



Recommendation with Subjective Tendency Based on Statistical Implicative Analysis

Hiep Xuan Huynh¹(✉), Cang Anh Phan², Tu Cam Thi Tran²,
and Hai Thanh Nguyen¹

¹ College of Information and Communication Technology, Can Tho University,
Can Tho 900000, Vietnam

{hxhiep,nthai.cit}@ctu.edu.vn

² Faculty of Information Technology, Vinh Long University of Technology Education,
Vinh Long 85000, Vietnam

{cangpa,tuttc}@vlute.edu.vn

Abstract. The recommendation systems have been investigating and applying in a vast of fields. The core of systems is the similarity measures and the dissimilarity measures. Many scientists have proposed various similarity measurements in different aspects, including the measures between the users and the users, the measures between the items and the items, the measures between users with the items. However, there are not much studies on the effects of statistical implicative in the recommendation system with subjective tendency. We mainly focus on showing the effects of the subjective tendency against the recommendation system's model through the prism of statistics implicative. Three specific approaches, including Independence, Dependence, and Equilibrium combined with the fifteen measures of the statistical bias are considered in our work. The experimental results evaluated on the Jester5k dataset compare the similarity measures and the interestingness measures based on the subjective tendency in recommendation systems.

Keywords: Statistical implicative analysis · Subjective tendency · Collaborative filtering · Interestingness measures · Similarity measures

1 Introduction

Recommendation systems [1–3] uses knowledge and data based on user's benefits and user's interests to provide appropriate advice/recommendations. The system supports and improves quality when the users make decisions to search and choose products online. The subjects that need to be interested in the recommendation system are the users, the items, and the user's feedback (called evaluates or ratings). The recommendation system can predict how a user evaluates an item, predicts the order (ranking) of the items in a list from the most

attractive to the least attractive for a user or an item (or the list of things) that is suitable.

Recommendation systems are divided into the main groups/forms: Content-Based Recommendation Systems (CBRS) [1,3,4], Collaborative Filtering Systems (CFS) [3,5-7], Knowledge-Based Recommendation Systems (KBRS) [3,4], Hybrid Recommendation System [3,4,8,9] and Context-Based Recommendation Systems (CBRS) [3,4,7]. CFS is commonly used. The collaborative filtering recommendation systems [10-14] suggest users to data items based on those previously rated by the other users (Fig. 1).

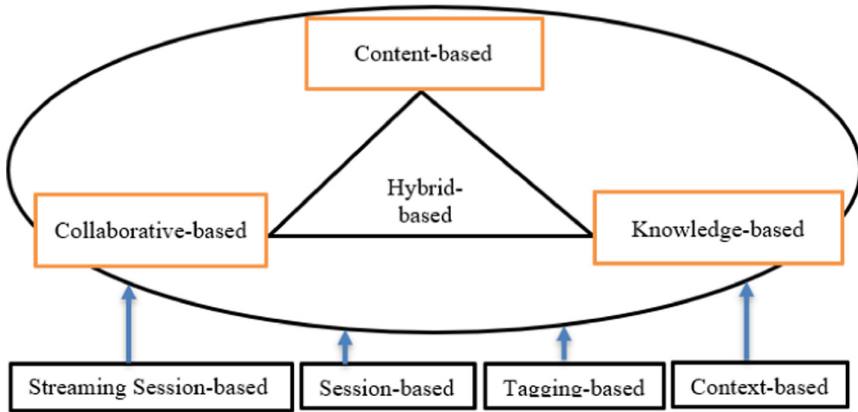


Fig. 1. Recommendation types.

Some other recommendation systems [15] have been developed based on a combination of some of the types listed above and with additional information such as context information, group information, etc. Proposing new consulting models and improving existing consulting methods is a primary research direction. Many methods in data mining and deep learning [16] are used in the consulting problem, such as the problem of classification, clustering [17], association rule mining [18], regression model [19,20]. As for the other methods, the advantage of the recommendation approach based on the association rule is transparency [1] - it can provide an in-depth explanation of the suggestion list for the users. In association rules that have two measures widely applied to evaluate item sets and generate association rule sets are Support and Confidence [14]. However, the quality of association rules and how suggestions are ranked in the recommendation systems should be done through objective similarity measures. The analysis of the effects in implication towards the subjective tendency has demonstrated an proposed approach to give more accurate and relevant recommendations. In this paper we propose a new recommendation model to predict users' missing ratings (ratings) for specific data items and propose items best suited to users based on some essential characteristics (subjective tendency) of

statistical implicative analysis approach [12,21]. In the proposed model, the similarity measure is used to filter the set of rules. Through the proposed model, advisory systems will be built.

