An Accurate Frequency Estimation Algorithm by Using DFT and Cosine Windows

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Abstract. Sinusoidal signal frequency estimation is one of the fundamental problems in signal processing, and it is widely used in wireless communication, signal processing, navigation, radar and so on. In this paper, an interpolation frequency estimation algorithm based on Discrete Fourier Transform (DFT) and cosine windows is proposed. Firstly, the sampling sequence of the signal is multiplied by a cosine window. Then, *N*-point DFT is used to search the position of the maximum spectral line and get the coarse estimation of frequency. Finally, the accurate frequency estimation is obtained by DFT interpolation of the maximum spectral line and the two Discrete-Time Fourier Transform (DTFT) samples on the left and right of the maximum spectral line. According to the simulation results, the performance of the proposed algorithm is better than that of MV-IpDTFT(3) algorithm, MV-IpDTFT(2) algorithm and Candan algorithm. The effect of harmonic interference on the frequency estimation results can be effectively suppressed.

Keywords: Frequency estimation · Interpolation · DFT · Cosine windows · Signal processing

1 Introduction

Frequency estimation is a fundamental subject that has been widely studied and applied in wireless communication, signal processing, navigation, radar and so on. For example, as the oscillation frequency produced by crystal oscillator may deviate from the nominal frequency, and the relative motion between the transmitter and receiver generally leads to Doppler frequency shift, the carrier frequency offset generally exists in communication system. The carrier frequency offset will destroy the orthogonality between subcarriers, resulting in inter subcarrier interference and system performance degradation. Therefore, it is necessary to estimate the carrier frequency offset

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accurately, which is very important for multi carrier system such as Orthogonal Frequency Division Multiplexing (OFDM) system. For another example, for a low orbit earth satellite, there is obvious Doppler frequency shift between the satellite and the ground station. The Doppler frequency shift must be accurately estimated by the satellite ground station, because its estimation accuracy determines the positioning and orbit determination accuracy of the satellite monitoring system. In satellite navigation receiver, carrier frequency estimation is a key problem in the process of satellite signal acquisition and tracking. These applications are based on single tone frequency sinusoidal signal model. It is necessary to estimate the frequency of sinusoidal signal. And the frequency estimation performance directly affects the performance of these applications.

Specifically, in the background of additive white Gaussian noise, well known frequency estimation algorithms can be divided into two categories, the time domain estimation algorithm and the frequency domain estimation algorithm. The time domain estimation algorithm includes the algorithm based on autocorrelation, the algorithm based on least squares and the algorithm based on maximum likelihood. The maximum likelihood algorithm has high accuracy and reaches the Cramer-Rao lower bound (CRLB), but it is computationally expensive and difficult for real-time realization [1]. The frequency domain estimation algorithms are mainly based on Discrete Fourier Transform (DFT) [2–7, 13]. They have relatively small amount of computation, and significant Signal-to-Noise Ratio (SNR) gain for sinusoidal signals. And they can be applied for real-time applications. For the frequency estimation algorithms based on DFT, the coarse estimation is carried out by searching the maximum spectral line position, and then the accurate estimation value is obtained according to different algorithms. Candan algorithm [2] used the maximum spectral line and its left and right spectral lines to achieve accurate estimation. The algorithm of Aboutanios and Mulgrew (A&M) used two spectrum lines for accurate estimation [3]. When $N = 1024$ and $SNR = 0$ dB, the estimation variance is only 1.0147 times of CRLB. Lei Fan algorithm [4] obtained the accurate frequency estimation by the interpolation of the primary spectral line and two auxiliary spectral lines located at arbitrary position within the main lobe or the first sidelobe of the frequency spectrum. The influence of the interval between the primary spectral line and the auxiliary spectral lines on the Mean Square Error (MSE) is analyzed through theoretical analysis and simulation experiments.

