Tier-Based Directed Weighted Graph Coloring Algorithm for Device-to-Device Underlay Cellular Networks

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Abstract. Device-to-Device (D2D) communication has been recognized as a promising technology in 5G. Due to its short-range direct communication, D2D improves network capacity and spectral efficiency. However, interference management is more complex for D2D underlaying cellular networks compared with traditional cellular networks. In this paper, we study channel allocation in D2D underlaying cellular networks. A tier-based directed weighted graph coloring algorithm (TDWGCA) is proposed to solve cumulative interference problem. The proposed algorithm is composed of two stages. For the first stage, the tier-based directed weighted graph is constructed to formulate the interference relationship among users. For the second stage, the maximum potential interference based coloring algorithm (MPICA) is proposed to color the graph. Different from the hypergraph previously investigated in channel allocation, our proposed graph reduces the complexity of graph construction significantly. Simulation results show that the proposed algorithm could better eliminate cumulative interference compared with the hypergraph based algorithm and thus the system capacity is improved.

Keywords: Device-to-Device communication · Channel allocation · Graph coloring

1 Introduction

Data traffic in cellular network increases significantly in recent years, which gives large pressure to base stations (BS). Device-to-Device communication, where two communication devices in proximity communicate directly with each other without relaying by the base station, has been recognized as a promising technique in 5G. D2D could offload data traffic of base stations. A D2D pair could transmit data on a dedicated channel in the overlay mode or reuse the spectrum of

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cellular user equipment (CUE) in the underlay mode [1]. In this approach, cellular devices can coexist with D2D devices in the same licensed channel. D2D communication increases overall spectral efficiency, network capacity and energy efficiency due to its short-range direct data transmission and spectrum reusing gain. However, underlay D2D causes severe interference to both cellular devices and other D2D devices sharing the same channel [2]. Interference management is a challenging problem in D2D underlaying cellular networks. Many existing researches aim to decrease the mutual interference between devices using the same channel.

Stackelberg game model was used in [3], where power allocation is modeled as a noncooperative game. Authors in [4] proposed a channel allocation algorithm which enables collision-free concurrent transmission. Channel allocation is formulated as one-to-one and many-to-one matching games in [5]. The algorithm proposed in [6] adopts VGG auction model to sell channels under the constraint of interference. The authors in [7] changed the resource allocation problem into a maximum weighted independent set (MWIS) problem and proposed a low complexity and distributed greedy approximation algorithm, called DistGreedy algorithm to solve MWIS problem. The DistGreedy algorithm can better exploit the opportunistic gains under fading channels. The authors in [8] formulate a multiobjective optimization problem (MOOP) to maximize the energy efficiency. The MOOP maximizes the rate and minimize the total transmit power of D2D transmitters simultaneously.

Graph theory is a practical tool in resource allocation. The authors in [9] used weighted bipartite graph and proposed an interactive algorithm to solve channel assignment problem. In [10], the authors proposed a heuristic graph-coloring resource allocation (GOAL) algorithm. An interference graph-based resource allocation (inGRA) algorithm is proposed in [11]. The authors proposed a novel greedy-based coloring algorithm based on interference graph in [12].

Traditional graph coloring algorithms only consider pair-wise interference model. It cannot well model cumulative interference caused by multiple devices. Hypergraph interference model is adopted in [13] and [14] to formulate cumulative interference relationship. A hypergraph based coloring algorithm was proposed in [13]. It first recognizes weak interferers and strong interferers to construct the graph and then colors the graph in a greedy manner. A directed hypergraph based algorithm in [14] takes asymmetric interference into account and used a centralized-distributed learning algorithm for channel allocation. In [13] and [14], the complexity of the algorithms increases significantly as the number of users increases.

In this paper, we study channel allocation for D2D underlaying cellular networks and present a novel tier-based directed weighted graph coloring algorithm. The algorithm is composed of two stages. For the first stage, a tier-based directed weighted graph (TDWG) is constructed to formulate interference relationship between user equipments (UEs). Since the structure of our proposed graph is similar to the traditional graph, the complexity of graph construction is greatly reduced compared with the hypergraph in [13]. For the second stage, the maximum potential interference based coloring algorithm (MPICA) is proposed to color the graph which considers cumulative interference elimination. We calculate each UE's maximum potential interference in different channels and select the UE which might be interfered most severely in the unchecked set to color. Then we color the UE in a greedy manner. Cumulative interference is eliminated better compared with the hypergraph algorithm and thus the network capacity increases.

The following sections are organized as follows. We introduce the system model and formulate the problem in Sect. 2. The proposed algorithm is presented in Sect. 3. Theoretical analysis are presented in Sect. 4. Simulation results are provided in Sect. 5 and conclusions are drawn in Sect. 6.

