



Assessing the Resilience of the Banking System in the Republic of Bulgaria Using Mathematical Models

Metodi Traykov¹  and Miglena Trencheva² 

¹ New Bulgarian University, Sofia, Bulgaria
mtraykov@nbu.bg

² South-West University “Neofit Rilski”, Blagoevgrad, Bulgaria
megy_tr2001@swu.bg

Abstract. This paper provides an in-depth evaluation of the resilience and stability of the banking system in the Republic of Bulgaria, utilizing innovative mathematical models for analysis. The study focuses on the period from 2016 to 2022, a time characterized by significant economic and financial fluctuations worldwide.

The research presents a rigorous assessment of the Bulgarian banking system’s stability, emphasizing the application of mathematical models including stress testing, factor analysis, and stochastic modeling. These models are employed to evaluate the systemic risk factors and determine the banking system’s resilience under various stress scenarios.

Our findings reveal that the Bulgarian banking system demonstrates notable resilience despite the challenging economic conditions. We attribute this robustness primarily to effective regulatory measures, responsible banking practices, and the system’s inherent structural strengths.

Nonetheless, the study also identifies potential vulnerabilities and provides strategic recommendations for further bolstering the system’s resilience. This research has significant implications for banking regulators, financial institutions, and policymakers, contributing to a more comprehensive understanding of the banking system’s resilience, thereby informing future regulatory and policy decisions.

Keywords: Banking System · Mathematical Models · resilience · Stability

1 Introduction

Capital adequacy is a key characteristic of the banking system because it gives investors and depositors confidence in the “safety and stability” of the banking system. Also, capital adequacy is important because the changes in the levels of capital adequacy act as a “brake” or “accelerator” for economic growth. The establishment of banking supervision ensures that the banks have adequate capital for their transaction matures.

Increasing the minimum capital threshold will guarantee that the banks will have more capital for their deposits, but this will slow the economy by limiting the volume of lending by banks. Decreasing the regulatory minimum capital will make the banks more competitive by using more aggressive lending, but it will decrease the overall confidence in the banking system's ability to withstand potential business cycle downturns [1–5].

In a fractional banking system, banks are required to keep a small portion of every borrowed lev on hand. This basic characteristic of the “fractional banking system” is known as the multiplier effect. In other words, the initial deposit creates many investments and loans. Determining capital levels in the banking system is analogous to using the gas pedal or brake in a car; increasing capital levels will lead to increased security at the expense of slowing economic activity. So, raising or lowering interest rates (or manipulating foreign exchange rates between currencies) affects the level of economic activity and decreases or increases the level of business and household economic growth in the economy as a whole [3, 5].

To achieve the “safety and stability” standards in the U.S., banks follow the CAMEL standard which has two main aspects:

1. An assessment of the banking experts using five key aspects of banking conditions and operations:
 - Capital adequacy;
 - Asset quality;
 - Management;
 - Earnings;
 - Liquidity;
2. A grade (based on a scale of 1 to 5) of banking stability and conditions according to mentioned above categories.

In addition, the Federal Reserve uses a complex rating system to evaluate banks by the following elements: earnings (E), subsidiary companies (P), bank subsidiaries (B), capital adequacy (consolidated), and other (non-bank) subsidiaries (O).

Banks are very flexible companies and they aim to maximize their profits using financial innovations. They adapt deposits and loans to the needs of firms and households. Competitiveness in deposits from customers requires paying the highest interest rate for a given maturity, and competitiveness in lending requires providing credit with the lowest interest rates [6].

The main focus of the article is on the interaction between the concepts of global financial stability and economic competitiveness. Also, we will consider the role of the Basel Agreement in harmonizing global standards and capital adequacy in the banking industry.

Capital adequacy management is a direct tool. The regulation of capital, by its nature, benefits some banks while reducing the competitiveness of others. The issues here pertain to the comparison between small banks and those involved with small businesses, mortgages, and consumers and corporations. However, the “harmonization” of international regulations lacks a clear academic thesis: regulating capital in the banking system is similar to regulating the economic activity in an individual economy. As a result of the global financial crisis (GFC), we already know that regulating the capital adequacy of banks is not sufficient to ensure their safety, and stability [7–10].

