Research on Image Enhancement Model Based on Variable Order Fractional Differential CLAHE

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Abstract. Image visual effects can be enhanced primarily through edge and texture enhancement or contrast enhancement. Image enhancement based on fractional differential can effectively enhance image details such as edge and texture using the weak derivative property of the 0–1-order fractional differential operator. Image enhancement based on gray statistics involves the redistribution of light and dark pixels to enhance the overall contrast of the enhanced image as well as the enlargement of the gray-level dynamic range, thereby improving the visual effect of the image effectively. To enhance the edge and texture information of the image, enhance the contrast of the image effectively, and then improve the visual effect of the image, an image enhancement model based on contrast limited adaptive histogram equalization incorporating a fractional differential operator is proposed. The image enhancement model incorporates a fractional differential operator into the adaptive limited contrast image enhancement model, which can enhance the image contrast and effectively enhance the edge and texture details of the image simultaneously. Experimental results show that the proposed variable-order fractional differential contrast-limited adaptive histogram equalization image enhancement model can significantly improve the contrast of the image compared with the traditional fractional differential image enhancement model; additionally, it can effectively enhance the edge and texture details of the image compared with the traditional image enhancement model, which is based on statistical methods.

Keywords: Fractional calculus · Image enhancement · Fractional gradient · Variable order · Histogram enhancement

1 Introduction

Image enhancement mainly emphasizes the local and non-local feature information of an image to achieve clearer images or highlight the edge texture and other important

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features of the image such that the image contrast can be enhanced; hence, the visual effect of the image will be enhanced for later applications in specific occasions. Two image enhancement methods exist: frequency and spatial methods. In the frequency method, an image is transformed into a two-dimensional discrete Fourier transform or cosine transform, and high-frequency information such as edge and texture is enhanced using a high-pass filter, rendering the enhanced image visual effects better or easier to be processed later [1, 2]. The spatial domain method can be classified into image enhancement based on a difference operator and that based on gray-level statistics [3, 4]. The difference operator uses the gray-level difference of neighboring pixels to extract the edge and texture features of the image and then adds the weighted sum to the original image to enhance the important features of the image, such as edges and textures, thereby achieving a better image visual effect. The gray-level statistical method can improve the dynamic range of gray values and image contrasts by redistributing the probability of light and dark pixel values in the image and hence improve the visual effects of the image.

In recent years, research focus on difference image enhancement has expanded from the traditional integer-order differential operator to the fractional-order sub-differential operator. Pu proposed a classical image enhancement model based on a fractional differential operator using the "weak derivative" and "nonlocal" characteristics of the fractional differential operator; as such, the fractional differential operator can enhance the high-frequency component of the image and retain the low- and medium-frequency information of the signal nonlinearly. Therefore, the application of a low-order fractional differential operator to image enhancement can enrich the weak edge and texture of the enhanced image; additionally, the detailed features of the smooth region of the image can be enhanced accordingly [5–7]. On this basis, scholars have proposed many improved image enhancement models based on fractional differential to solve the problems of fractional differential image enhancement models [8–13]. However, the enhancement effect of the fractional differential operator on low-contrast images is insignificant. This is because the fractional differential operator is essentially a difference operator. It mainly enhances the gray value of pixels in abrupt gray areas, such as edges and textures, and does not redistribute the local or nonlocal gray value distribution of the image. Therefore, the enhancement effect of the low-contrast image is poor. The image enhancement method based on pixel gray value statistics is a histogram enhancement technology, among which the histogram equalization method is widely used because of its simplicity and high efficiency [14–18]. The traditional histogram equalization image enhancement is a global image enhancement method that enhances an entire image in a unified scale. Therefore, this method can not only enhance the contrast of the background image, but also reduce the contrast of useful signals, and the gray level of the transformed image will be reduced owing to tradeoffs. Consequently, some details in the image will disappear, and the hierarchical sense of the processed image will be poor. To solve the problems in traditional histogram enhancement technology, Kare proposed a self-adaptive contrast-limited adaptive histogram equalization (CLAHE) image enhancement model [19], which uses block processing to perform histogram equalization to different degrees in different contrast regions of the image. To overcome noise in enlarged images during the enhancement process, a contrast is set.

A threshold is used to control the effects of noise, and a better contrast enhancement effect is obtained. On this basis, scholars have proposed many improvement methods [20, 21]. However, the effect of the CLAHE image enhancement model on images with rich texture details is insignificant. This is because the image enhancement model is based on the pixel statistical redistribution method, which cannot directly enhance the edge and texture details of the image; hence, the image enhancement ability is limited.

In summary, the fractional differential image enhancement model based on the difference method and the CLAHE image enhancement model based on statistical methods have their own advantages and disadvantages. The image enhancement model based on fractional differential can effectively enhance the edge and texture details of the image, but the effect of the model on low-contrast image enhancement is insignificant; hence, it cannot effectively improve the overall and local contrast of the image. Furthermore, the visual effect of the enhanced image is general. Meanwhile, the CLAHE operator can effectively improve the local and nonlocal contrast of an image; additionally, a contrast threshold is introduced to suppress noise amplification during image enhancement. However, the effect of the CLAHE operator on image enhancement with rich texture details is general, and the enhancement of image clarity is insignificant. To enhance the overall and local contrast of an image and effectively enhance the edge and texture details of the image, this study attempts to integrate a fractional-order differential operator into the contrast-constrained adaptive histogram enhancement model; additionally, a CLAHE image enhancement model based on a fractional differential-order local variable is proposed. The image model offers not only the advantages of the classical fractional differential enhancement operator, but also exhibits the ability of the CLAHE model to enhance the local and nonlocal enhancement effect of low-contrast images.