2 System Model and Problem Formulation

2.1 System Model

As shown in Fig. 1, we consider an isolated cellular network. The base station (BS) is located at the center. There are M D2D pairs and K cellular user equipments (CUEs) randomly located in the network. The set of D2D pairs is denoted by $D = [D_1, D_2, ..., D_M]$. The set of CUEs is denoted by $C = [C_1, C_2, ..., C_K]$. D_i^t and D_i^r represent the transmitter and receiver of D_i respectively. There are a total of N RBs, denoted by $RB = [RB_1, RB_2, ..., RB_N]$. Each RB occupies the same number of subcarriers. For simplicity, we use D_i or $i \in D$ to both denote the same D2D pair, C_k or $k \in C$ to denote the same CUE and channel n to denote RB_n.

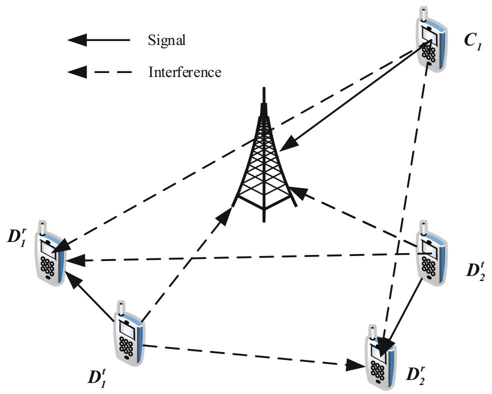

Fig. 1. System model for D2D communications underlaying cellular networks when sharing uplink resource.

If RB_n is allocated to C_k, the instantaneous signal-to-interference-plus-noise Ratio (SINR) of C_k on RB_n is denoted by

$$\gamma_{k,n}^c = \frac{P_{k,n}^c h_{k,b}^c}{\sum\limits_{i\in\phi_n, i\in D} P_{i,n}^d h_{i,b}^d + \sigma^2} \tag{1}$$

where $P_{k,n}^c$ represents the transmit power of C_k on RB_n, $h_{k,b}^c$ represents the channel gain of the cellular communication link from C_k to BS, ϕ_n represents the set of UEs which RB_n is allocated to, $P_{i,n}^d$ represents the transmit power of D_i^t and $h_{i,b}^d$ represents the channel gain of the interference link from D_i^t to BS. The thermal noise satisfies independent Gaussian distribution with zero mean and variance σ^2.

If RB_n is allocated to D_i, the instantaneous SINR of D_i on RB_n is denoted by

$$\gamma_{i,n}^d = \frac{P_{i,n}^d h_{i,i}^d}{\sum\limits_{k\in\phi_n, k\in C} P_{k,n}^c h_{k,i}^c + \sum\limits_{j\in\phi_n, j\in D, j\neq i} P_{j,n}^d h_{j,i}^d + \sigma^2} \tag{2}$$

where $h_{i,i}^d$ represents the channel gain of D2D communication link from D_i^t to D_i^r, $h_{k,i}^c$ represents the channel gain of interference link from C_k to D_i^r and $h_{j,i}^d$ represents the channel gain of interference link from D_j^t to D_i^r.

2.2 Problem Formulation

We assume that a CUE could utilize at most one RB and different CUEs could not share the same RB. CUEs wouldn't interfere with each other due to the characteristic of OFDM system. Consider the scenario where the number of D2D pairs is greater than that of CUEs. We investigate the case where a D2D pair could occupy at most one RB, but one RB could be allocated to multiple D2D pairs. Denote $\beta_{k,n}$ and $x_{i,n}$ as

$$\beta_{k,n} = \begin{cases} 1 & when\ RB_n\ is\ allocated\ to\ C_k \\ 0 & otherwise \end{cases} \tag{3}$$

$$x_{i,n} = \begin{cases} 1 & when\ RB_n\ is\ allocated\ to\ D_i \\ 0 & otherwise \end{cases} \tag{4}$$

Our objective is to maximize the network capacity by designing efficient algorithm with low complexity. Shannon capacity formula is used here to evaluate the network capacity.

$$max \sum_{n\in N} [\sum_{i\in D} log_2(1 + x_{i,n}\gamma_{i,n}^d) + \sum_{k\in C} log_2(1 + \beta_{k,n}\gamma_{k,n}^c)] \tag{5}$$

$$C1 : \sum_{n\in N} x_{i,n} \leq 1 \quad for\ \forall i \in D$$

$$C2: \sum_{n \in N} \beta_{k,n} \leq 1 \quad for\ \forall k \in C$$

$$C3: \sum_{k \in C} \beta_{k,n} \leq 1 \quad for\ \forall n \in RB$$

where $\gamma^c_{k,n}$ and $\gamma^d_{i,n}$ are given in (1) and (2) respectively. $C1$ and $C2$ implies that one D2D pair and one CUE could occupy at most one RB respectively. $C3$ means that one RB could be allocated to at most one CUE.