In the references mentioned above, the frequencies of sinusoidal signal are estimated in the additive white Gaussian noise background. However, in practical application, there are both additive white Gaussian noise and interference signals. For this situation, the appropriate window function can be selected for multiply the sampling sequence, and then the signal frequency is estimated based on DFT interpolation algorithm, which can effectively reduce the interference caused by useless spectrum components [8–12]. In [10], Candan derived the estimation expression of the algorithm in [2] when arbitrary window functions were used, and only needed to select different deviation correction coefficients according to the specific window function. When there is a single strong interference signal and additive Gaussian white noise, the algorithm can effectively reduce the impact of interference signal. In [11], Belega derived the frequency estimation expressions of Candan algorithm in [2] and AM algorithm in [3] when windows were used, obtaining MV-IpDTFT(2) algorithm and MV-IpDTFT(3) algorithm respectively, and derived the calculation formula of frequency estimation variance. Through the simulation experiment, the performance of different windowing algorithms in the background of harmonic interference and additive white Gaussian noise is analyzed.

In this paper, an accurate interpolation frequency estimation algorithm based on DFT and cosine windows is proposed. Firstly, the sampling sequence of the signal is multiplied by a cosine window. Then, N-point DFT is used to search the position of the maximum spectral line, and get the coarse estimation of frequency. Finally, the accurate frequency estimation is obtained by DFT interpolation of the maximum spectral line and the two Discrete-Time Fourier Transform (DTFT) samples on the left and right of the maximum spectral line. It can be seen from the simulation results, the performance of the proposed algorithm is better than the other methods, and the influence of harmonic interference on frequency estimation results is effectively suppressed.

2 Proposed Algorithm

This part mainly describes an accurate interpolation frequency estimation method based on DFT and cosine windows. The model of frequency sinusoidal signal in the background of additive white Gaussian noise is

$$s(t) = Ae^{\mathrm{j}(2\pi f_0 t + \theta_0)} + z(t) \tag{1}$$

where A is the amplitude of the complex sinusoid, f_0 is the frequency, θ_0 is the primary phase, $z(t)$ is the noise, N is the number of sampling and f_s is the sampling frequency. After sampling, we have

$$s(n) = Ae^{j(2\pi \frac{f_0}{f_s} n + \theta_0)} + z(n), n = 0, 1, 2, \ldots, N-1 \tag{2}$$

We perform DFT on $s(n)$, and get

$$S[k] = Ae^{\mathrm{j}\theta_0} \sum_{n=0}^{N-1} e^{\mathrm{j}2\pi \frac{f_0}{f_s} n} e^{-\mathrm{j}\frac{2\pi}{N} nk} + Z[k] \tag{3}$$

Let $S[l]$ denote the maximum spectrum line in the frequency spectrum indexed by the DFT. l is the index value of the maximum spectrum line, $\Delta f = f_s/N$ is frequency resolution, and $s_0(n)$ is signal without noise. For ease of expression, the DFT sample value at the position $f = (l+p)\Delta f$ can be denoted as:

$$S_p = \sum_{n=0}^{N-1} s_0(n) e^{-\mathrm{j}2\pi fn} \big|_{f=(l+p)\Delta f} \tag{4}$$

When there are both additive white Gaussian noise and narrowband interference signals, the time-domain windowing algorithm can be used to reduce the interference caused by useless spectrum components and improve the anti-interference performance of the estimation algorithm. Cosine windows are widely used in many references [10, 11], and their expression are

$$w(n) = \sum_{h=0}^{H-1} (-1)^h a_h \cos(2\pi \frac{h}{N} n), n = 0, 1, \ldots, N-1 \tag{5}$$

where $H \geq 1$ is the number of window coefficients a_h. Maximum sidelobe decay window (MSD window) belongs to cosine windows. When H is constant, the sidelobe decay rate of MSD window is as high as $6(2H-1)$ dB/octave. The coefficients of H-term MSD window can be expressed as:

$$a_0 = \frac{C_{2H-2}^{H-1}}{2^{2H-2}}, a_h = \frac{C_{2H-2}^{H-h-1}}{2^{2H-3}}, h = 1, 2, \ldots, H-1 \tag{6}$$