2 Theoretical Background

2.1 Fractional Calculus Theory

Fractional calculus theory is an extension of integral-order calculus theory. Leibniz initially established fractional calculus theory at the end of the 16th century. Subsequently, the development of fractional calculus theory and its application lagged behind. It was not until Riemann Liouville [22] introduced fractional calculus theory into Brownian motion analysis that fractional calculus theory was initially applied in practice. In recent years, fractional calculus theory has been widely used in signal processing and analysis, fractal theory, and fractional order PID controllers. To date, a unified definition for fractional calculus theory does not exist. Generally, different definitions are used based on applications. The definition of fractional Grümwald–Letnikov (G–L) is realized using a difference scheme; therefore, it is highly suitable for processing the gray values of discrete pixels in digital image processing. Furthermore, the fractional G–L definition is conducive to numerical calculations and can be regarded as the extension of the limit form of the integer-order differential.

According to the definition of the integer-order derivative for continuous functions $f(x)$, the first derivative is defined as shown in Eq. (1).

$$f^{'}(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \tag{1}$$

According to a similar definition, the definition of the n-order derivative of a continuous function $f(x)$ can be deduced, as shown in Eq. (2).

$$f^n(x) = \lim_{\Delta x \to 0} \frac{1}{(\Delta x)^n} \sum_{j=0}^{n} (-1)^j \binom{n}{j} f(x - j\Delta x) \tag{2}$$

According to the mathematical properties of the classical gamma function $\Gamma(x)$, $\binom{n}{j} = \frac{n\,!}{j\,!(n-j)!} = \frac{\Gamma(n+1)}{\Gamma(j+1)\Gamma(n-j+1)}$. Assuming that any real number v replaces a positive integer n and considering the special case of the fractional G–L definition in digital image processing, i.e., the distance between adjacent pixels is $\Delta x = 1$, a fractional G–L differential definition suitable for two-dimensional discrete signal processing is obtained. Equation (3) can be simplified to Eq. (4), where $g(i) = (-1)^i \frac{\Gamma(v+1)}{\Gamma(i+1)\Gamma(v-i+1)}$, and the symbol "*" implies a convolution operation.

$$f^v(x) = \lim_{n \to \infty} \sum_{i=0}^{n} (-1)^i \frac{\Gamma(v+1)}{\Gamma(i+1)\Gamma(v-i+1)} f(x-i) \tag{3}$$

$$f^v(x) = \lim_{n \to \infty} \sum_{i=0}^{n} g(i) f(x-i) = g(x) * f(x) \tag{4}$$

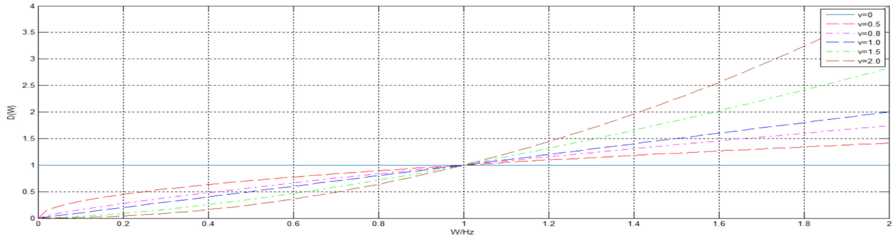
2.2 Amplitude Frequency Characteristics of Fractional Calculus Operators

To analyze the amplitude–frequency characteristics of the one-dimensional signal fractional calculus operator, the function $g(x)$ is transformed into a frequency space. Both sides of Eq. (4) are Fourier transformed to obtain Eq. (5), where the frequency domain function of the fractional calculus is $G(\omega)$, as shown in Eq. (6).

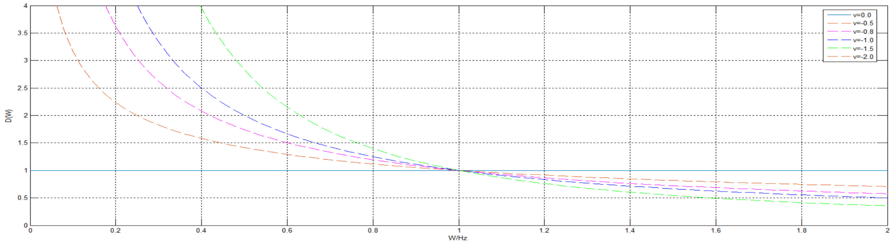
$$FT\left(f^v(x)\right) = FT\ (g(x) * f(x)) = G(\omega) \times F(\omega) \tag{5}$$

$$G(\omega) = FT\left((-1)^i \frac{\Gamma(v+1)}{\Gamma(i+1)\Gamma(v-i+1)}\right) = |\omega|^v e^{i\frac{v\pi}{2}\mathrm{sgn}(\omega)} \tag{6}$$