Note that the problem in (5) is NP-hard, which means we could not obtain the optimal result in polynomial time. We need to design an approximate algorithm with low complexity to solve the problem. In the following section, an improved graph coloring algorithm is presented.

3 Tier-Based Directed Weighted Graph Coloring Algorithm

In this section, we formulate the channel allocation problem as a coloring problem and propose a tier-based directed weighted graph coloring algorithm. We first present how to construct the tier-based directed weighted graph to recognize weak interferers with low complexity. Then we color the graph in a greedy manner which considers cumulative interference elimination. We assume that each UE only has local information.

3.1 Tier-Based Directed Weighted Graph Construction

The first step is to construct a tier-based directed weighted graph which corresponds to the network interference condition.

The graph is denoted by $G(V, E, W)$. Each CUE or D2D pair is represented by a vertex in the graph, the set of vertices is denoted by V. $E = [e_{i,j}]$ is the set of edges and $e_{i,j}$ denotes the directed edge from v_i to v_j. W is defined as $W = [w_{i,j}]$, where $w_{i,j}$ is the weight of $e_{i,j}$. Note that $w_{i,j}$ and $w_{j,i}$ might have different values due to the asymmetric interference effect.

It is assumed that C_k have local information and D_i is within its sensing range.

$$\eta_c q \leq \frac{P^c_k h^c_{k,b}}{P^d_i h^d_{i,b}} < \eta_c (q+1) \quad for\ i \in D \tag{6}$$

where η_c denotes the SINR threshold of CUE. P^c_k is the transmit power of C_k and P^d_i is the transmit power of D^t_i.

$q = floor(\frac{P^c_k h^c_{k,b}}{P^d_i h^d_{i,b} \eta_c})$ is derived from (6). If $q \in [0, Q-1]$, let $D_i \in L^k_q$ and form a directed edge $e_{i,k}$ from v_i to v_k with weight $w_{i,k} = \frac{1}{q+1}$. L^k_q is defined as interference q-tier. If we randomly select $(q+1)$ UEs from L^i_q and let them share the same channel with C_k, they together would cause strong cumulative interference to C_k.

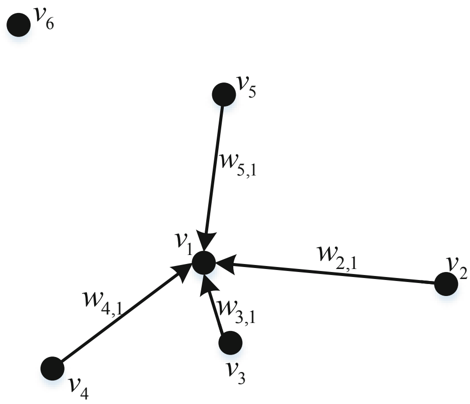

Fig. 2. A demonstration of directed edge formulation between UE1 and its neighboring nodes

Two CUEs could not share the same channel. For $\forall l \in C, l \neq k$, always form an edge from v_l to v_k with weight $w_{l,k} = 1$ and let $C_l \in L_0^k$.

Define $L^k = \bigcup_q L_q^k$. The set of neighboring nodes of vertex v_i is defined as $N(i) = \{j|w_{i,j} > 0\} \bigcup \{j|w_{j,i} > 0\}$.

For the receiver of a D2D pair, denoted by D_i^r, we define the following equations,

$$\eta_d q \leq \frac{P_i^d h_{i,i}^d}{P_j^d h_{j,i}^d} < \eta_d(q+1) \quad for\ j \in D \tag{7}$$

$$\eta_d q \leq \frac{P_i^d h_{i,i}^d}{P_k^c h_{k,i}^c} < \eta_d(q+1) \quad for\ k \in C \tag{8}$$

where η_d denotes the SINR threshold of a D2D pair.

From (7), we could calculate $q = floor(\frac{P_i^d h_{i,i}^d}{P_j^d h_{j,i}^d \eta_d})$. If $q \in [0, Q-1]$, let $D_j \in L_q^i$, and form a directed edge $e_{j,i}$ from v_j to v_i with weight $w_{j,i} = \frac{1}{q+1}$.

Likewise, $q = floor(\frac{P_i^d h_{i,i}^d}{P_k^c h_{k,i}^c \eta_d})$ is derived from (8). If $q \in [0, Q-1]$, let $C_k \in L_q^i$, and form a directed edge $e_{k,i}$ from v_k to v_i with weight $w_{k,i} = \frac{1}{q+1}$.