The article is organized in five parts. The first one introduces about the context and the problems need to be solved by the system as well as the approach should be taken to solve the above issues. The second part briefly describes the subjective tendency and the relevant contents are used for the recommendation system. The next one presents the proposed solutions and it's model to show the feasibility of the recommendation system based on statistical implicative analysis. The fourth part presents the experiments and the discussions, which focuses on presenting the effects of the tendencies of implications on the recommendation model. The finally part of the paper is the conclusion.

2 Subjective Tendency

2.1 Association Rule

Association rules [2,18,22,23] are relationships between the items. Let $T = \{t_1, t_2, t_3, \dots, t_n\}$ are n transactions (transaction t_i buy items $I(i_1, i_2, i_3, \dots, i_m)$, in which $t_i \subseteq I$). An association rule is implication of the form is $a \rightarrow b$ (with: a and b are two sets containing discrete elements so that $a \cap b = \phi$, and $a \subset I$, $b \subset I$). Set a (corresponding b) is associated with a subset of the transactions $A = T(a) = \{t \in T, a \subseteq t\}$ (corresponding $B = T(b)$). Set \bar{a} (corresponding \bar{b}) is associated with $\bar{A} = T(\bar{a}) = T - T(a) = \{t \in T, a \not\subseteq t\}$ (corresponding $\bar{B} = T(\bar{b})$). To accept or refuse to have b when appearing a . Normally, we only pay attention to the number of elements $n_{A\bar{B}}$, there is no direction to support law-making $a \rightarrow b$.

Each rule is described by parameters: the cardinal n of T is $n = \text{card}(T)$, the cardinals n_A of A is $n_A = \text{card}(A)$, and the cardinals n_B of B is $n_B = \text{card}(B)$, the number $n_{A \cap B} = \text{card}(A \cap B)$, and the number $n_{A \cap \bar{B}} = \text{card}(A \cap \bar{B})$.

For greater clarity, define the concepts of probability $p(A)$ (corresponding $p(B), p(A \cap B), p(A \cap \bar{B})$)) as the probability value of A (corresponding $B, A \cap B, A \cap \bar{B}$). This probability is calculated by the frequency of occurrence of A : $p(A) = \frac{n_A}{n}$ (corresponding $p(B) = \frac{n_B}{n}, p(A \cap B) = \frac{n_{A \cap B}}{n}, p(A \cap \bar{B}) = \frac{n_{A \cap \bar{B}}}{n}$).

2.2 Statistical Implicative Analysis

Statistical implicative tendency [24] is the attribute showing the relationship between the data (items and items, users and users, users and items). The observations and evaluations for specific situations in the change of interestingness value are a primary method to understand the exciting measures affecting the recommendation systems deeply [24].

Three particular situations including Independence/Dependence [24], Equilibrium [24]. Both the tendencies are called the subject (subjective tendency) of an objective interestingness measure. With n is the total number of occurrences; n_{AB} is the number of occurrences of both A and B ; n_A is the number

of occurrences of A ; n_B is the number of occurrences of B ; $n_{A\bar{B}}$ is the number of occurrences of A and $non - B$; $n_{\bar{B}}$ is the number of occurrences of $non - B$ (Fig. 2).

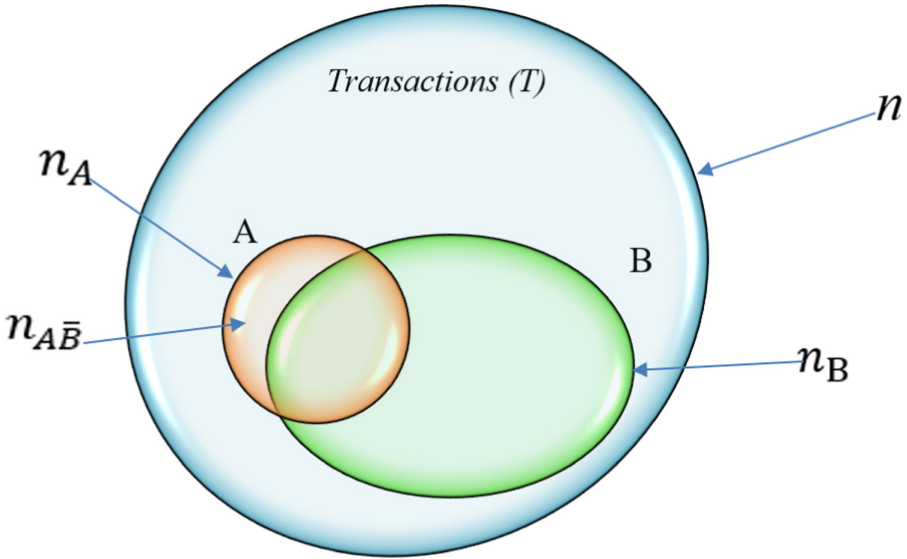


Fig. 2. The Venn diagram for an association rule $a \rightarrow b$ with 4 parameters $(n, n_A, n_B, n_{A\bar{B}})$.