Multiply $s(n)$ by cosine window, we have

$$s_w(n) = s(n) \cdot w(n), n = 0, 1, \ldots, N-1 \tag{7}$$

Then we perform DTFT on $s_w(n)$, and get

$$S_w(\lambda) = AW(\lambda - v)e^{j\varphi} \tag{8}$$

where $v = l + \delta$ is quantized value of the signal frequency, l is the index value of the maximum spectrum line. δ denotes the normalized frequency offset, and its value range is $[-0.5, 0.5]$. If it is assumed that the number of sampling points obtained is large enough ($N \gg 1$), then we have

$$W(\lambda) = \frac{N \sin(\pi\lambda)}{\pi} \sum_{h=0}^{H-1} (-1)^h a_h \frac{\lambda}{\lambda^2 - h^2} e^{-j\pi\lambda} = \tilde{W}(\lambda) e^{-j\pi\lambda} \tag{9}$$

in which

$$\tilde{W}(\lambda) = \frac{N \sin(\pi\lambda)}{\pi} \sum_{h=0}^{H-1} (-1)^h \alpha_h \frac{\lambda}{\lambda^2 - h^2} \geq 0 \tag{10}$$

Calculating the derivation of (10), we have

$$\tilde{W}'(\lambda) = \frac{N}{\pi} [\sin(\pi\lambda) + \pi\lambda \cos(\pi\lambda)] \sum_{h=0}^{H-1} \frac{(-1)^h a_h}{\lambda^2 - h^2} - \frac{2N}{\pi} \lambda^2 \sin(\pi\lambda) \sum_{h=0}^{H-1} \frac{(-1)^h a_h}{(\lambda^2 - h^2)^2} \tag{11}$$

In [4], the estimation formula for the normalized frequency offset δ is expressed as follows:

$$\hat{\delta} = \frac{N}{\pi}\tan^{-1}\left\{\mathrm{Re}\left[\frac{(|S_i| - |S_{-i}|)\cdot\sin(\frac{\pi i}{N})}{(|S_i| + |S_{-i}|)\cdot\cos(\frac{\pi i}{N}) - 2\cos(\pi i)\cdot|S_0|}\right]\right\} \tag{12}$$

When $i = 0.1$, denoting the curly bracket as U and replacing the corresponding spectral lines in (12) by $S_w(l+0.1)$, $S_w(l-0.1)$ and $S_w(l)$, we have

$$U = \frac{[|S_{\mathrm{w}}(l+0.1)| - |S_w(l-0.1)|]\cdot\sin(\frac{\pi}{10N})}{[|S_w(l+0.1)| + |S_w(l-0.1)|]\cdot\cos(\frac{\pi}{10N}) - 2\cos(\frac{\pi}{10})\cdot|S_w(l)|} \tag{13}$$

In (8), let $\lambda = l+0.1$, $\lambda = l-0.1$ and $\lambda = l$,we have

$$|S_w(l+0.1)| = |A|\,\widetilde{W}(0.1-\delta) \tag{14}$$

$$|S_w(l-0.1)| = |A|\,\widetilde{W}(-0.1-\delta) \tag{15}$$

$$|S_w(l)| = |A|\,\widetilde{W}(-\delta) \tag{16}$$

Substituting (14), (15) and (16) into (13) to simplify the formula, we have

$$U = \frac{[\tilde{W}(0.1-\delta) - \tilde{W}(-0.1-\delta)]\cdot\sin(\frac{\pi}{10N})}{[\tilde{W}(0.1-\delta) + \tilde{W}(-0.1-\delta)]\cdot\cos(\frac{\pi}{10N}) - 2\cos(\frac{\pi}{10})\cdot\tilde{W}(-\delta)} \tag{17}$$

