Orchestration Between Computational Thinking and Mathematics

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Abstract. The use of computational thinking (CT) in educational settings has increased in popularity in the last twenty years. The aim of the paper is there-by to unfold both CT and mathematics to develop an analytical model for analyzing mediating processes in teaching mathematics including CT. By applying a socio-cultural perspective, the mediation theory provides new opportunities for understanding the mediation process involved in the introduction of digital artifacts, such as robots in mathematic teaching. The research has a specific focus on the construction of knowledge through signs. The suggested model is intended as a tool to analyze the mediating processes that occur when using CT in mathematics and mathematics education. The proposed model includes two dimensions, where one of them concerns which perspective that is applied (either teacher- or student-centered), and the other dimension concerns the synergy between CT and mathematical concepts. These two dimensions produce different representations and are initiated in various ways.

Keywords: Computational thinking · Mathematics · Primary level · Mediating artifacts

1 Introduction

1.1 Computational Thinking in Mathematics

Over the last few years, researchers and educators have recognized computational thinking (CT) as a strong educational approach [1]. Wing (2006) introduced the prevailing discussion about CT. However, the concept of CT can be tracked back to Papert's work in the 1980s [2]. According to Wing, CT can be seen as "a way to solve problems, design systems, and understand human behavior by using the basic concepts of computer science" [3:33]. According to Wing's [3] definition, CT is a problem-solving method that can be used to explain a problem so both humans and computers can recognize the solution. Due to logical structure and mathematical modeling problems [4], a natural and historical connection links CT and mathematics. According to Pérez [5], CT can help students understand mathematical problems through programming, algorithmic thinking, and creating computational abstractions. According to Li et al. [6], CT is "a model of thinking that is more about thinking than computing" [6:4], and CT should be a part

E. Brooks et al. (Eds.): DLI 2021, LNICST 435, pp. 77–85, 2022.
https://doi.org/10.1007/978-3-031-06675-7_6

of students' abilities in the 21st century. Similarly, Chongo et al. [7] defined CT as "…a process of thinking and a tool for solving problems using computer concepts either with a computer (plugged in) or without one (unplugged)" [7:160]. These two definitions are similar because they relate to a cognitive and systematic process and clarify that CT can be used with or without a computer. Indeed, CT can be a thought process that is independent of technology [1]. Several researchers have concluded that students can learn and acquire knowledge from CT early in their learning careers. However, researchers must first focus on problem-solving and how students can express themselves using mobile learning, coding apps, and digital artifacts [19]. This paper considers the background that relates to how students can develop their mathematical understanding through CT. The paper draws on a socio-cultural perspective to focus on the thinking process and peer collaboration [7]. However, there is still a gap within the proposed theory about CT in mathematics and its practical utilization in practice, especially in low-level mathematics education. This article aims to use computational thinking and mathematics to establish an analysis model and recognize CT's utilization in math teaching. This article also emphases the meaning of signs that occur as part of students' development of mathematics concerning CT. This type of study is relevant for mathematics teachers and researchers because it can help them determine how teachers can incorporate digital artifacts to support CT concerning mathematics teaching.

This paper uses a socio-cultural viewpoint on learning to examine the relationship between CT and mathematics. These perspectives are presented in a model that combines both of them. To illustrate the analytical model, an empirical example is provided at the end of the theoretical framework.

1.2 A Sociocultural Perspective

In this paper, a socio-cultural perspective frames the model development process. Knowledge creation is entrenched in different contexts and is mediated by signs and artifacts that involve students and teachers in social activities. Cultural meanings regulate the relationships between humans and their surrounding environments through signs and artifacts developed by humans over time. Every function in children's cultural development will appear twice; first, at a social level between people and then on the individual level inside the child [8].

Language plays a vital role in this intermediary, which transfers from the outside to the inside through internalization. Internalization describes how humans reconstruct external influences as internal understanding. Mediation connects humans' perceptions of the world [8]. According to the socio-cultural point of view, humans are connected to the social and physical by signs and artifacts. This can be related to the higher mental processes that humans use to mediate understanding and definitions of being in the world [9]. Students often work to accomplish abstract ideas in mathematics education; here, cultural artifacts can make learning visible through signs and symbols [10]. In this context, looking further into semiotic mediation can be useful. How students or teachers mediate information, meaning, thoughts, and ideas through artifacts and symbols is central to semiotic mediation [11]. It is essential to recognize what a sign represents and its interpretation to understand the semiotic mediation process [12].