The Basel process is an essential and constructive element in achieving global financial stability, especially after the abandonment of the Bretton Woods system and the increasing deregulation of international capital flows. The agreements have contributed to the stability of the international banking system. Also, they have participated in the market to create financial risks in new markets and avoid regulations [11].

The Basel Agreement is often used in managing the global financial system and coordinating efforts necessary to achieve global financial stability between Ministries of Finance, central banks, and regulators. The Basel Agreement essentially represents a series of documents that are constantly under review. The first agreements and basic principles were established on the principle of the “home country”. The second set of documents relates to the harmonization of national standards for maintaining adequate capital in the banking sector [12–15].

Numerous violations have been established, some of which are given to national regulations by the “Right of Discretion”, to apply regulatory regulation or even to ignore it [16, 17].

2 Methodology

Mark Flannery says “There is a balancing act that politicians must perform between creating a truly safe system and maintaining its applicability. If you restrict banks too much, financial activity will decrease or shift to non-bank institutions, rendering the rules ineffective”. Mark Flannery has been tracking Basel regulations for two decades. He is a professor at the University of Florida in Gainesville.

In order to meet the needs of clients, banks adapt financial products or create entirely new financial instruments. Financial regulators need to be informed about such activities to assess whether new or modified guidelines and regulations will ensure standards of stability and safety. Banks develop financial instruments to trade on private interbank markets or public exchanges. They respond to competitive challenges and invest all the funds they control by reducing the amount of money held unused and required to be held in reserve, as per the requirements of banking regulators [18–20].

Bank competition is a process of reducing the advantages of business competitors (or increasing one’s advantages) at the global, national, or local level. In the banking sector, increasing someone’s competitiveness also affects the attractiveness of the bank as a target for acquisition. Other appropriate terms are “pro-competitive” and “anti-competitive” [21–26].

Let’s say that we are an investor faced with a stream of known future obligations. We want to own a portfolio of bonds through which we can meet these obligations. We would like to do this at minimal costs, but we also want to own a portfolio that is unlikely to encounter problems in case of changes in interest rates. Let’s have one obligation L , which must be paid over the last 5 years. If we can find safe zero-coupon bonds with a 5-year maturity with a face value of F , we can simply buy L/F bonds. In this case, we have two options, namely to face reinvestment risk (If the maturity of the bonds <5 years), and to face interest rate risk (if the maturity of the bonds >5 years). The perfect case for us is to find a zero-coupon bond that has a maturity equal to the date of each obligation. Unfortunately, this is impossible, and we need to find another way to protect our bond portfolio. A possible solution is immunization.

Let $P(\lambda)$ be a function that gives us a relation between the price of the bond and the yield.

Given a stream of cash flows at times t_0, t_1, \dots, t_n , then the duration of the stream can be define as follow:

$$D = \frac{PV(t_0)t_0 + PV(t_1)t_1 + PV(t_2)t_2 + \dots + PV(t_n)t_n}{PV}, \quad (1)$$

where [27, 28]:

- PV – the value of the entire stream;
- $PV(t_i)$ - the cash flow at a time $i = 0, 1, \dots, n$.

The duration appears as a weighted average of the times of the cash flows. The duration of a zero-coupon bond is the time to maturity. Using the yield as a discount rate to calculate the present values in a general bond, we will obtain Macaulay duration:

$$D = \frac{\sum_{k=1}^n \frac{k}{m} \frac{c_k}{(1+\lambda/m)^k}}{\sum_{k=1}^n \frac{c_k}{(1+\lambda/m)^k}}, \quad (2)$$

where m is coupon payments per year. In order to understand why the duration is useful, we must calculate the derivative of the price concerning the yield [28]:

$$\begin{aligned} \frac{dP}{d\lambda} &= \frac{d}{d\lambda} \left(\sum_{k=1}^n \frac{c_k}{(1+\lambda/m)^k} \right) = \sum_{k=1}^n c_k \frac{d}{d\lambda} \left[\frac{1}{(1+\lambda/m)^k} \right] \\ &= - \sum_{k=1}^n \frac{k}{m} \frac{c_k}{(1+\lambda/m)^{k+1}}. \end{aligned} \quad (3)$$

