A Game Theory Approach for Water Exchange in Eco-Industrial Parks: Part 1 - A Case Study Without Regeneration Units

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Abstract. Eco-Industrial Parks (EIP) offer to enterprises the possibility to make economic benefits and to minimize environmental impacts by sharing flows and increasing inter-enterprise exchanges. Part 1 of the paper presents a mathematical programming formulation for the optimal design of water exchange networks in an EIP without regeneration units, based on a single-leader multi-follower (SLMF) methodology. The goal of each follower is to minimize his operating cost, while the leader's goal is to reduce the consumption of freshwater in the EIP. Examples of EIPs are studied numerically. The results show that the SLMF methodology is very reliable in the multi-criteria scenarios of the EIP design, providing numerical Nash equilibrium solutions.

Keywords: Game theory · Nash equilibrium · Single-leader multi-follower game · Mixed-integer programming · Eco-Industrial Park

1 Introduction

In recent years, the term "climate change" has received a lot of attention. It not only adversely affects the lives of people at the present, but also threatens the environment in the future. The main cause of the Earth's climate change is the increase in activities that generate greenhouse gas waste, excessive exploitation of natural resources in industry. To protect the global environment as well as increase economic benefits, the concept of *Industrial Ecology* (IE) has emerged [2]. IE provides an innovative way to produce goods and services based on a circular model. Indeed, in these industrial ecosystems, resource consumption and waste generation are minimized by allowing the waste materials from one industry to serve as raw material for another. An application to this concept are EIP [7]: "A community of manufacturing and service businesses seeking enhanced environmental and economic performance through collaboration in managing environmental and resource issues including energy, water, and materials. By working together, the community of businesses seeks a collective benefit that

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is greater than the sum of the individual benefits each company would realize if it optimized its individual performance only". To achieve these objectives, optimization methods are required to design optimal exchange networks between enterprises. However, a major challenge in developing optimization methods is to deal with the network complexity of the solutions, especially when a lot of interconnections are considered, as is the case with EIP.

Modeling EIPs is somewhat a complex problem due to its size and the number of objectives to take into account. In the literature, there are two main approaches for designing and optimizing water-exchange networks in the EIP: multi-objective optimization (MOO) on one hand and game theory on the other hand. The MOO approach requires enterprises to coordinate their strategies and share information. This requirements usually does not exist in the optimal design of the exchange networks in the EIP as it indicated in the works of [9] and [11]. We refer the reader to [2–4,8] for the survey on MOO approach.

Another approach to design an optimal resource exchange network in an eco-industrial park is game theory. More precisely, the concept of the single-leader multi-follower game problem. Indeed, at the upper-level problem, there exists an EIP authority that wants to minimize resource consumption and generation of waste, while at the lower-level problem, each enterprise tries to minimize his operating cost. In SLMF game, the EIP authority makes decision first by anticipating the response of the enterprises. At the same time, all enterprises compete with each other in a generalized Nash game with the strategies of the EIP authority as exogenous parameters. It's worth mentioning that, at the lower level, enterprises play a generalized Nash equilibrium between them, so enterprises involved would be able to protect confidential information, without the need to share information with the other enterprises of the park. The existence of an optimal solution satisfying Nash equilibrium would ensure that none of each enterprise is prejudiced compared to others. We refer the reader to [6] for a survey of generalized Nash equilibrium problems and to [1,9–11] for a survey of SLMF approach.

In this work, the design and optimization of industrial water networks in eco-industrial parks are studied by formulating and solving single-leader multi-follower game problems. The approach is validated on a case study of water integration in EIP without regeneration units. The result shown that the equilibrium solution will provide economic benefits to enterprises participating in the EIP.

The remainder of this paper is organized as follows: Sect. 2 is dedicated the methodology and model formulation which briefly describes the problem addressed in this article and present a model for water exchange networks in EIPs without regeneration units, based on a single-leader-multiple-follower model. The reformatting of the EIP modeling problem as a mixed integer linear programming problem is addressed in Sect. 3. Section 4 is dedicated to numerical experiments on reasonably large EIP. Finally, general conclusions and future perspectives are presented in Sect. 5.