Figure 2 shows an example of edge formulation between UE1 and its neighboring nodes v_2, v_3, v_4, v_5. Neighboring nodes of v_1 are the strong interferers or weak interferers to v_1. v_6 is out of the sensing range of v_1.

It is worth mentioning that Q is a constant and the value of Q is optional. The network capacity will increase as Q increases. Different from the hypergraph based algorithm in [13], which goes through all the combinations of Q UEs to determine whether they cause cumulative interference to the specific UE, the proposed graph has a similar structure as the traditional graph. The complexity of graph construction wouldn't increase significantly as Q increases, thus reduces the complexity. Detailed complexity analysis is presented in Sect. 4.

Algorithm 1. Tier-based Directed Weighted Graph Construction

```
Initialize $L^i = \varnothing$, for $\forall i \in C \bigcup D$.
for each $c_k \in C$ do
    for each $d_j \in D$ do
        Calculate $q = floor(\frac{P_k^c h_{k,b}^c}{P_j^d h_{j,b}^d \eta_c})$
        if $q \in [0, Q-1]$ then
            Let $D_j \in L_q^k$ and form an edge $e_{j,k}$ with weight $w_{j,k} = \frac{1}{q+1}$.
        end if
    end for
    Let $l \in L_0^k$, for$\forall l \neq k, l \in C$ and form an edge $e_{l,k}$ with weight $w_{l,k} = 1$.
end for
for each $d_i \in D$ do
    for each $d_j \in D$ do
        Calculate $q = floor(\frac{P_i^d h_{i,i}^d}{P_j^d h_{j,i}^d \eta_d})$
        if $q \in [0, Q-1]$ then
            Let $D_j \in L_q^i$ and form an edge $e_{j,i}$ with weight $w_{j,i} = \frac{1}{q+1}$.
        end if
    end for
    for each $c_k \in C$ do
        Calculate $q = floor(\frac{P_i^d h_{i,i}^d}{P_k^c h_{k,i}^c \eta_d})$
        if $q \in [0, Q-1]$ then
            Let $C_k \in L_q^i$ and form an edge $e_{k,i}$ with weight $w_{k,i} = \frac{1}{q+1}$.
        end if
    end for
end for
```

3.2 Coloring Algorithm

After the tier-based directed weighted graph being constructed, we present the coloring algorithm to color $G(V, E, W)$. The set of $RB = [RB_1, RB_2, ..., RB_N]$ is represented by a set of colors $\zeta = [c_1, c_2, ..., c_N]$. Each RB_n is represented by a color c_n. Coloring vertex v_i in c_n is equivalent to allocating RB_n to UE i. Some definitions are formulated below.

Definition 1. *$WP_{i,n}$ is defined as*

$$WP_{i,n} = \sum_{j \in L^i \bigcap (\Phi \bigcup \psi_n)} w_{j,i} \tag{9}$$

where Φ is the set of UEs which are waiting to be colored and ψ_n is the set of UEs which have been allocated to RB_n. $WP_i = [WP_{i,1}, WP_{i,2}, ..., WP_{i,N}]$ is the vector that stores $WP_{i,n}$.

$\Phi \bigcup \psi_n$ is the set of UEs which might cause interference to v_i on channel n. The vertex with higher $WP_{i,n}$ has higher probability of being strongly interfered by its neighboring nodes on channel n.

After constructing the graph, initialize $WP_{i,n} = \sum_{j \in L^i} w_{j,i}$, for $\forall n \in RB$. If v_i's neighboring node v_j is colored in c_n before coloring v_i, v_i updates

$$WP_{i,ch} = WP_{i,ch} - w_{j,i}, for\ \ \forall ch \neq n, ch \in RB \tag{10}$$

Definition 2. *Available color set (ACS) is represented by* $A_i = [a_{1,i}, a_{2,i}, ..., a_{N,i}]$. $a_{n,i} \in [0,1]$ *represents the availability of channel n on* v_i.

$a_{n,i} = 0$ means v_i would suffer strong cumulative interference if RB_n is allocated to UE i. Thus c_n is not available to v_i. $a_{n,i} = 1$ means v_i is not interfered by its neighboring nodes on channel n. Before coloring, initialize $a_{n,i} = 1$. If v_i's neighboring node v_j is colored in c_n before v_i, v_i updates

$$a_{n,i} = max(0, a_{n,i} - max(w_{i,j}, w_{j,i})) \tag{11}$$

The pseudo code of the proposed algorithms are presented in Algorithm 1 and Algorithm 2.