Figure 1 shows the amplitude–frequency characteristic curves of the fractional differential operator and the fractional integral operator based on the fractional G–L definition. Subgraph (a) represents the amplitude–frequency characteristic curve corresponding to the differential order defined by the fractional-order G–L in the range of $v \in \{0.0,\ 0.5,\ 0.8,\ 1.0,\ 1.5,\ 2.0\}$. Direct observation shows that the fractional-order G–L calculus operator with a positive order can enhance the high-frequency part of the signal ($\omega > 1$). Furthermore, the enhancement amplitude will increase rapidly with the increase in the differential order, but the enhancement amplitude is not as significant as that of the high-order integer differential. The subpositive fractional G–L differential operator can enhance the middle- and low-frequency parts of the signal ($\omega < 1$) to a certain extent, but the high-order sub-integer differential operator has a certain degree of attenuation effect on the low- and medium-frequency information. Subgraph (b) shows that the fractional-order range $v \in \{0.0, -0.5, -0.8, -1.0, -1.5, -2.0\}$ defined by G–L is the corresponding amplitude–frequency characteristic curve. Direct observation shows that the fractional-order G–L calculus operator with a negative order had a certain degree of nonlinear attenuation on the high-frequency part of the signal ($\omega > 1$), and the attenuation amplitude decreased gradually with the decrease in the differential order. Compared with the high-order integral operator, the fractional-order G–L calculus operator can retain more high-frequency information. The fractional-order G–L calculus operator with a negative order had a certain degree of nonlinear enhancement on the middle- and low-frequency parts of the signal ($\omega < 1$), and the enhancement amplitude increased significantly with the decrease in the differential order.



a. Fractional differential operator



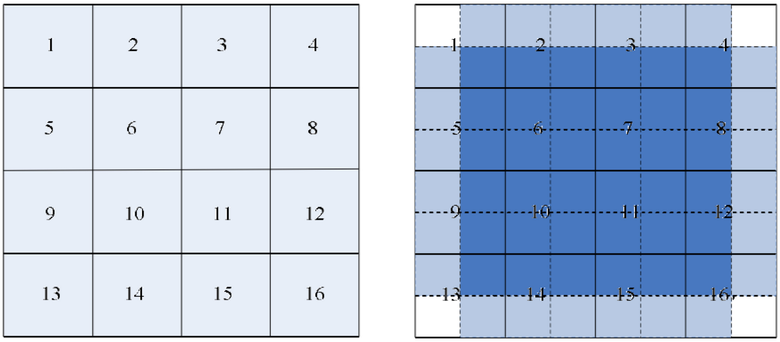
b. Fractional integral operator

Fig. 1. Amplitude–frequency characteristic curve of fractional order G–L calculus operator

2.3 CLAHE Model

To solve the block effect and real-time problem of the traditional histogram enhancement model, Karel proposed a contrast-constrained adaptive histogram image enhancement algorithm. The core idea of the CLAHE algorithm is to use the image segmentation mechanism. Before calculating the cumulative histogram of each image block, a clipping threshold is determined to increase the amplitude of the original image cumulative histogram. The clipping part cannot be omitted but is redistributed to the original image histogram according to certain rules. Finally, bilinear interpolation is used to improve the timeliness of the CLAHE algorithm, and the block effect is eliminated. The core steps of the CLAHE algorithm are as follows.

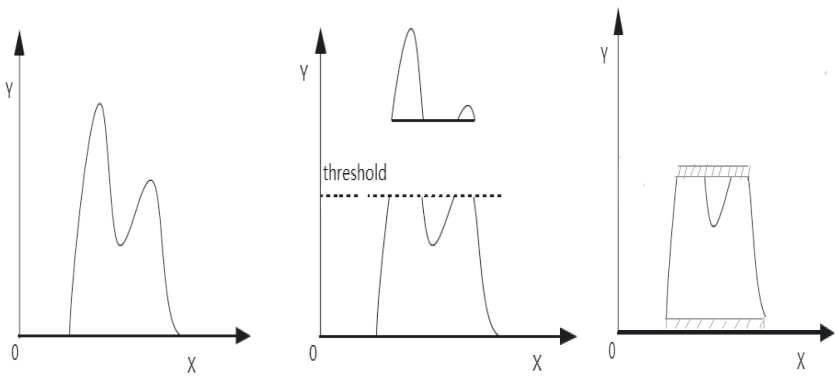
Step 1. The image is divided into n * n image non-overlapping sub-blocks of the same size, as shown in in Fig. 2(a).



a. Block image b. Sub-block image partition

Fig. 2. Image block diagram for n = 4

Step 2. The histogram of the current image sub-block is calculated, and the clipped image histogram is determined according to the image gray threshold, as shown in Fig. 3. The number of pixels in the image block whose gray histogram is greater than the histogram clipping threshold is accumulated and stored in the exceeding threshold vector. Subsequently, the histogram is redistributed repeatedly to satisfy the clipping threshold. Finally, the image sub-block is processed using traditional histogram equalization.



a. Histogram of original image b. Histogram after clipping c. Histogram after reallocation

Fig. 3. Schematic diagram of image sub block histogram clipping process

Step 3. According to the area divided by subgraph (b) in Fig. 2, interpolation calculation is performed using different methods. Specifically, as shown in Fig. 4, for the pixels in the dark region, bilinear interpolation was performed according to the gray value of the pixel in the four neighborhood regions. For the pixel in the light color region, the gray value was linearly interpolated according to the gray value of the pixel in the photographic neighborhood. The gray value of the pixel in the colorless region was determined according to the histogram mapping of the sub-block. The specific interpolation method is expressed in Eq. (7).