In the developed model, mediations play a vital part in the use of tools and signs in mathematics and CT. Therefore, it is essential to reflect on the background of each sign. The semiotic perspective is helpful because it examines the relationships between signs that are included in the learning context of introductory CT in mathematics.

1.3 Knowledge Creation

According to Brandsford et al. [13], every student brings a viewpoint to the learning process. Learning can be understood as a process that allows students to construct new knowledge based on already known knowledge. Therefore, teachers must pay attention to the understanding that students already have. Otherwise, students' knowledge and experiences in the classroom may be different from their teacher's intentions. It is difficult for students to learn new concepts without considering their existing knowledge [8, 13].

In the process of knowledge creation, a student's learning must be visible to the teacher so that the teacher can help each student with misunderstandings. Brandsford [13] believes that by participating in discussions, asking questions, and using artifacts, students can thoroughly discover content that motivates learning and gain a deep understanding of mathematical concepts. With this in mind, students who study mathematical concepts by themselves will recognize the process of integrating CT into mathematics, CT-related symbols, and mathematical concepts [14, 15]. During growth, students develop spontaneous concepts, as well as scientific concepts with the support of adults or knowledgeable peers [8].

1.4 Connections Between Computational Thinking and Mathematics

Researchers have found that some inherent characteristics of CT can help students contextualize certain mathematical content by both working with and without digital artifacts [16]. During an investigation of the course guide and CT definition, Barcelos et al. [17] highlighted high-level skills related to CT:

- *Alternating between different semiotic representations*, which involves translating a model that is expressed as one symbolic representation into another symbolic representation. These representations can take many forms, including charts, tables, verbal expressions, algorithmic representations, or drawings.
- *Establishing relationships and identifying patterns* in situations that require students to identify patterns, use decomposition, or establish a formation rule. In mathematics, this skill is often applied to numerical regularities and abstraction used in problem-solving.
- *Building descriptive and representative models* by means of math or algorithmic language that describes and exposes students' considerations of a problem.

Determining the skills that are connected to CT and mathematics provides an opening for analyzing and understanding the relationship between CT and mathematics.

Recently, researchers have found that it is important for students to develop their own opinions and understandings regarding engagement in computation [6, 18]. By dealing with the connection between spontaneity and scientific concepts, students can start to recognize mathematical concepts in depth. CT highlights the meaning of students' thinking about the fundamentals of mathematics [6]. Mediation offers a connection to social and cultural processes and personal higher mental developments. From this point of view, students can internalize the intermediary process provided by teachers who are grounded in cultural, social, and institutional strengths.

2 Orchestration Between CT and Mathematics

Developing the theoretical basis for this paper led to the development of the following model: "Orchestration between CT and mathematics" (Fig. 1). This model consists of two dimensions, one of which is based on the interplay between CT and mathematics. The second dimension is established on the interplay concerning teacher- and student-centered perspectives. The interplay between CT and math emerges through theoretical perspectives, which are essential to how teachers mediate and construct teaching activities to support their students' development of mathematical, scientific concepts. "Orchestration" relates to teachers' function in facilitating various activities that mediate the creation of student symbols and the relationship between signs and scientific concepts [15]. If teachers can use CT to convey mathematics content to promote and coordinate various teaching activities, then it is appropriate to include CT in mathematics teaching [15].

The future of applying CT in math is based on the perspective concerning teacher-centered and student-centered activities that provide various content in traditional math lessons or more problem-based methods. Signs that are associated with CT and mathematical concepts appear in multiple forms. Teachers can understand the relationship that connects CT and math and examine students' learning conditions [15]. The cross-tabulation of these dimensions indicates the four-domain model of CT methods in mathematics teaching. When adopting this model, a teacher should examine the meaning of combining CT and math teaching. The model illustrates areas of relationships between CT and math learning. The model may be utilized as an analytic instrument for understanding the connection among CT and math and can also be used as a tool for using digital artifacts and CT to mediate activities in mathematics. Additionally, teachers can use the model to construct activities that focus on various aspects and symbolic representations of mathematical learning.