In the graph construction stage, each UE senses it's neighboring nodes and uses (6), (7) or (8) to determine L^i. A tier-based directed weighted graph could be constructed using Algorithm 1.

For each vertex in the graph, initialize $a_{n,i} = 1$ and use (9) to calculate $WP_{i,n}$, for $\forall i \in C \bigcup D, \forall n \in RB$.

Algorithm 2. Maximum Potential Interference based Coloring Algorithm

Initialize $a_{n,i} = 1$, for $\forall i \in C \bigcup D, n \in RB$.
for each $i \in C \bigcup D$ **do**
  Calculate $WP_{i,n}$ using (9), for $\forall n \in RB$
end for
Initialize $r = 1$, $\varphi_r = \varnothing$.
repeat
  if $\varphi_r \neq \varnothing$ **then**
    Use (12) to determine x_r
  else
    Use (13) to determine x_r
  end if
  if $A_{x_r} == \mathbf{0}_{1*N}$ **then**
    leave vertex x_r uncolor.
  else
    $n = \underset{n}{\text{argmax}}(a_{n,x_r})$.
    Color vertex x_r in c_n.
  end if
  Use (10) to update WP_i and use (11) to update A_i, for $i \in N(x_r)$
  Update $\varphi_r = \bigcup_{h=1}^{r-1} N(x_h) - \bigcup_{h=1}^{r-1} x_h$.
  r=r+1
until $r > M + K$

Let x_r denotes the rth vertex to be colored. Different from the hypergraph based algorithm in [13] or the traditional graph-based algorithm which chooses the node having maximum degree in the subgraph as the next node to be colored, we select the node which might receive maximum potential interference to color.

Define $\varphi_r = \bigcup_{h=1}^{r-1} N(x_h) - \bigcup_{h=1}^{r-1} x_h$. φ_r is the set of unchecked neighboring nodes of the previously colored vertices.

When $\varphi_r \neq \varnothing$, x_r is determined by

$$x_r = \underset{i \in \varphi_r}{\operatorname{argmax}}(\max_{n \in \alpha_i} WP_{i,n}) \tag{12}$$

When $\varphi_r = \varnothing$, x_r is selected from the rest of the unchecked vertices.

$$x_r = \underset{i \in D \bigcup C - \bigcup_{h=1}^{r-1} x_h}{\operatorname{argmax}} (\max_{n \in \alpha_i} WP_{i,n}) \tag{13}$$

where $\alpha_i = \{n | a_{n,i} > 0\}$ is denoted as the available channel of v_i.

When vertex x_r is chosen, check A_{x_r}. If $a_{n,x_r} = 0$, for $\forall n \in N$, leave vertex x_r uncolor. Otherwise, select the color with maximum value in A_{x_r}. If A_{x_r} has multiple maxima, randomly select one of them and color vertex x_r using the corresponding color. If vertex x_r is colored in c_n, the neighboring nodes of vertex x_r update their ACS vectors and WP vectors. For example, assume vertex x_r and v_j are neighbors, then update $a_{n,j} = \max(0, a_{n,j} - max(w_{j,x_r}, w_{x_r,j}))$ and $WP_{j,ch} = WP_{j,ch} - w_{x_r,j}$, for $ch \neq n, ch \in \forall RB$.

After x_r is checked. Let $r = r + 1$. Repeat the process until all the vertices are checked.

4 Theoretical Analysis

The tier-based directed weighted graph coloring algorithm is processed in a greedy manner. There is no accurate complexity analysis for greedy based graph coloring algorithm. We focus on the worst case complexity of the algorithm.

The algorithm is composed of two stages, i.e. the graph construction stage and the coloring stage.

For graph construction stage, each UE calculates the SIR with its neighboring UEs. Note that two CUEs could not share the same channel and any two CUEs automatically form an edge. It's unnecessary for a CUE to calculate the interference from other CUEs. The complexity is proportional to $O(MK + M^2)$.

For graph coloring stage, we first need to initialize each vertex's A_i vector and WP_i vector. The time complexity is $O(M + K)$. When a specific vertex is colored in c_n, its neighboring node should update $a_{i,n}$ and $WP_{i,n}$. The time complexity is proportional to the number of edges. The worst case is that any two vertices form two directed edges. The maximum number of edges is $(M + K)(M + K - 1)$. In the coloring process, the complexity is proportional to $O((M + K)^2)$.

The overall complexity of the proposed algorithm is $O((M + K)^2))$.

Our proposed algorithm takes quadratic polynomial time, which is similar to the graph based channel allocation algorithm. This is because the structure of our proposed graph is similar to the traditional graph. The complexity of the hypergraph based channel allocation algorithm in [13] is cubic given by $O((M+K)^3)$ when $Q=2$, and the complexity increases significantly as Q increases. While the time complexity of the proposed algorithm is still $O((M+K)^2)$ when Q increases. The proposed algorithm reduces the complexity significantly compared with hypergraph based algorithm.