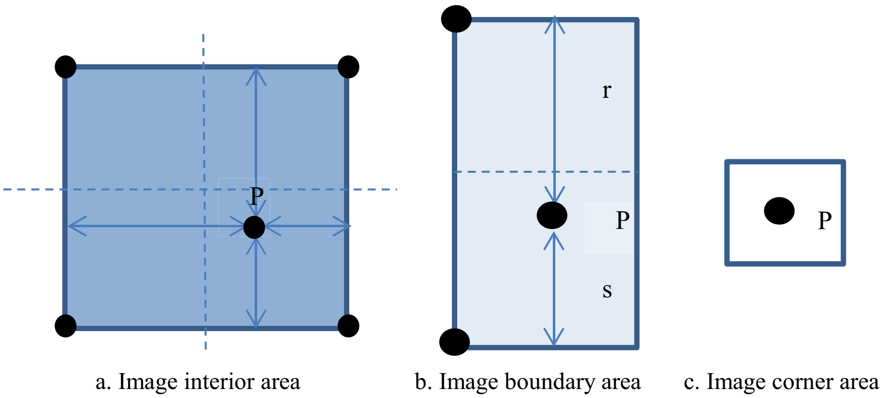


Fig. 4. Schematic diagram of pixel interpolation method in different regions

$$\mathrm{P_{new}} = \begin{cases} \frac{\mathrm{s}}{s+r}\left(\frac{y}{x+y}f_{i,j}(P_{old}) + \frac{x}{x+y}f_{i+1,j}(P_{old})\right) + \frac{\mathrm{r}}{s+r}\left(\frac{y}{x+y}f_{i,j+1}(P_{old}) + \frac{x}{x+y}f_{i+1,j+1}(P_{old})\right) \\ \frac{\mathrm{s}}{s+r}f_{i,j}(P_{old}) + \frac{\mathrm{r}}{s+r}f_{i,j}(P_{old}) \\ \mathrm{P_{old}} \end{cases} \quad (7)$$

3 Fractional Differential Contrast-Limited Adaptive Histogram Equalization (FCLAHE) Model

To enhance the edge and texture details of the image, effectively enhance the image contrast, and improve the visual effect of the image, a model combining the advantages of the fractional-order differential operator and the CLAHE model is proposed herein, known as the variable-order FCLAHE image enhancement model.

3.1 Image Detail Measure

(1) Fractional gradient modulus of the image

The image gradient is described as a measure in a bounded variation function or distribution space, denoted as BV(Ω), in which discontinuous jump features such as edges and textures are allowed. Therefore, the space BV(Ω) is often used to describe the global characteristics of images in an image processing model based on a variational method [23]. In this study, based on the total variation and AAA BV(Ω) AA space and combined with fractional calculus theory, the fractional total variation and fractional BV(Ω) space were extended.

Suppose Ω is a bounded subset of the image plane and image $f \in L^1(\Omega)$. If the distribution derivative of an image f can be expressed by a finite-vector-valued random measure on a bounded subset, i.e., when $\forall\phi = (\phi_1, \phi_2) \in C_0^1(\Omega)^2$ and $\left|\phi_{L^\infty(\Omega)}\right| \le 1$, then the fractional Green's formula $\int_\Omega f \times div^v \phi d\Omega == \int_\Omega \overline{(-1)^v D^v f} \cdot \phi d\Omega$ is satisfied. In this case, $D^v f = (D_x^v f, D_y^v f)$ is a finite vector value measure on Ω, from which expressions of fractional total variation and fractional step degree can be obtained, as shown in Eqs. (8) and (9).

$$\int_\Omega \left|D^v f\right| d\Omega = \sup \left\{ \begin{array}{l} \int_\Omega f div^v \phi d\Omega : \phi = (\phi_1, \phi_2) \\ \in C_0^1(\Omega)^2, \left|\phi_{L^\infty(\Omega)}\right| \le 1 \end{array} \right\} \tag{8}$$

$$D^v f = \nabla^v f = \left(\frac{\partial^v f}{\partial x_1^v}, \frac{\partial^v}{\partial x_2^v} \right) \tag{9}$$

Based on Eq. (9) and considering the "eight-neighborhood" of the image, the fractional ladder degree modulus function of the image can be obtained, as shown in Eq. (10).

$$\left|\nabla^v f\right| = \sqrt{\left(\begin{array}{l} \left(\frac{\partial^v f}{\partial x_+^v}\right)^2 + \left(\frac{\partial^v f}{\partial x_-^v}\right)^2 + \left(\frac{\partial^v f}{\partial y_+^v}\right)^2 + \left(\frac{\partial^v f}{\partial y_-^v}\right)^2 + \\ \left(\frac{\partial^v f}{\partial x_{45^\circ}^v}\right)^2 + \left(\frac{\partial^v f}{\partial x_{135^\circ}^v}\right)^2 + \left(\frac{\partial^v f}{\partial x_{275^\circ}^v}\right)^2 + \left(\frac{\partial^v f}{\partial x_{315^\circ}^v}\right)^2 \end{array} \right)} \tag{10}$$

Based on Eq. (10), the fractional gradient modulus of the "sub-block image" can be obtained, as shown in Eq. (11).