Carrying out the first order Taylor series expansion for $\tilde{W}(0.1-\delta)$, $\tilde{W}(-0.1-\delta)$ and $\tilde{W}(-\delta)$ near 0.1, −0.1 and 0 respectively, and ignoring the higher order term, we have

$$\tilde{W}(0.1-\delta) = \tilde{W}(0.1) + \tilde{W}'(0.1)(-\delta) = \tilde{W}(0.1) - \tilde{W}'(0.1)\delta \tag{18}$$

$$\widetilde{W}(-0.1-\delta) = \widetilde{W}(-0.1) + \tilde{W}'(-0.1)(-\delta) = \widetilde{W}(0.1) + \tilde{W}'(0.1)\delta \tag{19}$$

$$\tilde{W}(-\delta) = \tilde{W}(0) + \tilde{W}'(0)(-\delta) \tag{20}$$

Substituting (18), (19) and (20) into (17), we have

$$U = \frac{\tilde{W}'(0.1)\cdot\sin(\frac{\pi}{10N})}{\cos(\frac{\pi}{10})\cdot\tilde{W}(0) - \tilde{W}(0.1)\cdot\cos(\frac{\pi}{10N})}\cdot\delta \tag{21}$$

The estimation expression of δ can be obtained as

$$\hat{\delta} = \frac{\cos(\frac{\pi}{10}) \cdot \tilde{W}(0) - \tilde{W}(0.1) \cdot \cos(\frac{\pi}{10N})}{\tilde{W}'(0.1) \cdot \sin(\frac{\pi}{10N})} \cdot U = r \cdot U \tag{22}$$

$\tilde{W}(0)$ and $\tilde{W}(0.1)$ can be calculated according to (10), and $\tilde{W}'(0.1)$ can be calculated according to (11). Substituting $\tilde{W}(0)$, $\tilde{W}(0.1)$ and $\tilde{W}'(0.1)$ into (22), we have

$$r = \frac{\cos(\frac{\pi}{10}) \cdot \alpha_0 - \cos(\frac{\pi}{10N}) \cdot \sin(\frac{\pi}{10}) \cdot \frac{1}{\pi} \cdot \sum_{h=0}^{H-1} (-1)^h \frac{\alpha_h \cdot 0.1}{0.01 - h^2}}{\left\{ [\frac{1}{\pi}\sin(\frac{\pi}{10}) + \frac{1}{10}\cos(\frac{\pi}{10})] \sum_{h=0}^{H-1} \frac{(-1)^h \alpha_h}{0.01 - h^2} - \frac{1}{50\pi}\sin(\frac{\pi}{10}) \sum_{h=0}^{H-1} \frac{(-1)^h \alpha_h}{(0.01 - h^2)^2} \right\} \cdot \sin(\frac{\pi}{10N})} \tag{23}$$

For two term MSD windows, we have $r_{2MSD} = 134.97$. For three term MSD windows, we have $r_{3MSD} = 232.91$. We take the iterative procedures as shown in Table 1.

Table 1. The proposed algorithm steps.

Algorithm: Proposed an interpolation frequency estimation algorithm	
1	Get $s_w(n) = s(n) \cdot w(n)$, $n = 0, 1, \ldots, N-1$
2	Perform N-point DFT of $s_w(n)$
3	Find l and let $l\Delta f$ be the rough estimation of signal frequency
4	Calculate $S_w(l+0.1)$ and $S_w(l-0.1)$, via $S_w(l+p) = \sum_{n=0}^{N-1} s_w(n) e^{-j2\pi n \frac{l+p}{N}}, p = \pm 0.1$
5	Calculate $\hat{\delta}_1$ with $S_w(l)$, $S_w(l+0.1)$ and $S_w(l-0.1)$, via (22)
6	Calculate $S_w(l+\hat{\delta}_1)$, $S_w(l+\hat{\delta}_1+0.1)$ and $S_w(l+\hat{\delta}_1-0.1)$, via $S_w(l+p) = \sum_{n=0}^{N-1} s_w(n) e^{-j2\pi n \frac{l+p}{N}}, p = \hat{\delta}_1, \hat{\delta}_1 \pm 0.1$
7	Calculate $\hat{\delta}_2$ with $S_w(l+\hat{\delta}_1)$, $S_w(l+\hat{\delta}_1+0.1)$ and $S_w(l+\hat{\delta}_1-0.1)$, via (22)
8	The frequency estimate is $\hat{f} = (l + \hat{\delta}_1 + \hat{\delta}_2)\Delta f$

3 Simulation Results

In order to verify the performance of the proposed algorithm, we carry out simulation analysis in the presence of harmonic interference and additive white Gaussian noise. Meanwhile, for the sake of finding the difference between the performance of the proposed algorithm and that of the competitive algorithms, this part are conducted to compare the performance of the proposed algorithm with that of Candan algorithm [10], MV-IpDTFT(2) algorithm [11] and MV-IpDTFT(3) algorithm [11].