2 Methodology and Model Formulation

2.1 Problem Statement

Let n denote the given number of enterprises, $I := \{1, \ldots, n\}$ denote the index set of enterprises, and 0 denote the sink node. The sink node is a place to store contaminated wastewater. Thus, we define $I_0 = \{0\} \cup I$. Each enterprise has its own pre-defined water input requirement and quality characteristics, as well as the quantity and quality of available output wastewater. For each enterprise, the resource consumption can be freshwater, and/or wastewater from other enterprises. Indeed, the polluted water from an enterprise can be sent to the sink node, and/or sent to other enterprises. The objective of the model is to determine a network of connections of water streams among them so that both the total freshwater consumption and the annualized operating cost of each enterprise in the park are minimized, while satisfying all process and environmental constraints.

2.2 Minimizing Operating Costs

Each enterprise $i \in I$ receives the wastewater from other enterprises in the EIP. Nevertheless, for technical constraints on the process P_i, the pollutant concentration delivered by the other enterprises cannot exceed a certain maximum value denoted here by $C_{i,\text{in}}$ [ppm]. On the other hand enterprise i has a given contaminant load M_i [g/h] that needs to be diluted before exiting the enterprise. To do so, enterprise i needs to use an amount of fresh water z_i [T/h] such that the outlet pollutant concentration is less than a limit concentration $C_{i,\text{out}}$ [ppm]. Actually considering that enterprise i will optimize his process, it is assumed that each enterprise $i \in I$ will only consumes the exact amount of freshwater it needs to satisfy $C_{i,\text{out}}$, and therefore, its output pollutant concentration is always equal to this constant.

There is an exchange of materials between the enterprises in EIPs. On the other hand, let E be the configuration for the water exchange network in EIPs such that $(i, j) \in E$ then the enterprise i can send his wastewater to enterprise j. Especially if the enterprise i uses the connection $(i, 0)$, it means that it is discharging polluted water outside the park.

Defining the *stand-alone* and *complete* configuration, respectively, as follows

$$E_{\text{st}} := \{(i, 0) : i \in I\} \quad \text{and} \quad E_{\max} := \{(i, j) \;:\; i \in I,\, j \in I_0\},$$

thus a valid configuration E must satisfy that $E_{\text{st}} \subset E \subset E_{\max}$. We denote by $\mathcal{E}$ the set of all valid configurations for the EIP. Furthermore, for any $E \in \mathcal{E}$, we denote by $E^c = E_{\max} \setminus E$ the family of connections that are not present in E.

In term of variables, each enterprise $i \in I$ sends polluted water to $j \in I_0$, taken into account by variable $F_{i,j}$ [T/h]. In addition, we set $F = (F_{i,j} \;:\; (i, j) \in E_{\max})$, the full vector of fluxes through the configuration.

Furthermore, for each enterprise $i \in I$, we denote $F = (F_i, F_{-i})$ where $F_i = (F_{i,j} \;:\; j \in I)$ and $F_{-i} = (F_{k,j} : k \in I \setminus \{i\}, j \in I)$, to emphasize the vector of fluxes between enterprise i. Then, for a fixed network E, the EIP model without regeneration units must satisfy the following constraints:

1. Water mass balance constraint for an enterprise $i \in I$:

$$z_i + \sum_{(k,i)\in E} F_{k,i} = \sum_{(i,j)\in E, j\neq 0} F_{i,j} + F_{i,0}. \tag{1}$$

2. Contaminant mass balance constraint for an enterprise $i \in I$:

$$M_i + \sum_{(k,i)\in E} C_{k,\text{out}} F_{k,i} = C_{i,\text{out}} \left(\sum_{(i,j)\in E, j\neq 0} F_{i,j} + F_{i,0} \right). \tag{2}$$