$$FGM_{I_k} = \sum_{(i,j) \in I_k} \left|\nabla^v f_{i,j}\right| \tag{11}$$

(2) Image texture measurements

The autocorrelation function $x(t)$ is typically used to represent the relationship between random signals at any time point. It is generally used to describe the cross-correlation of specific signals and the correlation degree at different times in the same sequence. Furthermore, it is typically used to search for repetitive patterns. Because the image texture roughness is proportional to the autocorrelation function, an autocorrelation function is introduced herein to mathematically describe the image texture feature value; hence, the image texture measurement can be obtained, as shown in Eq. (12).

$$ITM_{\varepsilon,\eta}(f_k) = \frac{\sum\limits_{(i,j)\in f_k} f(i,j)f(i-\varepsilon,j-\eta)}{\sum\limits_{(i,j)\in f_k} \left[f(i,j)\right]^2} \quad \varepsilon,\eta \in [1,D] \text{ D is the offset distance} \tag{12}$$

3.2 Variable-Order Weight Function

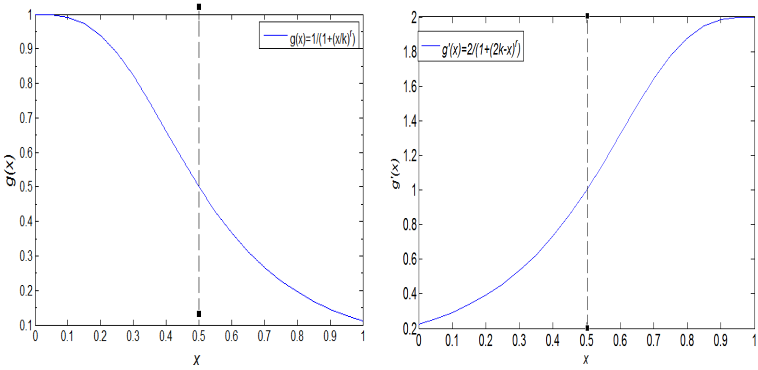
As shown by the amplitude–frequency characteristic curve of the fractional differential operator, the fractional differential operator can more effectively enhance the "weak texture" details of the image than the integer-order differential operator in the low-frequency part of the image; additionally, it can nonlinearly enhance the "strong texture" and edge information of the image in the high-frequency part of the image, but the lifting amplitude is lower than that of the high-order integer differential operator. Therefore, a coincidence score must be constructed for the weight function of the properties of the differential-order image enhancement operator. In Eq. (13), using the special properties of the special function, $g(x)$ as shown in Fig. 5(a), the boundary of function $g(x)$ is approximately $x = k$, the definition domain is $x \in [0,\ 1]$, and the value domain is $g(x) \in [0,\ 1]$. In the definition domain, the function $g(x)$ decreased with the increase in x. In this study, the normalized image eigenvalues were in the range of 0 to 1, and the corresponding fractional differential-order values were between 0 and 2 because the fractional differential operator in this interval can enhance the image details more reasonably. Therefore, the range of the function $g(x)$ was extended to 0–2, and the symmetric function $x = k$ about $g^{'}(x)$ was derived, as shown in Eq. (14). As shown in Fig. 5(b), the function $g^{'}(x)$ is related to $x = k$ (the boundary), definition domain $x \in [0,\ 1]$, and value domain $g'(x) \in [0,\ 2]$. Within the definition domain, the value of function $g^{'}(x)$ increased with x, and the increase range of the function value was controlled by parameter r.

$$g(x) = \frac{1}{1 + (x/k)^r} \tag{13}$$

$$g^{'}(x) = \frac{2}{1 + ((2k - x)/k)^r} \tag{14}$$

3.3 Description of FCLAHE Algorithm

Step 1: The Image is Classified into Two Levels. Based on the CLAHE algorithm, the image is divided into N*N sub-blocks, and the sub-block image is further divided into M*M block "secondary sub-block images" by combining the template scale of the



(a) $g(x)$ (b) $g'(x)$

Fig. 5. Schematic diagram of weight function within $x \in [0,\ 1]$ range

traditional fractional differential mask operator. If the domain Ω is a bounded open subset of the real space R^2, then the image can be represented as $f = \{f_\Omega : \Omega \in R^2 \to R\}$. As shown in Fig. 6, the entire image f_Ω in the definition domain Ω is blocked, and it is numbered in the line order to obtain the line vector set $[f_1, f_2, f_3 \ldots f_{N*N}]$ of N* N * N "sub-block images." Hence, the row vector set $[f_{k_1}, f_{k_2}, f_{k_3} \ldots f_{k_{M*M}}]$ of M * M "secondary sub-block images" can be obtained after the current "sub-block image" f_k, $k \in 1, N*N$ is divided into two levels (Fig. 7).

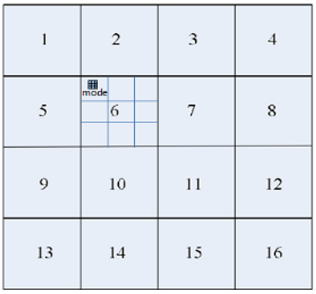


Fig. 6. N = 4, M = 3 block diagram

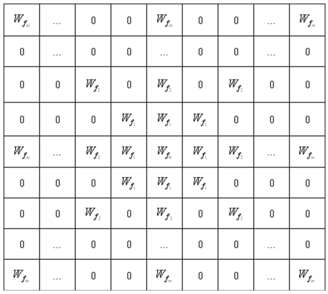


Fig. 7. The fractional order G - L operator

Step 2: Calculate the "Sub-Block Image" Eigenvalue. The fractional step modulus value of each "sub-block image" was calculated using Eq. (11) and then stored into the fractional ladder modulus value vector FGM_{blk} of the "sub-block image." Using the image texture formula show in Eq. (12), the texture measure value of each "sub-block image" was calculated and then stored into the texture measure vector ITM_{blk} of the "sub-block image." Subsequently, the fractional ladder degree modulus vector FGM_{blk} and image texture measure vector ITM_{blk} were normalized by a sigmoid function in the global range, as shown in Eq. (15). Finally, the weighted sum of the two types of image eigenvalues was calculated to obtain the "sub-block image" eigenvalue vector SIF_{blk}, as shown in Eq. (16).