When there are harmonic interference signals, the Total Harmonic Distortion (THD) is used to describe the intensity of the interference signal, and it is defined as follows

$$THD = \sqrt{\frac{\frac{1}{2}\sum_{M=2}^{M_{max}} A_M^2}{\frac{1}{2}A_1^2 + \frac{1}{2}\sum_{M=2}^{M_{max}} A_M^2}} \tag{24}$$

where A_1 is the amplitude of fundamental tone wave and A_M $(M = 2, 3, \cdots, M_{\max})$ is the amplitude of each harmonic. In the following simulation experiment, it is assumed that there are second, third and fourth harmonics, and the ratio of amplitudes is 4:2:1. The phase of fundamental tone and each harmonic is uniformly distributed on interval $[0, 2\pi]$.

When $N = 128$, $SNR = 50dB$, $THD = 5\%$, Fig. 1 show the MSE of δ with respect to the signal frequency quantization value v of the proposed algorithm and MV-IpDTFT(3) algorithm [11]. Two iterations are carried out for both algorithms. The quantized value v is in the interval $[2.51, 12.1]$. We calculate a point with a step of

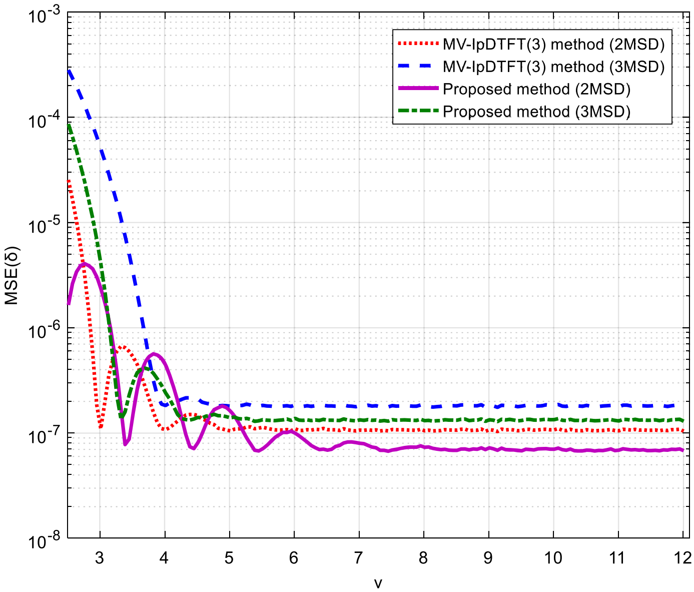

Fig. 1. Simulated MSE of the proposed method and MV-IpDTFT(3) method with respect to v $(N = 128, SNR = 50dB, THD = 5\%)$

1/16. It can be seen from Fig. 1, when $v > 5.5$, the MSE(δ) of the proposed algorithm with two term MSD window is the lowest. When $2.51 \leq v \leq 5.5$ and $|\delta|$ is close to 0.5, the MSE(δ) of the proposed algorithm with two term MSD window is the lowest. When $2.51 \leq v \leq 5.5$ and $|\delta|$ is close to 0, the MSE(δ) of MV-IpDTFT(3) algorithm with two term MSD windows is the lowest.