3. Inlet/outlet concentration constraints for an enterprise $i \in I$:

$$\sum_{(k,i)\in E} C_{k,\text{out}} F_{k,i} \leq C_{i,\text{in}} \left(z_i + \sum_{(k,i)\in E} F_{k,i} \right). \tag{3}$$

4. Positivity of fluxes and null fluxes outside the connections: we need all the fluxes to be positive:

$$\begin{cases} F_{i,j} \geq 0, & \forall (i,j) \in E \\ z_i \geq 0, & \forall i \in I. \end{cases} \tag{4}$$

   Of course, we also put

$$\forall (i,j) \in E^c,\ F_{i,j} = 0, \tag{5}$$

   that is, the effective fluxes can only pass through the existing connections in E.

By combining Eqs. (1) and (2) we obtain:

$$M_i + \sum_{(k,i)\in E} C_{k,\text{out}} F_{k,i} = C_{i,\text{out}} \left(z_i + \sum_{(k,i)\in E} F_{k,i} \right), \quad \forall i \in I. \tag{6}$$

From the Eq. (6) the freshwater consumption of the enterprise $i \in I$ is defined by

$$z_i(F_{-i}) = \frac{1}{C_{i,\text{out}}} \left(M_i + \sum_{(k,i)\in E} \left(C_{k,\text{out}} - C_{i,\text{out}}\right) F_{k,i} \right). \tag{7}$$

Thus, each enterprise i wants to minimize his operating cost, given by

$$\text{Cost}_i(F_i, F_{-i}, E) = A \left[c \cdot z_i(F_{-i}) + \gamma_{i,0} F_{i,0} + \sum_{(k,i)\in E} \gamma_{k,i} F_{k,i} + \sum_{(i,j)\in E, j\neq 0} \gamma_{i,j} F_{i,j} \right], \tag{8}$$

where A [h] stands for the annual EIP operating hours, c [\$/T] for the purchase price of freshwater, $\gamma_{i,0}[\$/T]$ for the price of wastewater discharge, and $\gamma_{p,q}[\$/T]$ for the cost of sending wastewater from enterprise p to q.

With all these considerations, each enterprise's $i \in I$ optimization problem is given by $P_i\left(F_{-i}, E\right)$

$$\begin{aligned} &\min_{F_i} \operatorname{Cost}_i\left(F_i, F_{-i}, E\right) \\ &s.t. \begin{cases} \text{Equations (1)-(3)-(5)}, \\ z_i(F_{-i}) \geq 0, \\ F_i \geq 0. \end{cases} \end{aligned} \tag{9}$$

For a network $E \in \mathcal{E}$, the family of equilibria for E at the lower-level problem is given by $\operatorname{Eq}(E)$. Furthermore,

$$F \in \operatorname{Eq}(E) \Longleftrightarrow \forall i \in I,\ F_i \text{ solves the problem } P_i\left(F_{-i}, E\right).$$

Remark 1. *If no enterprise sends wastewater to enterprise $i \in I$, the amount of freshwater consumed by enterprise i must be*

$$z_i = \frac{M_i}{C_{i,\text{out}}}.$$

Then, the cost of stand-alone configuration is given by

$$\operatorname{STC}_i = A \cdot (c + \gamma_{i,0}) \frac{M_i}{C_{i,\text{out}}}.$$

2.3 Minimizing Consumption of Natural Resources

In the model, the EIP authority tries to optimize the total freshwater consumption, and so he wants to minimize the objective function

$$Z(F) = \sum_{i \in I} z_i(F_{-i}). \tag{10}$$

The authority must ensure a relative improvement of $\alpha \in]0, 1[$ in the costs, with respect to the stand-alone operation, that is,

$$\operatorname{Cost}_i(F_i, F_{-i}, E) \leq \alpha \cdot \operatorname{STC}_i \quad \forall i \in I. \tag{11}$$