$$\begin{cases} FGM_{blk} = \frac{1}{1+e^{-FGM_{blk}}} \\ ITM_{blk} = \frac{1}{1+e^{-ITM_{blk}}} \end{cases} \tag{15}$$

$$\begin{aligned} &SIF_{blk} = k_1 FGM_{blk} + k_2 ITM_{blk} \; k_1 + k_2 = 1 \\ &k_1 \text{ and } k_2 \text{are weight adjustment coefficients} \end{aligned} \tag{16}$$

Step 3: Calculate characteristic value of "secondary sub-block image". As in step 2, the fractional step modulus vector of the "sub-block image" can be obtained using the fractional step modulus formula shown in Eq. (11) and the image texture measurement formula shown in Eq. (12). Subsequently, the same normalization process was performed. Finally, the weighted sum of the two image eigenvalues was calculated to obtain the "secondary sub-block image" eigenvalue vector SIF_{sub_blk}.

Step 4: Determine the Order of Variable-Order Fractional Differential. The "sub-block image" eigenvalue vector SIF_{blk} and the "secondary sub-block image" eigenvalue vector SIF_{sub_blk} are weighted and summed again; consequently, the local and nonlocal image eigenvalues SIF of the current processing module can be obtained, as shown in Eq. (17). Using the $g'(x)$ property of the weight function deduced in this study, i.e., it is an increasing function in the real number field, and that the enhancement amplitude increases with the independent variable, the fractional differential-order function with variable order can be obtained, as shown in Eq. (18).

$$SIF = k_3 SIF_{blk} + k_4 SIF_{sub\_blk} \; k_3 \text{ and } k_4 \text{ are weight coefficients}' \; k_3 + k_4 = 1 \tag{17}$$

$$v = \frac{2}{1 + ((2k - SIF)/k)^r} k \text{ and } r \text{ are adjustment parameters} \tag{18}$$

Step 5: Variable-Order Fractional Differential Enhancement. Using the fractional G–L differential operators in [6], as shown in Eqs. (19) and (20), and extending them to eight directions in the image, we can obtain the fractional G–L differential mask operator based on the fractional-order G–L differential mask operator, as shown in Fig. 7, where the coefficient of the mask operator is expressed by Eq. (21). Finally, the current sub-block image f_k and variable-order mask operator are convoluted to obtain the edge and texture details of the current sub-block image.

$$\begin{aligned} &{}_a^G D_t^v f(x, y)_x \triangleq f(x, y) + (-v) f(x-1, y) + \frac{-v(-v+1)}{2} f(x-2, y) \\ &+ \frac{-v(-v+1)(-v+2)}{6} f(x-3, y) + \cdots + \frac{\Gamma(-v+m)}{\Gamma(m+1)\Gamma(-v)} f(x-m, y) \end{aligned} \tag{19}$$

$$\begin{aligned} &{}_a^G D_t^v f(x, y)_y \triangleq f(x, y) + (-v) f(x, y-1) + \frac{-v(-v+1)}{2} f(x, y-2) \\ &+ \frac{-v(-v+1)(-v+2)}{6} f(x, y-3) + \cdots + \frac{\Gamma(-v+n)}{\Gamma(n+1)\Gamma(-v)} f(x, y-n) \end{aligned} \tag{20}$$

$$\begin{cases} W_{f_0} = 1 \\ W_{f_1} = -v \\ W_{f_2} = \frac{v(v-1)}{2} \\ \cdots\cdots \\ W_{f_m} = \frac{v(v-1)(v-2)\ldots(v-m+1)}{m!} \end{cases} \tag{21}$$

Step 6: FCLAHE Image Enhancement. Redistributing the current sub-block image according to certain rules, the histogram distribution of f_k shows the occurrence times of all pixels in the original histogram within the clipping threshold. Subsequently, the redistributed histogram was equalized to obtain a better contrast in the sub-block image. Finally, the image details extracted by the CLAHE method and fractional differential were weighted and summed, as shown in Eq. (22); the function min() ensures that the gray value of the processed image is within the specified range.

$$f_k = \min\left(k_5 \sum_{i \in f_k} \left(f_{k_i} * \frac{W_v}{\max(W_v)}\right) + k_6 CLAHE(f_k),\ 1\right)$$
$$k_5 \text{ and } k_6 \text{ are weight adjustment coefficients} \tag{22}$$

4 Experimental Simulation and Comparative Analysis

4.1 Evaluation Parameters

In a subjective evaluation, the human eye directly views the enhanced image, and the enhanced image mainly focuses on the human eye's feeling, to reflect the human visual perception. Hence, the image is more vivid. Because the human eye is sensitive to the texture details and edge parts of the image, direct observation was performed in this study to contrast the difference between visual light and shade, as well as the difference in binary images with edge and texture features. The objective index was evaluated by constructing the relevant evaluation function of important objective image evaluation features based on the subjective feeling of the human eyes. Subsequently, according to the evaluation function, the numerical quantization results based on some image features were obtained. In this study, the edge preserving coefficient, average gradient, contrast, and image entropy were used to evaluate the objective enhancement effect of image enhancement operators.