If the modified duration (D_m) is $D/(1+\lambda/m)$, then

$$\frac{dP}{d\lambda} = -D_m P. \quad (4)$$

The D_m is related to the slope of the curve, representing the yield. To obtain this curve we must use the prices at a given point in time. In our context, this is the price elasticity of the bond concerning changes in yield. In this case, we can use a first-order approximation:

$$\delta P \approx -D_m P \delta \lambda. \quad (5)$$

We can get an even better approximation with the help of a second-order approximation [28–30]. This is done by defining convexity:

$$C = \frac{1}{P} \frac{d^2 P}{d\lambda^2}. \quad (6)$$

So, we have

$$C = \frac{1}{P(1+\frac{\lambda}{m})^2} \sum_{k=1}^n \frac{k(k+1)}{m^2} \frac{c_k}{(1+\frac{\lambda}{m})^k} \quad (7)$$

The unit of measure for convexity is time squared. Here the slower decrease in value when the required return increases is represented by greater convexity. Also, if the required return decreases, then the greater convexity means a faster increase in the corresponding value. So, the second-order approximation is based on both convexity and duration:

$$\delta P \approx -D_M P \delta \lambda + \frac{PC}{2} (\delta \lambda)^2. \tag{8}$$

If the yield is the same for all bonds, it can be shown that the duration of the portfolio is simply the weighted average of all durations [25–31].

3 Result

Due to the complexity of portfolios held by financial institutions, standardized methods for measuring the risk to which a bank may be exposed are necessary, especially for those banks that play major roles in the financial markets. Complex financial instruments such as options, and swaps, as well as various types of structured loans, collateralized mortgage obligations (CMOs), and collateralized debt obligations (CDOs), are required. The different asset classes within a portfolio have different risk evaluation methods, and combining these risks in a meaningful way can be challenging (see Fig. 1, 2 and 3). It is natural to determine firm-wide risk using the distribution of potential losses at a fixed future moment (each unit within a financial institution contributes to profit or loss). This will give us a meaningful measure of firm-wide risk, which provides the value of the risk.

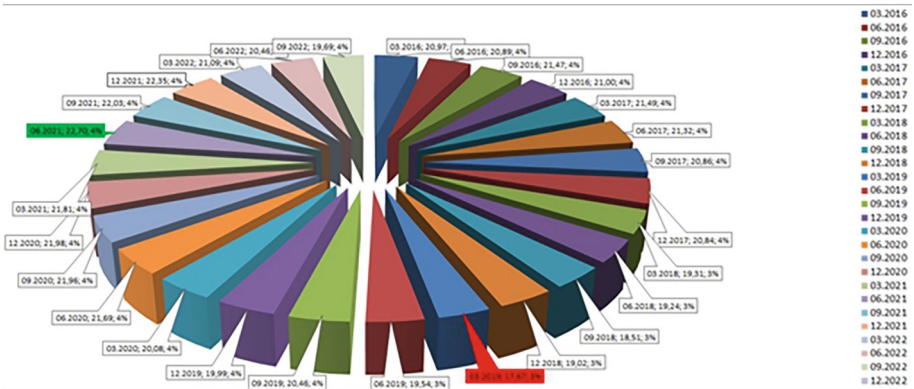


Fig. 1. Base Tier 1 Equity Ratio (%) In First Group

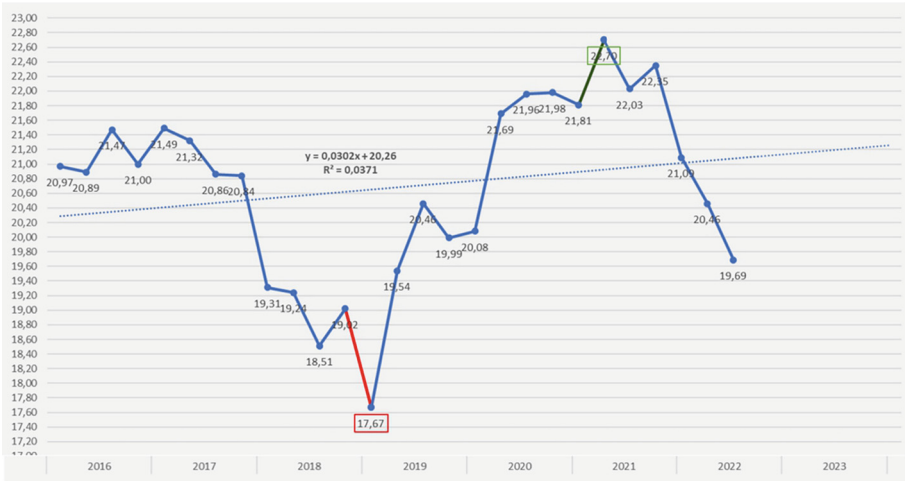


Fig. 2. The estimated value for the ratio of the basic equity of Tier 1 (%) for the end of 2023 is 21.23.