Thus, the optimization problem of the EIP authority is given by

$$\begin{aligned} &\min_{F \in \mathbb{R}^{|E_{\max}|}, E \in \mathcal{E}} Z(F) \\ &s.t. \begin{cases} F \in \operatorname{Eq}(E), \\ \operatorname{Cost}_i(F_i, F_{-i}, E) \leq \alpha \cdot \operatorname{STC}_i, \qquad \forall i \in I. \end{cases} \end{aligned} \tag{12}$$

3 Mixed-Integer Programming Reduction

As we can observe the EIP authority problem (12) is mathematical programming with equilibrium constraints, which is hard to solve. In the following, however, this type of problem will be reformulated as a single mixed-integer programming problem.

3.1 Characterization of Equilibria

In order to characterize the equilibrium set $\mathrm{Eq}(E)$ as a system of equalities and inequalities, for each enterprise $i \in I$, we define the set

$$E_{i,\mathrm{act}} := \left\{ (i,j) \in E \ : \ \gamma_{i,j} = \gamma_i^* := \min_{(i,k) \in E} \gamma_{i,k} \right\}. \tag{13}$$

Theorem 2. *For any valid exchange network $E \in \mathcal{E}$ and denoting $S(E)$ by the set*

$$S(E) = \left\{ F \ : \forall i \in I, \begin{array}{l} z_i + \sum_{(k,i)\in E} F_{k,i} = \sum_{(i,j)\in E} F_{i,j} \\ \sum_{(k,i)\in E} C_{k,\mathrm{out}} F_{k,i} \leq C_{i,\mathrm{in}} \left(z_i + \sum_{(k,i)\in E} F_{k,i} \right) \\ z_i(F_{-i}) \geq 0 \\ F_i \geq 0 \\ F_i \big|_{E^c_{i,\mathrm{act}}} = 0 \end{array} \right\} \tag{14}$$

then, one has $S(E) = \mathrm{Eq}(E)$. Furthermore, any optimal solution (F, E) of the mathematical programming problem

$$\begin{aligned} &\min_{F \in \mathbb{R}^{|E_{\max}|}, E \in \mathcal{E}} \quad Z(F) \\ &s.t. \begin{cases} F \in S(E), \\ \mathrm{Cost}_i(F_i, F_{-i}, E) \leq \alpha \cdot \mathrm{STC}_i, \qquad \forall i \in I. \end{cases} \end{aligned} \tag{15}$$

is an optimal solution of the SLMF problem (12).

For the proof of Theorem 2, we prefer the reader to Appendix A in Part 2 of the paper [5].

3.2 Mixed-Integer Formulation

Constraint $F_i\big|_{E^c_{i,\mathrm{act}}} = 0, \forall i \in I$ depends on the configuration of the water exchange network and is therefore difficult to implement numerical experiments. Thus, we will reduce the single optimization problem (12) to the mixed-integer programming problem.

Let $(i, j) \in E_{\max}$, we define

$$C(i, j) := \{(i, k) \in E_{\max} \ : \ \gamma_{i,k} = \gamma_{i,j}\} \tag{16}$$

the *arc class* of (i, j). Furthermore, we denote by $\mathcal{C}_i = \{C(i, j) \ : \ (i, j) \in E_{\max}\}$ the family of all arc classes of enterprise i.

If there exists one class $C(i, j) \in \mathcal{C}_i$ such that $E_{i,\text{act}} \subseteq C(i, j)$, then we will call it the *active class* of E of the enterprise i, and we will denote it by $C_i(E)$.