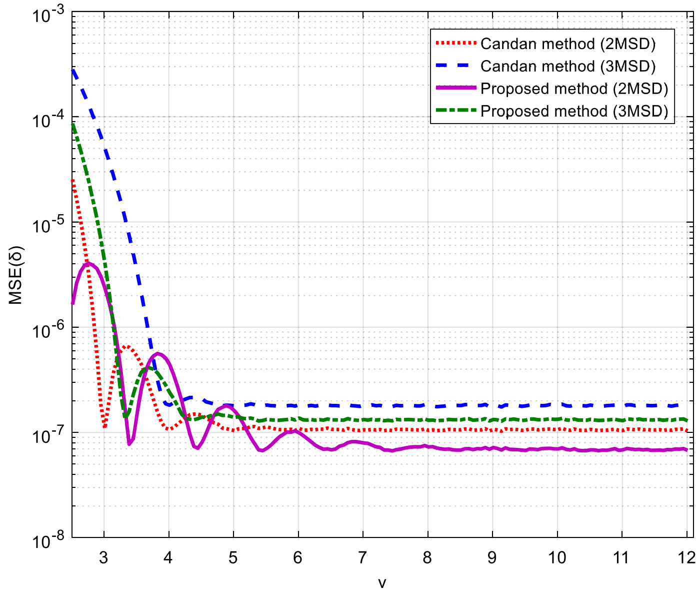

Fig. 2. Simulated MSE of the proposed method and Candan method with respect to v ($N = 128,\ SNR = 50dB,\ THD = 5\%$)

When $N = 128,\ SNR = 50dB,\ THD = 5\%$, Fig. 2 show the MSE of δ with respect to the signal frequency quantization value v of the proposed algorithm and Candan algorithm [10]. Two iterations are carried out for both algorithms. The quantized value v is in the interval $[2.51,\ 12.1]$. We calculate a point with a step of 1/16. It can be seen from Fig. 2, the MSE(δ) of Candan algorithm is similar to that of MV-IpDTFT(3) algorithm. When $v > 5.5$, MSE(δ) of the proposed algorithm with two term MSD window is the lowest. When $2.51 \leq v \leq 5.5$ and $|\delta|$ is close to 0.5, the MSE (δ) of the proposed algorithm with two term MSD window is the lowest. When $2.51 \leq v \leq 5.5$ and $|\delta|$ is close to 0, the MSE(δ) of Candan algorithm with two term MSD windows is the lowest. It can also be seen from Fig. 1 and Fig. 2 that for the same estimation algorithm, when H is smaller, the MSE(δ) is lower, and the estimation performance is better.

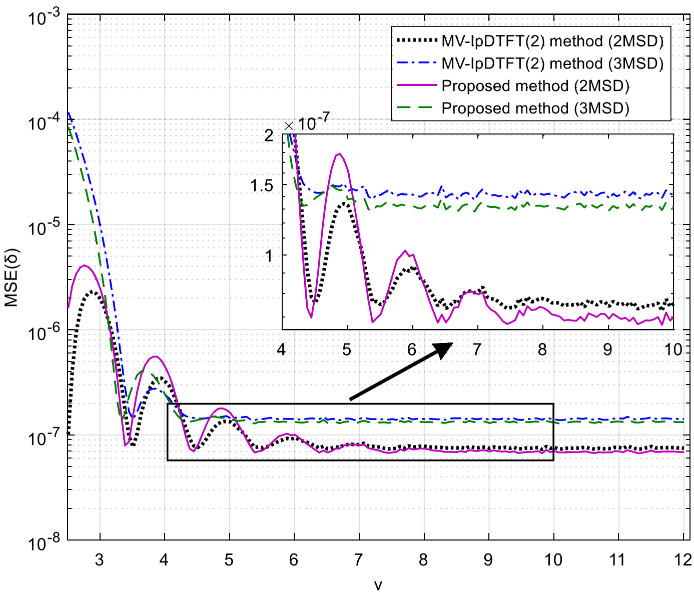

Fig. 3. Simulated MSE of the proposed method and MV-IpDTFT(2) method with respect to v ($N = 128$, $SNR = 50dB$, $THD = 5\%$)

When $N = 128$, $SNR = 50dB$, $THD = 5\%$, Fig. 3 show the MSE of δ with respect to the signal frequency quantization value v of the proposed algorithm and MV-IpDTFT(2) algorithm [11]. Two iterations are carried out for both algorithms. The quantized value v is in the interval [2.51, 12.1]. We calculate a point with a step of 1/16. It can be seen from Fig. 3, when $v > 6$, MSE(δ) of the proposed algorithm with two term MSD window is the lowest. When $v \le 6$, in most cases, the MSE(δ) of MV-IpDTFT(3) algorithm and the proposed algorithm with two term MSD windows are lower than the other algorithms. It can also be seen from Fig. 3 that for the same estimation algorithm, when H is smaller, the MSE(δ) of frequency estimation is lower, and the estimation performance is better.