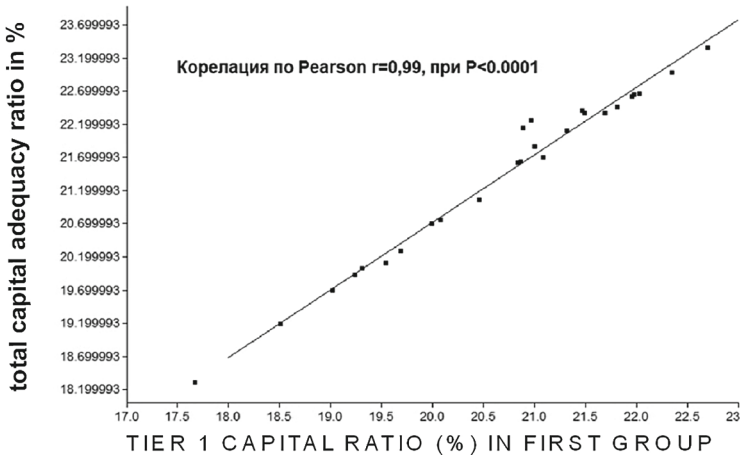


Fig. 3. Correlation between base tier 1 equity ratio (%) and total capital adequacy ratio (%)

3.1 Multiple Regression Analysis

A model ($z = f(x, y)$) of the dependence of Financial result (FR, z in the model) on Credits (Cr, x in the model) and receivables from the non-governmental sector (RNS, y in the model) was found:

A model ($z = f(x, y)$) of the dependence of Fixed Capital (CS, z in the model) on Credits (Cr, x in the model) and receivables from the non-governmental sector (RNS, y in the model) was found:

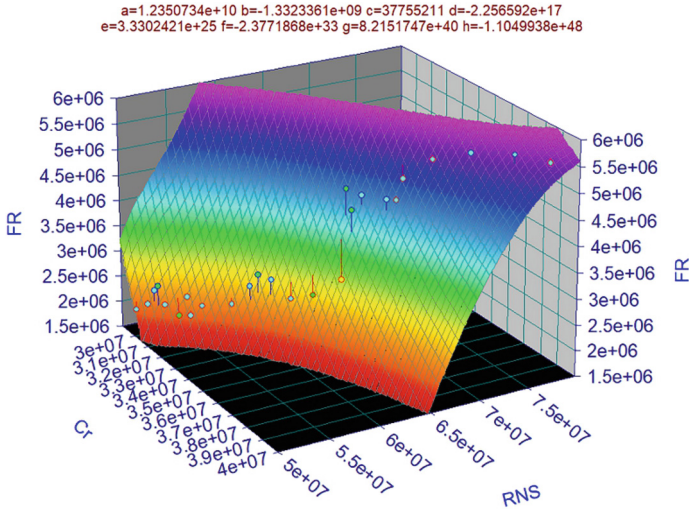


Fig. 4. Model of the dependence on Financial result

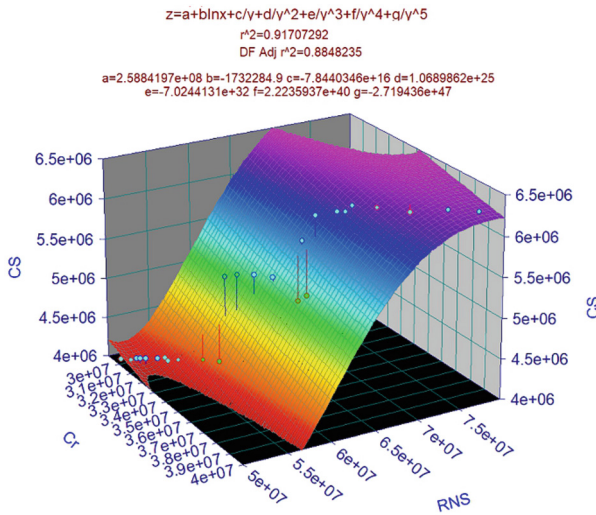


Fig. 5. Model of the dependence of Fixed Capital

A model ($z = f(x, y)$) of the dependence of capital and reserves (*CandR*, z in the model) on Credits (Cr , x in the model) and receivables from the non-governmental sector (RNS, y in the model) was found:

The banking system of the Republic of Bulgaria has demonstrated a notable degree of resilience in the past eight years. Systemic stability has been a critical focus, considering the interconnected nature of financial institutions and their collective exposure to market risk. The application of regression analysis and stress testing has been instrumental in

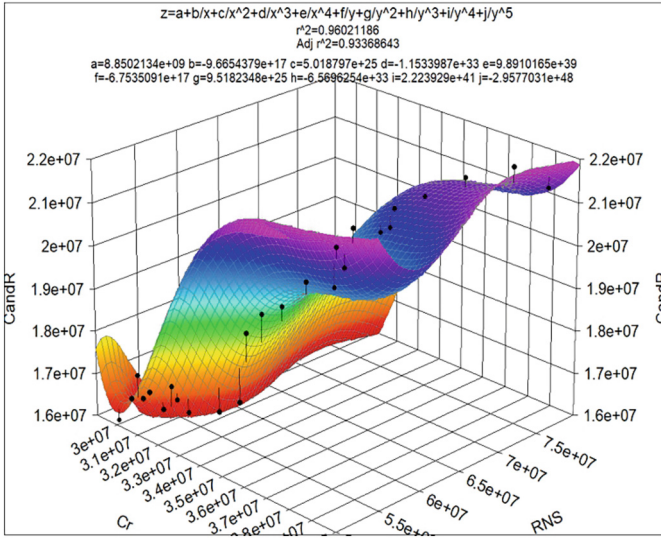


Fig. 6. Model of the dependence of capital and reserves

quantifying this stability. By simulating numerous hypothetical scenarios, these tools have helped in assessing the potential impact of various risk factors on the banking sector (see Fig. 1, 2, 3 and 4).

The Bulgarian banking sector has also showcased considerable progress in credit management, which is a crucial aspect of banking stability. The ratio of non-performing loans has seen a significant reduction, reflecting the effectiveness of credit risk management strategies. The use of credit scoring models and machine learning algorithms has been instrumental in this regard. These technologies have enabled banks to predict borrower behavior with greater accuracy, thereby facilitating more informed lending decisions.

In terms of liquidity, Bulgarian banks have maintained adequate levels of financial reserves. This factor is crucial for ensuring that banks can meet their short-term obligations. Time series analysis and other mathematical models have been used to forecast future liquidity needs and thereby maintain financial solvency.

Moreover, capital adequacy, measured by the Capital Adequacy Ratio (CAR), has remained well above the regulatory minimum. This indicates that Bulgarian banks possess sufficient capital to absorb a reasonable amount of losses before becoming insolvent.

In conclusion, the Bulgarian banking system has demonstrated commendable stability over the past eight years, as evidenced by robust systemic stability, effective credit management, adequate liquidity, and high capital adequacy. These factors, combined with the innovative use of mathematical models and techniques, suggest a positive trajectory for the Bulgarian banking sector moving forward.

It is important to differentiate the Capital Adequacy Requirement (CAR), and the Capital Reserve Requirements (CRR) of a bank. CAR represents how banks should

manage their capital or how they should finance themselves, particularly in terms of the mix between debt and equity on their balance sheets. CRR represents the minimum reserve that each bank must hold (see Fig. 4 and Fig. 5).

The mandatory minimum reserves, or liquidity requirements, represent a small portion of the deposits that need to be held in highly liquid forms, such as cash. The cash reserve ratio is the ratio of reserves in the form of physical cash, precious metals, or government notes. Here we want to note that liquid securities enable banks to adjust their portfolio by holding government securities at any time without affecting profitability (see Fig. 6).

These two concepts are often opposing. Capital requirements are a ratio of assets to certain types of liabilities or capital. On the other hand, reserve requirements are a ratio of certain obligations held against certain types of assets.