Now, let $D = \bigcup_{i \in I} \mathcal{C}_i$, the set of all arc classes of enterprises. We introduce the boolean variable $y = (y_C)_{C \in D} \in \{0, 1\}^{|D|}$ in the following way: for each enterprise $i \in I$ and each arc class $C \in \mathcal{C}_i$, we set

$$y_C = \begin{cases} 1 & \text{if } C \text{ is the active class of } i, \\ 0 & \text{otherwise.} \end{cases}$$

With this new boolean variable, we set the following constraints:

1. For each enterprise $I \in I$,

$$\sum_{C \in \mathcal{C}_i} y_C = 1, \tag{17}$$

   namely, only one class is active.
2. For each enterprise $I \in I$,

$$\sum_{(i,j) \in C} F_{i,j} \leq K \cdot y_C, \quad \forall C \in D, \tag{18}$$

   where K is a constant large enough. This constraint ensures that, if $(i, j) \notin C$, then $F_{i,j} = 0$.

From the new boolean variable $y \in \{0, 1\}^{|D|}$, we set up the graph associated to y as

$$E(y) = \left(\bigcup \{C \ : \ y_C = 1\} \right) \cup \{(i, 0) \ : \ i \in I\}. \tag{19}$$

Now, let's consider the following Mixed-Integer optimization problem:

$$\begin{aligned} &\min_{F \in \mathbb{R}^{|E_{\max}|}, y \in \{0,1\}^{|D|}} \quad Z(F) \\ &s.t. \left\{ \text{Equations (1)-(3)-(4)-(11)-(17)-(18).} \right. \end{aligned} \tag{20}$$

Theorem 3. *If (F, E) is an optimal solution of problem (15), then (F, y^E) is an optimal solution of problem (20), where $y^E \in \{0, 1\}^{|D|}$ is defined as*

$$y_C^E = \begin{cases} 1 & \textit{if } C = C_i(E) \textit{ for some } i \in I, \\ 0 & \textit{otherwise.} \end{cases}$$

If (F, y) is an optimal solution of problem (20), then $(F, E(y))$ is an optimal solution of problem (15).

For the proof of Theorem 3, we prefer the reader to Appendix B in Part 2 of the paper [5].

3.3 Null Class as Exit Option

We can observe that the stand-alone configuration E_{st} is always a feasible configuration for the EIP model. However, with the constraint (11), the problem may become infeasible. Therefore, we need to take into account the possibility of excluding some enterprises from the network when the EIP authority does not ensure to satisfy constraint (11) for such enterprises.

Now, for each enterprise $i \in I$, we introduce a boolean variable $y_{i,\mathrm{null}} \in \{0,1\}$ such that

$$y_{i,\mathrm{null}} = \begin{cases} 1 & \text{if } i \text{ breaks the constraint (11),} \\ 0 & \text{otherwise.} \end{cases}$$

With this boolean variable, we will add the following constraints to problem (20).

1. For each enterprise $i \in I$,
$$y_{i,\mathrm{null}} + \sum_{C \in \mathcal{C}_i} y_C = 1, \tag{21}$$
2. For each enterprise $i \in I$,
$$\sum_{(i,j) \in C(i,0)} F_{i,j} \leq K \cdot (y_{C(i,0)} + y_{i,\mathrm{null}}), \tag{22}$$
$$\sum_{(i,j) \in E_{\max}, j \neq 0} F_{i,j} \leq K \cdot (1 - y_{i,\mathrm{null}}). \tag{23}$$
Constraints (22) and (23) are to ensure that, if the enterprise breaks the constraint, then the enterprise e do not share polluted water with other enterprises and will use the connection $(i,0)$.
3. For each enterprise $i \in I$,
$$\sum_{(k,i) \in E_{\max}} F_{k,i} \leq K \cdot (1 - y_{i,\mathrm{null}}). \tag{24}$$
This constraint is to ensure that, if the enterprise breaks the constraint (11), then no enterprises can send him any polluted water.
4. For each enterprise $i \in I$,
$$\mathrm{Cost}_i(F_i, F_{-i}, E(y)) \leq \alpha_i \cdot \mathrm{STC}_i \cdot (1 - y_{i,\mathrm{null}}) + \mathrm{STC}_i \cdot y_{i,\mathrm{null}}. \tag{25}$$