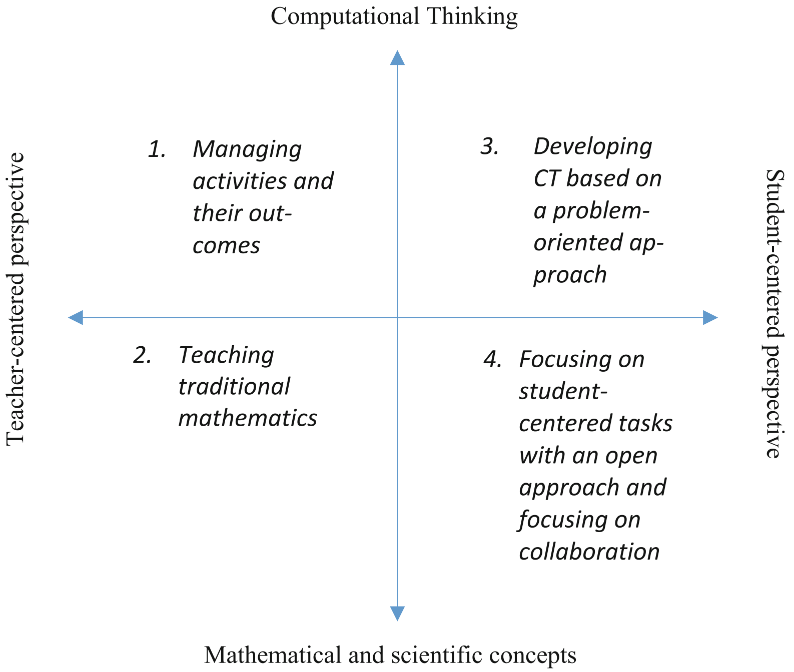

Fig. 1. Orchestration between computational thinking and mathematics

The model presents 4 squares, every referring to diverse teaching activities that can take place during a teaching sequence. Square 1 illustrates the focus on managing activities and student outcomes; here, the teacher controls the activities. Square 2 portrays the more traditional approach to mathematics, which incorporates a textbook-driven approach. The teacher also manages these activities. Square 3 presents the focus on developing CT based on a problem-oriented approach in which tasks are student-centered [6]. Square 4 presents the focus on student-centered tasks in which tasks incorporate an open approach and focus on student collaboration [13, 18].

2.1 A Case with Beebot

This case utilizes data from a second-grade class. The class used Beebots as a digital artifact. The tasks explained in this case are the first and third tasks of a teaching sequence. The students worked in groups of two with one Beebot for each pair; there were ten teams in the class. This example involves tasks and activities that encouraged the students to classify geometric figures and use appropriate words.

An iterative process of didactic cycles followed the teaching sequences.

The first didactic cycle involved the robots. The students had to become familiar with the robots and understand how they worked.

The second didactic cycle required the students to have their robots make a square, and then they had to investigate how small or large the robots could make the square.

According to CT, the students were asked to create an algorithm for constructing the square.

During *the third didactic cycle*, the students were asked to work with the geometric properties of polygons. The task (Fig. 2) was to make the robots land on, for example, all squares, triangles, etc. and describe the characteristics of the individual polygon.

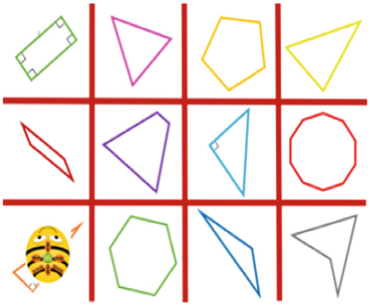

Fig. 2. The task involving geometric properties

The fourth didactic cycle involved a problem-solving approach; the students had to figure out which type of polygon the robots could make. The students were required to investigate the type – from one-sided to ten-sided polygon – that each robot could make and see if they could make any generalizations from it [21].

The next section covers the first and third didactic cycles to unfold the use of the developed model.

2.2 Familiarity with the Robot

Based on the model in Fig. 1, the first part is characterized by Square 1 and 3.