For example of association rule on **Jester transaction** are showed in the Table 1.

Table 1. An example of hand activities

TID	Items
1	{Shaking Hand, Swiping Up}
2	{Thumb Down, Stop Sign, Shaking Hand}
3	{Thumb Down, Thumb Up, Doing other things}
4	{Shaking Hand, Stop Sign}
5	{Thumb Down, Stop Sign, Shaking Hand}
6	{Thumb Down, Doing other things}
7	{Thumb Down, Thumb Up, Shaking Hand, Swiping Up, Stop Sign, Doing other things}
8	{Shaking Hand, Swiping Up, Doing other things}
9	{Swiping Up, Stop Sign}

Corresponding to Table 1, set of transactions $T = \{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9\}$, set of items $I(i_1, i_2, i_3, i_4, i_5, i_6)$ Supposing for an association rule $a \rightarrow b$ with $a = \{Thumb\ Down, Stop\ Sign\}$, $b = \{Thumb\ Up\}$.

Table 2. An example of hand activities represented in a matrix

TID	Thumb down (i_1)	Thumb up (i_2)	Stop sign (i_3)	Shaking hand (i_4)	Swiping up (i_5)	Doing other things (i_6)
t_1	0	0	0	1	1	0
t_2	1	0	1	1	0	0
t_3	1	1	0	0	0	1
t_4	0	0	1	1	0	0
t_5	1	0	1	1	0	0
t_6	1	0	0	0	0	1
t_7	1	1	1	1	1	1
t_8	0	0	0	1	1	1
t_9	0	0	1	0	1	0

Form an example of hand activities in Table 2, $A = \{t_2, t_5, t_7\}$, $B = \{t_3, t_7\}$, $A\bar{B} = \{t_2, t_5\}$. Thus, with the association rule $a \rightarrow b$ in example at Table 1, the result: $n = 9$; $n_A = 3$; $n_B = 2$; $n_{A\bar{B}} = 2$.

2.3 Interestingness Measure

Interestingness measure of an association rule [2, 23–25] is based on an objectively interestingness measure, which will then be calculated based on the number of the rule $f(a \rightarrow b) = f(n, n_A, n_B, n_{A\bar{B}}) \in R$. For the convenience of the calculation process, converting between the numeric parameters of a rule, we can use the transforms as follows:

$$n_{AB} = n_A - n_{A\bar{B}}, n_{\bar{A}} = n - n_A, n_{\bar{B}} = n - n_B, n_{\bar{A}\bar{B}} = n_B - n_A + n_{A\bar{B}},$$

$$n_{\bar{A}\bar{B}} = n - n_B - n_{A\bar{B}}$$

Example: Interesting value for measure $Lift = \frac{n(n_A - n_{A\bar{B}})}{n_A n_B}$ with the parameters ($n = 9$; $n_A = 3$; $n_B = 2$; $n_{A\bar{B}} = 2$) are taken from Sect. 2.3

$$f(a \rightarrow b) = f(n, n_A, n_B, n_{A\bar{B}}) = \frac{n(n_A - n_{A\bar{B}})}{n_A n_B} = \frac{9 * (3 - 2)}{3 * 2} = \frac{3}{2} = 1.5$$

2.4 Tendency Evaluating

Subjective tendency [21, 24] were studied in the Independence/Dependence and Equilibrium context. The recommendation system is mainly based on similarity. The similarity measures are influenced by three properties (Independence/Dependence and Equilibrium). The study focuses on the influence of the

recommendation system by the similarity measures, according to the subjective tendency.

Independence/Dependence. This tendency can only occur when the premise and outcome of a rule are combined independently. This situation only happens when $n_{AB} = \frac{n_A n_B}{n}$ or $n_{A\bar{B}} = \frac{n_A n_{\bar{B}}}{n}$, if the interestingness value is a constant.

$$f(a \rightarrow b) = f\left(n, n_A, n_B, \frac{n_A n_{\bar{B}}}{n}\right) = constant$$

Example: Interesting value for measure $Lift = \frac{n(n_A - n_{A\bar{B}})}{n_A n_B}$ is applied by Independence/Dependence tendency.