We denote by $\overline{D} = D \cup \{\mathrm{Null}_i \; : \; i \in I\}$, where Null_i is the null class, associated to $y_{i,\mathrm{null}}$, and $D_0 = D \setminus \{C(i,0) \,:\, i \in I\}$. Denoting

$$\mathrm{STC}_i(y_{i,\mathrm{null}}) := \alpha_i \cdot \mathrm{STC}_i \cdot (1 - y_{i,\mathrm{null}}) + \mathrm{STC}_i \cdot y_{i,\mathrm{null}}.$$

With all the foregoing, problem (20) becomes

$$\min_{F \in \mathbb{R}^N, y \in \{0,1\}^{|\overline{D}|}} \quad Z(F)$$
$$s.t. \begin{cases} \text{Equations (1)-(3)-(4)-(11)-(21)-(22)-(23)-(24),} \\ \displaystyle\sum_{(i,j) \in C} F_{i,j} \leq K \cdot y_C, \qquad \forall C \in D_0. \end{cases} \tag{26}$$

4 Numerical Experiments

4.1 Case Study

Now, we simulate numerical examples of the model described in Sect. 2. We assume that

$$\gamma_{i,j} = \begin{cases} \delta & \text{if } j \in I, \\ \beta & \text{if } j = 0. \end{cases} \tag{27}$$

From (27), we can observe that each enterprise i has to pay for both water receipt and water deposit. Therefore, for each enterprise $i \in I$, the set $\mathcal{C}_i = \{C_{i,p}, C_{i,0}\}$ where

$$C_{i,p} = \{(i,j) \in E_{\max} \ : \ j \in I\} \quad \text{and} \quad C_{i,0} = \{(i,0)\}.$$

Now, for each enterprise $i \in I$, we introduce three integer variables, $y_{i,p}, y_{i,0}$, $y_{i,\text{null}} \in \{0,1\}$ as follows:

- If $y_{i,p} = 1$, it means the connections in $C_{i,p}$ are included in the network.
- If $y_{i,0} = 1$, it means the connection $(i,0)$ is the only exit connection for i, and i participates in the EIP.
- If $y_{i,\text{null}} = 1$, it means the connection $(i,0)$ is the only exit connection for i, and i does not participate in the EIP.

Note that only one of these integer variables takes the value 1, i.e., $y_{i,\text{null}} + y_{i,p} + y_{i,0} = 1, \forall i \in I$, and in doing so it determines the network E to be implemented and the operation that each enterprise can do within this network. We denote by $y \in \{0,1\}^{3n}$ the vector of all integer variables of all enterprises.

Since the optimization problem (26) can have several solutions and to get a solution with more enterprises involved, we replace $Z(F)$ by

$$Z(F) + \text{Coef} \cdot \sum_{i \in I} y_{i,\text{null}}, \tag{28}$$

where Coef ≥ 0 is a coefficient to penalize the objective function. Then, the optimization problem (26) becomes:

$$\begin{aligned} \min_{F,y} \quad & Z(F) + \text{Coef} \cdot \sum_{i \in I} y_{i,\text{null}} \\ s.t. \ & \begin{cases} \text{Equations (1)-(3)-(4)-(11)}, & \\ y_{i,\text{null}} + y_{i,p} + y_{i,0} = 1, & \forall i \in I, \\ \displaystyle\sum_{(i,j) \in C_{i,p}} F_{i,j} \leq K \cdot y_{i,p}, & \forall i \in I, \\ F_{i,0} \leq K \cdot (y_{i,0} + y_{i,\text{null}}), & \forall i \in I, \\ \displaystyle\sum_{(i,j) \in E_{\max}, j \neq 0} F_{i,j} \leq K \cdot (1 - y_{i,\text{null}}), & \forall i \in I, \\ \displaystyle\sum_{(k,i) \in E_{\max}} F_{k,i} \leq K \cdot (1 - y_{i,\text{null}}), & \forall i \in I. \end{cases} \end{aligned} \tag{29}$$

We use input data on the scale of an EIP of 15 enterprises. The data is given in Table 1 and Table 2. Data is partially inspired from [4]. Additionally, it is supposed that the EIP operates A = 1 h.