The students worked together in pairs, and each team had one robot. During the first task, the students were required to familiarize themselves with the robots. The teacher provided a short overview of the different buttons on the robots, and after that, the students were asked to figure out what the different buttons were used for, which aligned with Square 1. Subsequently, the teacher followed up on this activity during the class discussion. The following excerpt illustrates what some of the groups were struggling with:

Student 1: We clicked a whole lot, but it did not do what we wanted it to.
Student 2: It was because we forgot to press "delete." Then we clicked two forwards and two to the side, two backward and two to the side.
Teacher: What did you think it was doing? A square?
Student 1: Yes, but it did not. It just started driving around and going backward [the student moves his body to reveal what the robot had done] because we forgot to press "delete."
Teacher: If you had clicked on "delete," would it then have made a square? Did you try it?

Student 3: You must press two forward, one left, two forward, one left, two forward, one left, two forward, and one left.

This is an excellent example of what the students were struggling with during the first task. Many had trouble remembering to clear the robot after ending the activity and had problems creating a new activity. Student 1 also used his gesture to show the movement of the robot, which helped him make the movement more understandable [21]. During the first task, the students worked in Square 3 to investigate how the Bee-bots worked. During the class discussion, the teacher became aware of the students' misunderstandings. Giving the students time to explore the Beebots and following up in conversation with help from teachers and other students helped the struggling students focus more on the mathematics in the later exercises. The switch between Square 1 and 3 helped the students investigate the robots' features and still be aware of some of their misconceptions.

2.3 Mediating Artifact

In the third teaching cycle, students performed two activities. For the first task, they were asked to deal with the geometric properties of the polygons (see Fig. 2). Students must let the robot land on all squares, triangles, etc., and describe the characteristics of each polygon. When students sort the polygons, it helps them focus on the features of each polygon. This allows them to increase their knowledge of individual polygons. Students must establish a connection with the robot through CT to create an overview of the polygons on the worksheet and create an algorithm to move the robot from one triangle to another [21]. The students use the robots as a mediating artifact to describe the characteristics of the polygons. Some of the students used gestures to convey the directions in which the robots should turn. The task can be related to Square 4 since the students investigated how to characterize the different polygons from a student-centered perspective.

Students must also classify different polygons according to criteria related to right angles, acute angles, etc. This helps to support students' research on various attributes such as "right angle" and "equal length side." The classification of the polygons in Fig. 2 according to the new standard helps support students' reasoning at a higher level of abstraction [20, 21]. This also enables students to distinguish polygons and understand the standard features of polygons that do not seem to have the same characteristics. By classifying polygons, students learned that different polygons could have the same characteristics. When students use the robot as an intermediate workpiece, they work with CT to program the robot. Through CT, students try to make the robot move in a different order; for example, make the robot move to all triangles (Fig. 2). If the robot does not land on the required polygon, the students will constantly debug and correct their code during the task. In this way, students receive CT training when they introduce their work to the classroom and solve reward tasks [21].

The students used various signs to construct the meaning of the activity. The mediation process enhanced the students' understanding of the signs that appeared in various activities, which transformed into the psychological development of thinking and memory [9]. This activity embodies the transition from the teacher- to the student-centered perspective. In this case, the students used CT to develop their understanding of mathematical science concepts in relation to geometry.

3 Concluding Remarks

Using BeeBots, the students worked on new forms of geometric representation, and the learning content was presented in a new way that helped develop the students' mathematical and scientific understanding. The teacher worked to create an interaction between spontaneous and mathematical scientific concepts at the beginning of the lesson. The students understood the task through this interaction, so they were later able to perform it by mediating the task's computational and mathematical intentions. The mediation thus became essential for the teacher to help the students in their acquisition of mathematical and scientific concepts. The teacher was given a central role in the mediation process since it was significant. Therefore, CT that is implemented in composing an intermediary process can help students understand their math and CT better.

The teacher's conversation with the students during the class discussion also became necessary for the mediation process to support the students' development of scientific and mathematical concepts. However, little knowledge is available about how teachers can include CT in mathematics teaching and how CT can support students' mathematical and scientific understanding and problem-solving processes. The developed model can help teachers who implement CT in math. The model can also be expended to analyze the mediating processes that appear when CT interacts with math. The model-based consideration reveals the common sense of applying CT in mathematics teaching; however, more research is needed to study the model further.

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