Our regression model confirms that the Modigliani-Miller theorem is valid for the banking sector in the US, as more leverage leads to an increase in the beta coefficient of the banking system. To see the link between the capital asset pricing model and the Modigliani-Miller theorem, we would need to trace this effect using the required return on equity. We may achieve this using our regression model:

$$\text{Cost of Equity} = \text{Risk - Free Rate} + \beta_{\text{equity}} * \text{Market Risk Premium}$$

$$\text{Cost of Equity} = \text{Risk - Free Rate} + (\beta_2 * \text{leverage}) * \text{Market Risk Premium}$$

3.2 Conclusion

Our study shows that an increase in dividend payout symbolizes a decrease in firms' cost of potential loss in capital markets and thus increases investor confidence in the firm. As a result, the dividend payout ratio should have a positive relationship with leverage. The dividend payout in relation to the pecking order theory is considered related to the financial deficit. This means that an increase in paid dividends is due to the greater need for external financing. Following the pecking order theory, debt financing is first in line to meet these needs, and therefore the level of leverage increases.

Acknowledgements. This work supported by the project of the Bulgarian National Science Fund (Ministry of Education and Science of the Republic of Bulgaria), entitled: "Analysis of the main indicators and regulators for banking activities in the Republic of Bulgaria", contract N° KII-06-M35/3 from 30.12.2019, led by Assist. Prof. Miglena Trencheva.

References

1. Abogun, S., Olaniyi, T., Ijaiya, M., Fagbemi, T.: Earnings persistence of Nigerian listed banks. *J. Siasat Bisnis* **24**(2), 168–178 (2020)
2. Astuti, Y.: Analisis CAR dan ISR terhadap ROA Perbankan Syariah yang Terdaftar di JII Periode 2015–2019. *Mabsya* **3**, 116–127 (2021)

3. Aswal, D., Sharma, D.: Determinants of stock prices of the banking sector with reference to private sector banks. *Analele Universităţii Constantin Brâncuşi din Târgu Jiu : Seria Economie* **5**, 5–14 (2020)
4. Scherer, B., Martin, D.: *Modern Portfolio Optimization with NuOPT™, S-PLUS®, and S+Bayes™*. Springer, New York (2005). <https://doi.org/10.1007/978-0-387-27586-4>
5. Djuraidah, A., Silvianti, P., Yaman, A.: Analisis Risiko Operasional Bank XXX dengan Metode Teori Nilai Ekstrim. *Statistika* **11**(2), 115–126 (2011)
6. Filip, B.: A comparative analysis on banking systems' profitability between western European and cee countries. *J. Public Adm. Finan. Law* **5**, 168–181 (2016)
7. Gîrlea, M.: Modelul de business bancar: tendinţe actuale. *Studia Universitatis Moldaviae: Stiinte Exacte si Economice. Studia universitatis moldaviae* **2**(102), 102–107 (2017)
8. Hartono, A.: Pengukuran kinerja keuangan dengan metode eagles (Studi Kasus Pada Bank BUMN Yang Listing Di BEI Tahun 2011–2013). *Ekuilibrium: J. Ilmiah Bidan Ilmu Ekonomi* **10**, 55–68 (2016)
9. Horbachov, P., Makarichev, A.: Estimation of delay on signalized intersections of urban streets with a three-phase signal. *Avtomobil'nyj Transport (Har'kov)* **44**, 30–39 (2019)
10. Isayas, Y.: Determinants of banks' profitability: empirical evidence from banks in Ethiopia. *Cogent Econ. Finan.* **10**, 2031433 (2022)
11. Kartal, M.: Yeniden Yapılandırma Çalışmalarının Bankacılık Sektörünün Gelişimine Etkileri: Türkiye İncelemesi (effects of restructuring efforts to the development of banking sector: Turkey examination). *İşletme Araştırmaları Dergisi* **11**, 3172–3189 (2019)
12. Katranzhy, L., Podskrebko, O., Krasko, V.: Modelling the dynamics of the adequacy of bank's regulatory capital. *Baltic J. Econ. Stud.* **4**, 188–194 (2018)
13. Klepczarek, E.: Determinants of European banks' capital adequacy. *Comp. Econ. Res.* **18**, 81–98 (2015)
14. Kozarević, E., Polić, N., Perić, A.: Financial system development progress in western Balkans. *Banks Bank Syst.* **12**, 7–19 (2017)
15. Hall, M.: Implementation of the BIS “rules” on capital adequacy assessment. A comparative study of the approaches adopted in the UK, the USA and Japan. *PSL Q. Rev.* **45**, (2013)
16. Minasyan, V.: New ways to measure catastrophic financial risks: “VaR to the power of t” Measures and How to Calculate Them. *Финансы: теория и практика* **24**, 92–109 (2020)
17. Taleb, N.: *Dynamic Hedging: Managing Vanilla and Exotic Options*. Wiley, New York (1996)
18. Nguyen, V., Liu, D.: The impact of ownership structure on Vietnamese commercial banks' profitability. *Int. J. Econ. Financ. Issues* **10**(3), 187–194 (2020)
19. Noreen, U., Alamdar, F., Tariq, T.: Capital buffers and bank risk: empirical study of adjustment of Pakistani banks. *Int. J. Econ. Financ. Issues* **6**, 1798–1806 (2016)
20. Pambuko, Z., Pramesti, D.: The effectiveness of bank aceh syariah conversion decisions. *Economica: J. Ekon. Islam* **11**, 1–23 (2020)
21. Pantos, S.: Designing stress tests for UK fast-growing firms and Fintech. *Risks* **11**(2), 1–22 (2023)
22. Rebonato, R.: *Interest-Rate Option Model*, 2nd edn. Wiley, Chichester (2000)
23. Rebonato, R.: *Modern Pricing of Interest-Rate Derivatives: The LIBOR Market Model and Beyond*. Princeton University Press, Princeton (2002)
24. Rahman, M., Chowdhury, A., Mouri, D.: Relationship between risk-taking, capital regulation and bank performance: empirical evidence from Bangladesh. *Eurasian J. Bus. Econ.* **11**, 29–57 (2018)
25. Ranka, M., Milica, N., Marijana, L.: Relationship between macroeconomic aggregates and bank performance. *Megatrend Revija* **13**, 131–146 (2016)
26. Rehman, Z., Muhammad, N., Sarwar, B., Raz, M.: Impact of risk management strategies on the credit risk faced by commercial banks of Balochistan. *Financ. Innov.* **5**, 1–13 (2019)

