging visits. Considering the burden of upfront investment for procuring a high-end recharging station [28] and the cost of using a third-party-based recharging station that is commonly larger than the basic electricity cost [29], this result indicates that implementing drones with EVs could be a viable solution for last-mile logistics.

6 Conclusions

This study considered the cooperation between EVs and drones in last-mile logistics. Here, a firm must deliver parcels to a set of customer nodes using a set of EV-drone tandems, where the EVs could visit a set of recharging stations to recharge a battery if needed. The main contribution of this study is to define E-VRPD with simultaneous coordination between EVs and drones and the limited energy of EVs. In addition, a sequential decomposition algorithm with an improvement phase is proposed as a solution approach. The algorithm works by decomposing E-VRPD into four sub-problems: clustering, traveling salesman problem, recharging insertion and drone insertion. The solution from this phase is then used as a starting point of the improvement phase with VND.

Our numerical experiments confirmed the effectiveness of the algorithm and showed the relevancy and potential benefits of implementing an E-VRPD system. In this regard, it has been shown that the inclusion of drones into EV-based delivery systems could improve system performance by reducing the total completion time. Future steps to further explore the E-VRPD system are to develop a mathematical formulation to model E-VRPD and compare it to an ICEV-based system from a financial perspective. Also, other extensions may be added to the model to reflect real-life scenarios.

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