4 Conclusion

In this paper, an interpolation frequency estimation algorithm based on DFT and cosine windows is proposed. Firstly, the sampling sequence of the signal is multiplied by a cosine window. Then, N-point DFT is used to get the coarse estimation of frequency. Finally, the accurate frequency estimation is obtained by DFT interpolation of the maximum spectral line and the two DTFT samples on the left and right of the maximum spectral line. It can be seen from the simulation results, the performance of the proposed algorithm is better than that of MV-IpDTFT(3) algorithm, MV-IpDTFT(2) algorithm and Candan algorithm. When $2.51 \le v \le 5.5$ and $|\delta|$ is close to 0.5, the MSE (δ) of the proposed algorithm with two-term MSD window is lower than MV-IpDTFT

(3) algorithm and Candan algorithm. When $v > 6$, MSE(δ) of the proposed algorithm with two-term MSD window is lower than the competing algorithms. The proposed algorithm can actively suppress the effects of the harmonic interference signal. It can be used in complex applications with both additive white Gaussian noise and harmonic interference.

References

1. Rife, D.C., Boorstyn, R.R.: Single-tone parameter estimation from discrete-time observations. IEEE Trans. Inform. Theory **55**(9), 591–598 (1974)
2. Candan, C.: Analysis and further improvement of fine resolution frequency estimation method from three DFT samples. IEEE Sig. Process. Lett. **20**(9), 913–916 (2013)
3. Aboutanios, E., Mulgrew, B.: Iterative frequency estimation by interpolation on fourier coefficients. IEEE Trans. Sig. Process. **53**(4), 1237–1242 (2005)
4. Fan, L., Qi, G.Q.: Frequency estimator of sinusoid based on interpolation of three DFT spectral lines. Sig. Process. **144**, 52–60 (2018)
5. Yang, C., Wei, G.: A noniterative frequency estimator with rational combination of three spectrum lines. IEEE Trans. Sig. Process. **59**(10), 5065–5070 (2011)
6. Djukanović, S., Popović-Bugarin, V.: Efficient and accurate detection and frequency estimation of multiple sinusoids. IEEE Access **7**, 1118–1125 (2019)
7. Serbes, A.: Fast and efficient sinusoidal frequency estimation by using the DFT coefficients. IEEE Trans. Commun. **67**(3), 2333–2342 (2019)
8. Andria, G., Savino, M., Trotta, A.: Windows and interpolation algorithms to improve electrical measurement accuracy. IEEE Trans. Instrum. Meas. **38**(4), 856–863 (1989)
9. Belega, D., Dallet, D., Petri, D.: Accuracy of sine wave frequency estimation by multipoint interpolated DFT approach. IEEE Trans. Instrum. Meas. **59**(11), 2808–2815 (2010)
10. Candan, C.: Fine resolution frequency estimation from three DFT samples: case of windowed data. Sig. Process. **114**, 245–250 (2015)
11. Belega, D., Petri, D.: Frequency estimation by two- or three-point interpolated Fourier algorithms based on cosine windows. Sig. Process. **117**, 115–125 (2015)
12. Shin, D., Kwak, C., Kim, G.: An efficient algorithm for frequency estimation from cosine-sum windowed DFT coefficients. Sig. Process. **166**, 245–250 (2020)
13. Fan, L., Qi, G.Q., Xing, J., Jin, J.Y., Liu, J.Y., Wang, Z.S.: Accurate frequency estimator of sinusoid based on interpolation of FFT and DTFT. IEEE Access **8**, 44373–44380 (2020)