(1) Edge Preservation Index (EPI)
The edge preserving index indicates that the enhancement operator maintains the horizontal or vertical edge of the image. The higher the EPI value, the better is the edge preserving ability of the operator. The formula for the edge retention coefficient is shown in Eq. (23).

$$EPI = \sum_{i=1}^{row}\sum_{j=1}^{col}\left|\begin{matrix}\Delta_{level} f_{after}(i,j)+ \\ \Delta_{vertical} f_{after}(i,j)\end{matrix}\right| \Bigg/ \sum_{i=1}^{row}\sum_{j=1}^{col}\left|\begin{matrix}\Delta_{level} f_{befor}(i,j)+ \\ \Delta_{vertical} f_{befor}(i,j)\end{matrix}\right| \tag{23}$$

(2) Average Gradient (AG)

The AG value of the image can describe detailed contrast and texture changes in the image as well as reflect the clarity of the image to a certain extent. The formula to calculate the AG value is shown in Eq. (24).

$$AG = \frac{1}{M * N}\sum_{i=1}^{row}\sum_{j=1}^{col}\sqrt{\Delta_{level} f(i,j)^2 + \Delta_{vertical} f(i,j)^2} \tag{24}$$

(3) Entropy (E)

The E of an image describes the average amount of information contained in the image. The higher the E value, the more information it contains, and the richer are the edge texture details of the image. The formula to calculate the image E is as follows:

$$Entropy(f_{i,j}) = -\sum_{L=1}^{NUM\_GL} P\left(f_{i,j}\right)\log P\left(f_{i,j}\right) \tag{25}$$

p is the probability function of the image pixel

(4) Contrast (C)

C represents the ratio of black to white, i.e., the gradient from black to white. The larger the ratio, the more gradients exist from black to white, and the better is the texture level of the detail. The formula for C is as follows:

$$C = \left|\sum_{i=1}^{row}\sum_{j=1}^{col}\Delta f(i,j)\right| \Big/ Number \tag{26}$$

Here, "number" indicates the number of differences between the gray values of eight adjacent regions of the image.

4.2 Experimental Results Analysis

The variable-order fractional order CLAHE image enhancement model proposed herein involves many parameters. The weighted coefficient or threshold parameter of the model can be determined using the empirical value or average proportion after many experiments. In this study, the number of first-order blocks was N = 4, the number of second-order blocks was M = 3, the offset distance of the image texture was D = 5, and the weights of the fractional-step eigenvalue and texture eigenvalue of the "sub-block image" and "secondary sub-block image" were $k_1 = 0.5$ and $k_2 = 0.5$, respectively. For the variable-order fractional differential, the weights of the two-level sub-block image eigenvalues were $k_3 = 0.5$ and $k_4 = 0.5$, and the variable-order fractional differential function $k = 0.5$, $r = 2$. The first four coefficients were used as the estimated values of the fractional differential. The weights of the FCLAHE contrast enhancement image and fractional differential detail image were $k_5 = 1.1$ and $k_6 = 1$, respectively.

To verify the advantages of the FCLAHE image enhancement model proposed herein, the current classic image enhancement methods were compared, including the Laplace image enhancement (Laplace), traditional fractional-order G–L image enhancement, histogram equalization (HE), and CLAHE methods. The experimental results show that superior performance of the proposed method based on comparing the images obtained from different image enhancement methods and the texture characteristics of the image. The experimental results show that the proposed FCLAHE image enhancement method had a higher contrast and edge texture than those of existing image enhancement methods, as indicated by the calculated quantitative values of objective indicators of the image after different enhancement methods, including the EPI, AG, E, and C. Furthermore, it demonstrated clearer and better visual effects.

Figure 8 shows the contrast of different enhancement models to enhance the Barbara image. Direct observation of the red circle part of the Barbara image shows that the Laplace operator, fractional G–L operator, and FCLAHE image enhancement model proposed herein significantly improved the texture details of the rattan chair information of the Barbara image compared with the HE and CLAHE image enhancement methods. Because the Laplace operator is a second-order differential operator, the edge and texture amplitude of the enhanced image improved significantly. However, if the differential order of the Laplace operator is extremely large, the strong edge information in the image will be enhanced excessively, thereby resulting in the false "bright line" phenomenon on the arm edge of the character image, which may cause image distortion. Direct observation of the Barbara blue circle shows that the CLAHE and FCLAHE image enhancement models can improve the local contrast information of the image compared with the Laplace, fractional-order G–L, and HE methods because it considers the local and nonlocal statistical information of the image, resulting in better bright-contrast visual effects and more visible low-contrast details. Figure 9 shows the contrast effect of the texture features of image Barbara enhanced by different enhancement models. Direct observation shows that the method proposed herein can enhance the detailed texture features of the image more effectively than other image enhancement methods, as well as effectively improve the local and overall contrasts of the image.