Table 1. Parameters of the network.

Enterprise i	$C_{i,\text{in}}$(ppm)	$C_{i,\text{out}}$(ppm)	M_i(g/h)
1	0	50	1000
2	0	100	2500
3	10	50	1500
4	10	100	5000
5	30	300	20000
6	40	600	5000
7	20	200	5000
8	50	400	15000
9	50	100	10000
10	50	600	20000
11	110	450	15000
12	200	400	10000
13	500	1100	30000
14	300	3500	15000
15	600	2500	25000

Table 2. Associated costs.

Parameter	Value ($/tonne)
c	0.13
β	0.22
δ	0.01

All simulations below performed using Julia `Julia v1.0.5` programming language, using `CPLEX V12.10.0` as a solver.

4.2 Results and Discussion

In this section, we present the numerical results with the above data. The resulting optimized EIP network is shown in Fig. 1, and it corresponds to $\alpha = 0.90$ and Coef = 1. This optimal network provides operating cost of each enterprise and total freshwater consumption that are lower than a stand-alone network as shown in Table 3.

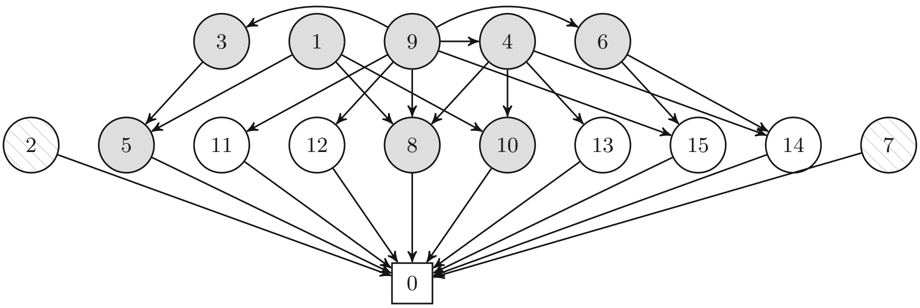

Fig. 1. The optimal configuration in the case $\alpha_i = 0.90$ and Coef $= 1$. Dashed nodes are operating in stand-alone node and gray nodes are active consuming freshwater.

Table 3. Summary of results of the EIP.

Enterprise	Freshwater stand-alone (T/h)	Freshwater in EIP (T/h)	Cost_i stand-alone (MMUSD/hour)	Cost_i in EIP (MMUSD/hour)	% Reduction in Cost_i
1	20.00	20.00	7.00	2.80	60.00
2	25.00	25.00	8.75	8.75	0.00
3	30.00	33.75	10.50	4.80	54.28
4	50.00	50.00	17.50	7.11	59.36
5	66.67	35.13	23.33	21.00	10.00
6	8.33	5.36	2.92	0.82	71.84
7	25.00	25.00	8.75	8.75	0.00
8	37.50	16.74	13.12	11.81	10.00
9	100.00	100.00	35.00	14.00	60.00
10	33.33	13.74	11.67	10.01	14.18
11	33.33	0.00	11.67	9.86	15.51
12	25.00	0.00	8.75	7.67	12.38
13	27.27	0.00	9.54	6.90	27.71
14	4.28	0.00	1.50	1.08	28.12
15	10.00	0.00	3.50	2.73	21.89
Total	495.72	324.72	173.50	118.09	31.94

For this case study, when the enterprises operate stand alone, then the entire system consumes a total 495.72 (T/h) of freshwater. The optimal design obtained with SLMF approach allowed to reduce its freshwater requirement to 324.72 (T/h) which is equivalent to a reduction of 34.5%. Furthermore, the water demand of enterprises 11, 12, 13, 14, and 15 are entirely supplied by other enterprises.