5 Simulation Results

Table 1. Simulation parameters

Cellular layout	Isolated cell
Cell radius	500 m
D2D pair distance	20 m–60 m
D2D pair transmit power	13 dBm
CUE transmit power	23 dBm
Noise power spectral density	−174 dBm/Hz
Channel bandwidth per RB	1.25 MHz
Pathloss model (UE to UE)	148 + 40 lg(d(km))
Pathloss model (UE to BS)	128.1 + 37.6 lg(d(km))
Threshold η_c	10 dB
Threshold η_d	20 dB

To evaluate the performance of the proposed tier-based directed weighted graph coloring channel allocation algorithm, we conduct the simulations in this section. Consider an isolated cell. D2D pairs and CUEs are randomly distributed in the cell. Each D2D pair or CUE has a fixed transmit power. The distance between the transmitter and receiver of one D2D pair is uniformly distributed between 20 m to 60 m. The channel is frequency flat. The simulation parameters are presented in Table 1.

In Fig. 3, we show the network capacity as a function of D2D pairs M with $K=10$ CUEs and $N=20$ channels. The hypergraph based resource sharing method (HBRSM) in [13] is evaluated with $Q=2$, while our proposed algorithm is evaluated with both $Q=2$ and $Q=10$. The network capacity increases as M grows. When $Q=2$, our proposed algorithm and the hypergraph based algorithm could both eliminate the independent interference generated by one neighboring UE and cumulative interference generated by two neighboring UEs. The network capacity gain of our proposed algorithm is due to the change of coloring order. In our proposed algorithm, the UE chosen to be allocated next

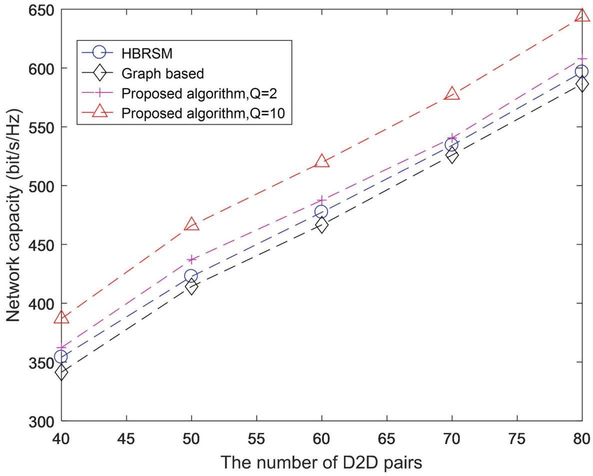

Fig. 3. The network capacity with the number of D2D pairs, K = 10, N = 20

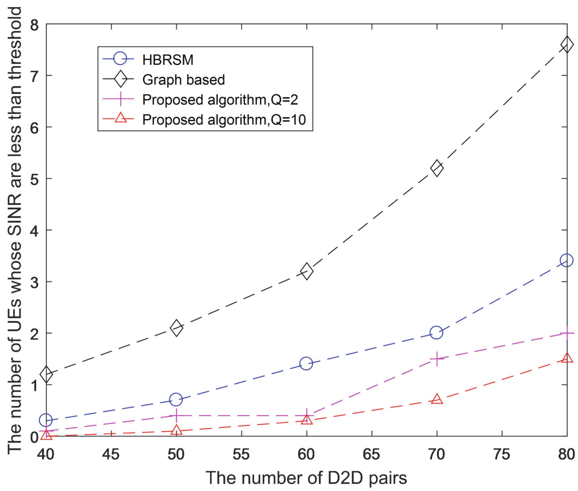

Fig. 4. The number of UEs whose SINR are below η, K = 10, N = 20

is selected from the neighboring set of the previously allocated UEs and the UE who might be interfered most in the set is colored. Further growth of network capacity when $Q = 10$ is due to cumulative interference being eliminated more accurately.