27. Rosawati, Y., Pinem, D.: Pengaruh dana pihak ketiga, permodalan, aktiva produktif dan likuiditas terhadap jumlah penyaluran kredit perbankan. *Ekon. dan Bisnis* **4**, 157–172 (2017)
28. Shreve, S.: *Stochastic Calculus for Finance*, vol. I & II. Springer, New York (2003). <https://doi.org/10.1007/978-0-387-22527-2>
29. Sundaresan, S.: *Fixed Income Markets and Their Derivatives*. South Western College Publishing, Cincinnati (1997)
30. Setiawan, R., Putri, N., Rachmansyah, A.: Determinant net interest margin pada bank perkreditan rakyat indonesia. *J. Riset Bisnis dan Manajemen* **12**, 1–9 (2019)
31. Susila, G.: Pengaruh kualitas aktiva produktif, capital adequacy ratio, dan loan to deposit ratio terhadap profitabilitas pada lembaga perkreditan desa. *J. Ilmu Sosial dan Humaniora* **6**, 108–114 (2017)
32. Sutrisno, S.: Risiko, Efisiensi dan Kinerja pada Bank Konvensional di Indonesia. *J. Ilmiah Akuntansi dan Bisnis*, 111–116 (2017)
33. Visita, L.: The effect of inflation, profit-loss sharing loan, and capital adequacy towards performance of Indonesian Islamic banks. *DIJB (Diponegoro Int. J. Bus.)* **2**, 57–63 (2019)
34. Wang, S., Liu, Q., Yuksel, S., Dincer, H.: Hesitant linguistic term sets-based hybrid analysis for renewable energy investments. *IEEE Access* **7**, 114223–114235 (2019)
35. Yehorycheva, S., Kolodiziev, O., Prasolova, S.: Actual problems of the capital stability management in the Ukraine’s banking system. *Banks Bank Syst.* **12**, 60–67 (2017)