Figure 10 shows the contrast curves of the AG, edge retention coefficient, C, and E values of the two images above enhanced using the Laplace, fractional-order G–L, HE, CLAHE, and FCLAHE methods. Direct observation shows that the FCLAHE method improved the low-frequency weak edge and texture details of the image more effectively

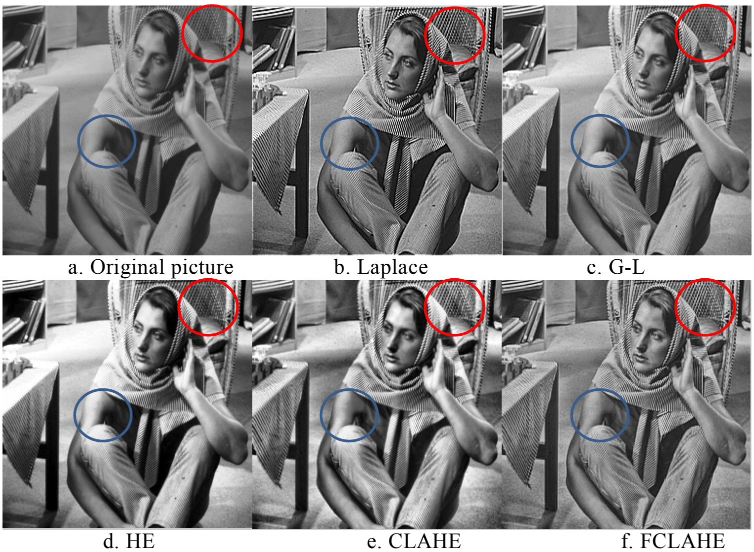


Fig. 8. Pictures obtained using different image enhancement models

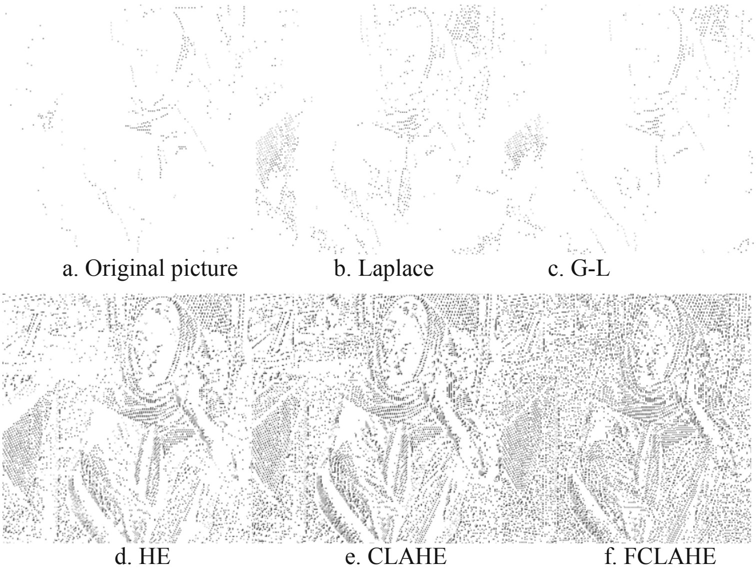


Fig. 9. Texture feature map of different image enhancement methods

compared with the difference or statistical image enhancement methods. Moreover, it exhibited the ability of the nonlocal adaptive histogram operator to improve the local and nonlocal contrasts of the image; consequently, the evaluation index of the enhanced image was better than those of other classical image enhancement methods.

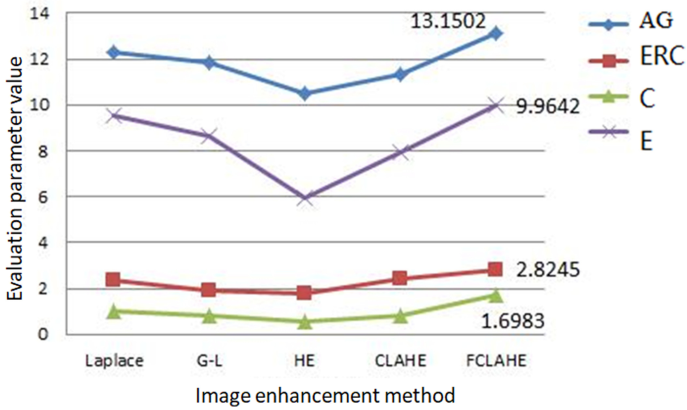


Fig. 10. Evaluation data corresponding to different image enhancement models

5 Conclusion

The fractional differential image enhancement model and its improved model typically utilizes the "weak derivative" and "nonlocal" properties of the fractional differential operator; hence, it can effectively enhance the edge and texture information of an image. However, the effect of this model on low-contrast image enhancement is not ideal. The CLAHE image enhancement model utilizes the probability redistribution of light and dark pixels to enhance the overall contrast of an enhanced image and expands the dynamic range of the gray level, which can effectively improve the light and dark contrasts of the image. However, the effect of the CLAHE model on images with rich texture details is insignificant. Considering the advantages and disadvantages of the difference and statistical methods for image enhancement, a variable-order fractional order CLAHE image enhancement model based on the existing fractional-order differential image enhancement model and the classical CLAHE model was proposed. The model uses the blocking mechanism of the CLAHE model, in which the current sub-block image is divided into two levels. The order of the fractional differential operator is determined by the linear weighted value of the fractional ladder degree modulus and the image texture measure of the current image sub-block and secondary block images. The simulation results show that the proposed FCLAHE method afforded different degrees of improvement compared with other classical image enhancement models owing to the combination of the fractional differential operator and the CLAHE operator's excellent characteristics, whether in subjective direct observations or based on objective evaluation data.

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