In Fig. 4, we show the number of UEs whose SINR are less than SINR threshold η after channel allocation. For our proposed algorithm, the number of UEs whose SINR are less than η is approximate zero when $M < 60$. Graph based algorithm has the worst performance, because it doesn't consider cumulative interference. When $M > 70$, the number of UEs which receive strong cumulative

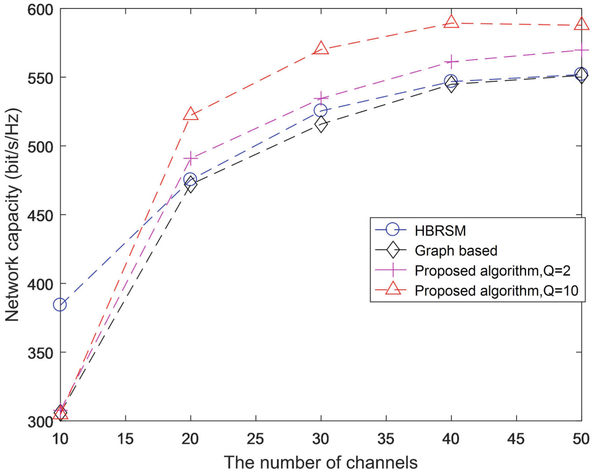

Fig. 5. Network capacity with the number of channels, K = 10, M = 60

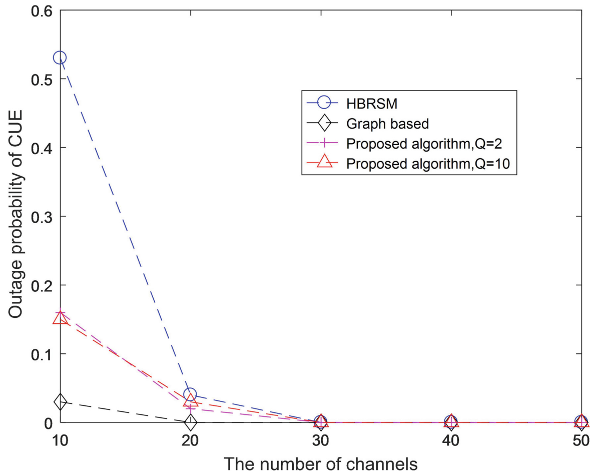

Fig. 6. Outage probability Of CUE with the number of channels, K = 10, M = 60

interference using the hypergraph based algorithm increases rapidly. Our proposed algorithm shows better performance due to better cumulative interference elimination method.

In Fig. 5, the network capacity as a function of the number of channels N is presented. As the number of channels grows, cumulative interference decreases. The capacity gap between the hypergraph based method and graph based algorithm is decreased. When $N > 40$, the performance of the hypergraph based algorithm and graph based algorithm is nearly the same. When $N = 40$, the

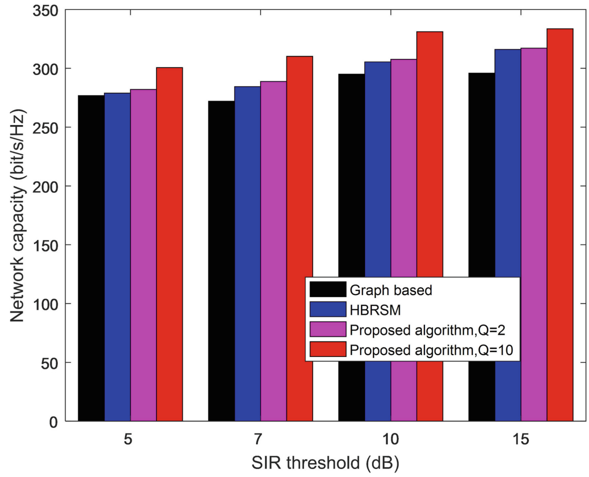

Fig. 7. The network capacity with η, K = 10, N = 20, $\eta = \eta_c = \eta_d$

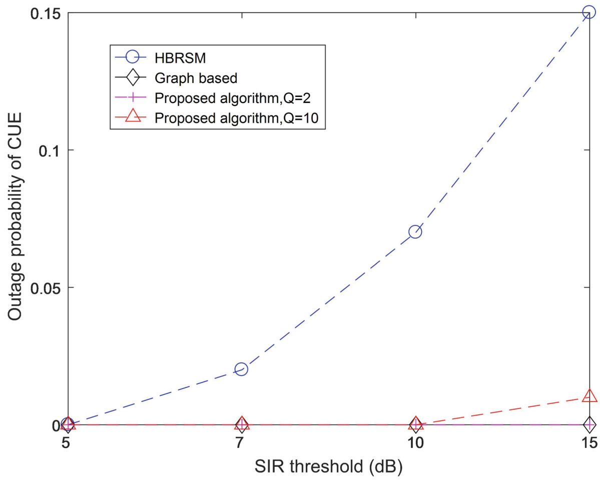

Fig. 8. Outage probability Of CUE with η, K = 10, N = 20,$\eta = \eta_c = \eta_d$

network capacity of our proposed algorithm with $Q = 2$ is 16.4 bit/s/Hz higher than the hypergraph based algorithm, the gap increases to 44.5 bit/s/Hz when Q = 10.