When working in the EIP, the EIP authority ensures that each enterprise gets at least a 10% reduction in costs. Total operating cost is reduced compared to the stand-alone case, as expected, from 173.50 ($/h) to 118.09 ($/h), i.e., 31.94% reduction. Nevertheless, exploring Table 3, we can observe that the cost reduction is not uniform across the enterprises. In the optimized network, enterprise 6 achieves the highest percentage reduction of operating cost corresponding to 71.84% while enterprises 5 and 8 have the lowest reduction corresponding to 10.00% with respect to the stand-alone configuration. Nevertheless, the EIP authority cannot guarantee that the enterprises 2 and 7 reduce their operating costs by at least 10%, so they operate stand-alone.

Moreover, the benefit obtained varies with the different values of α. Figure 2 is a box plot to describe the distribution of the cost reduction of participating enterprises in the park with parameters α in the interval $[0.80, 0.99]$.

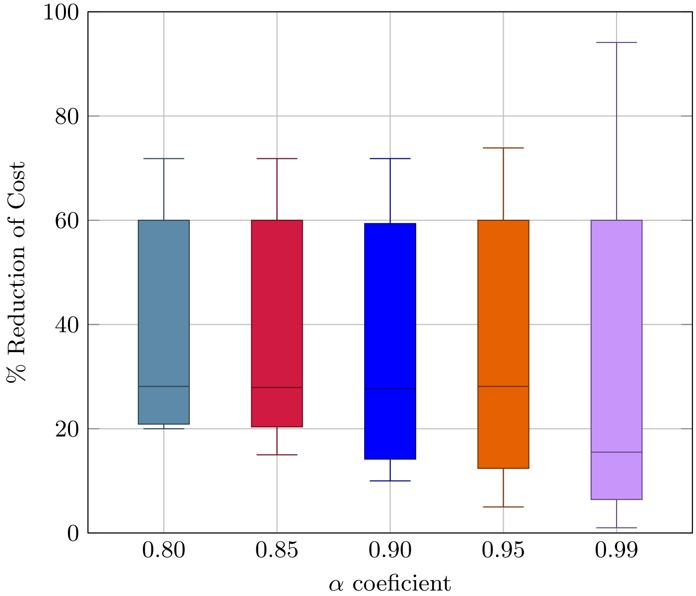

Fig. 2. % Reduction of cost with $\alpha \in [0.80, 0.99]$ and Coef = 1.

A box plot is a standardized way of displaying the distribution of data based on a five number summary: lower limit, lower quartile (Q_1: 25th Percentile), median (Q_2: 50th Percentile), upper quartile (Q_3: 75th Percentile), and upper limit. Let us choose $\alpha = 0.90$ to analyze the distribution of the cost reduction of enterprises, and other parameters α can be analyzed analogously. All enterprises will reduce less than or equal to 71.84% (upper limit). All enterprises can reduce operating costs by at least 10% (lower limit). At least 75% of enterprises can reduce costs by 14.18% or higher. The lower (resp. upper) quartile Q_1 (resp. Q_3) is 14.18% (resp. 59.36%) which is equivalent to that there are 25% (resp. 75%) of enterprises can reduced their operating costs less than or equal to 14.18% (resp. 59.36%), while the median reduction Q_2 in this case is 27.71%, so exactly haft the enterprises are reduced lesser or higher 27.71% compared to their stand-alone operating costs.

5 Conclusion and Perspectives

In this work, we have presented a game theory methodology to optimize the water exchange networks in the EIP. The approach is validated on a case study of water integration in the EIP without regeneration units. Using this methodology the results show that the game theory methodology is very reliable in the multi-criteria scenarios of the EIP design. More precisely, the EIP authority ensures that each enterprise gets at least a 10% reduction in costs on one hand, on the other hand the total freshwater consumption and total operating cost in the optimal configuration have been reduced by 34.5% and 31.94%, respectively.

A methodology taking into account in this present work only the single contaminant case is implemented. Thus, we would like to address the EIP design with multi contaminant case in the future.

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