Figure 6 shows the outage probability of CUEs as a function of the number of channels. The outage probability of CUEs decreases as the number of channels increases. The hypergraph based algorithm has the highest CUE outage probability when few channels are available. Combine Figs. 5 and 6, when few channels are available in the network, the CUEs in the hypergraph based method are least likely to be allocated a channel compared with the other two algorithms. D2D pairs usually contribute more to the network capacity compared with CUEs because of its short-range direct communication. The hypergraph based algorithm sacrifices some of the CUEs in exchange for DUEs in order to gain high network capacity. While the proposed algorithm guarantees that most of the CUEs would be allocated a channel.

In Figs. 7 and 8, when η_c and η_d increase simultaneously, the network capacity first increases then becomes saturated using the graph based method and the proposed method. Network capacity increases as η grows using the hypergraph based method, but the outage probability of CUEs also increases.

6 Conclusion

In this paper, we studied channel allocation in D2D underlaying cellular networks. The neighborhood interference was formulated as a tier-based directed weighted graph. We proposed maximum potential interference based coloring algorithm to color the graph in a greedy manner. The proposed algorithm aims to eliminate cumulative interference and thus increases network capacity. Complexity of the proposed algorithm also reduce significantly compared with the hypergraph based algorithm. Simulation results showed that our proposed algorithm has better performance compared with the graph based algorithm and the hypergraph based algorithm.

References

1. Asadi, A., Wang, Q., Mancuso, V.: A survey on device-to-device communication in cellular networks. IEEE Commun. Surv. Tutor. **16**(4), 1801–1819 (2014)
2. Doppler, K., Rinne, M., Wijting, C., Ribeiro, C.B., Hugl, K.: Device-to-device communication as an underlay to LTE-advanced networks. IEEE Commun. Mag. **47**(12), 42–49 (2009)
3. Yin, R., Zhong, C., Yu, G., Zhang, Z., Wong, K.K., Chen, X.: Joint spectrum and power allocation for D2D communications underlaying cellular networks. IEEE Trans. Veh. Technol. **65**(4), 2182–2195 (2016)
4. Zhao, H., Ding, K., Sarkar, N.I., Wei, J., Xiong, J.: A simple distributed channel allocation algorithm for D2D communication pairs. IEEE Trans. Veh. Technol. **67**(11), 10960–10969 (2018)
5. Kazmi, S.M.A., et al.: Mode selection and resource allocation in device-to-device communications: a matching game approach. IEEE Trans. Mob. Comput. **16**(11), 3126–3141 (2017)
6. Zhang, F., Zhou, X., Sun, M.: Constrained VCG auction for spatial spectrum reuse with flexible channel evaluations. In: 2017 IEEE Global Communications Conference (GLOBECOM 2017), pp. 1–6, Singapore (2017)
7. Joo, C., Lin, X., Ryu, J., Shroff, N.B.: Distributed greedy approximation to maximum weighted independent set for scheduling with fading channels. IEEE/ACM Trans. Netw. **24**(3), 1476–1488 (2016)
8. Mili, M.R., Tehrani, P., Bennis, M.: Energy-efficient power allocation in OFDMA D2D communication by multiobjective optimization. IEEE Wirel. Commun. Lett. **5**(6), 668–671 (2016)

9. Zhang, H., Wang, T., Song, L., Han, Z.: Graph-based resource allocation for D2D communications underlaying cellular networks. In: 2013 IEEE/CIC International Conference on Communications in China - Workshops (CIC/ICCC), pp. 187–192, Xi'an (2013)
10. Cai, X., Zheng, J., Zhang, Y.: A graph-coloring based resource allocation algorithm for D2D communication in cellular networks. In: 2015 IEEE International Conference on Communications (ICC), pp. 5429–5434, London (2015)
11. Zhang, R., Cheng, X., Yang, L., Jiao, B.: Interference graph-based resource allocation (InGRA) for D2D communications underlaying cellular networks. IEEE Trans. Veh. Technol. **64**(8), 3844–3850 (2015)
12. Zhao, L., Wang, H., Zhong, X.: Interference graph based channel assignment algorithm for D2D cellular networks. IEEE Access **6**, 3270–3279 (2018)
13. Zhang, H., Song, L., Han, Z.: Radio resource allocation for device-to-device underlay communication using hypergraph theory. IEEE Trans. Wirel. Commun. **15**(7), 4852–4861 (2016)
14. Sun, Y., Du, Z., Xu, Y., Zhang, Y., Jia, L., Anpalagan, A.: Directed-hypergraph-based channel allocation for ultradense cloud D2D communications with asymmetric interference. IEEE Trans. Veh. Technol. **67**(8), 7712–7718 (2018)