$$\begin{aligned} f(a \rightarrow b) &= f\left(n, n_A, n_B, \frac{n_A n_{\bar{B}}}{n}\right) = \frac{n\left(n_A - \frac{n_A n_{\bar{B}}}{n}\right)}{n_A n_B} = \frac{n\left(\frac{nn_A}{n} - \frac{n_A n_{\bar{B}}}{n}\right)}{n_A n_B} \\ &= \frac{n\left(\frac{nn_A - n_A n_{\bar{B}}}{n}\right)}{n_A n_B} = \frac{nn_A - n_A n_{\bar{B}}}{n_A n_B} = \frac{n_A(n - n_{\bar{B}})}{n_A n_B} \\ &= \frac{(n - n_{\bar{B}})}{n_B} = \frac{(n - n_{\bar{B}})}{n_B} = \frac{n_B}{n_B} = 1 \end{aligned}$$

The result of the interestingness measure value of Lift is affected by Independence/Dependence tendency.

Equilibrium. This tendency is the state of an object in which all the values that act on it are balanced. In other words, Equilibrium only can occur when the exciting value of the rule is a constant.

$$f(a \rightarrow b) = f\left(n, n_A, n_B, \frac{n_A}{n}\right) = constant$$

Example: Interesting value for measure $Lift = \frac{n(n_A - n_{A\bar{B}})}{n_A n_B}$ is applied by Equilibrium tendency.

$$\begin{aligned} f(a \rightarrow b) &= f\left(n, n_A, n_B, \frac{n_A}{n}\right) = \frac{n\left(n_A - \frac{n_A}{n}\right)}{n_A n_B} = \frac{n\left(\frac{nn_A}{n} - \frac{n_A}{n}\right)}{n_A n_B} \\ &= \frac{n\left(\frac{nn_A - n_A}{n}\right)}{n_A n_B} = \frac{nn_A - n_A}{n_A n_B} = \frac{n_A(n - 1)}{n_A n_B} = \frac{(n - 1)}{n_B} \\ &= \frac{(n - 1)}{n_B} \neq constant \end{aligned}$$

The result of the interestingness measure value of Lift is not affected by Equilibrium tendency.

Table 3 shows fifteen measures for these criteria, with the value of 1 is satisfied and a value of 0 is unsatisfied. The fifteen measures are considered Collective Strength, Confidence, Conviction, Gini_Index, Implication Index, Laplace, Least Contradiction, Lerman, Sebag & Schoenauer, Lift/Interest Factor, Jaccard, Support, Kappa, Jmeasure, Causal Support to evaluate the variation of interestingness values from Independence value or Equilibrium value. The interestingness measure will be evaluated as the change tendency from Independence/Dependence values or Equilibrium value.

Table 3. Subjective tendency with 15 interestingness measures [24]

Number	Interestingness measures	Independence/dependence	Equilibrium
1	Collective strength	1	0
2	Confidence	0	1
3	Conviction	1	0
4	Gini_Index	1	0
5	Implication index	1	0
6	Laplace	0	1
7	Least contradiction	0	1
8	Lerman	1	0
9	Sebag & Schoenauer	0	1
10	Lift/interest factor	1	0
11	Jaccard	0	0
12	Support	0	0
13	Kappa	1	0
14	J-measure	1	0
15	Causal support	0	0

3 Subjective Tendency Recommendation

3.1 Rating Matrix

The data model can be organized as a table of values in a rating matrix where presents the user’s ratings for items. The value which is not rated is denoted as “-”. For example, Table 4 exhibits the users with ratings for the products. Based on some computation, a recommendation can provide the rating score for the $user_x$ with the corresponding item column.

Table 4. Rating matrix with users and items

	$item_1$	$item_2$	$item_3$	$item_4$	$item_5$
$user_1$	4	1	-	4	2
$user_2$	-	5	3	-	-
$user_3$	-	2	3	-	3
$user_4$	-	1	4	-	2
$user_x$	4	-	-	-	3

The rating matrix of the dataset is used in a recommendation system model built with the influence of statistical attributes.

Table 5. The rating matrix of the dataset in a recommendation system

	<i>item</i> ₁	<i>item</i> ₂	<i>item</i> ₃	<i>item</i> ₄	<i>item</i> ₅
<i>u</i> ₃₄₆₄	2.75191	NA	3.58355	NA	NA
<i>u</i> ₁₅₀₀₅	NA	NA	NA	NA	NA
<i>u</i> ₉₆₅₈	NA	NA	NA	NA	NA
<i>u</i> ₁₃₃₉₆	NA	NA	NA	NA	NA
<i>u</i> ₉₅₆₅	NA	NA	NA	NA	NA

Table 5 shows a rating matrix of 5 (users/rows) \times 5 (items/columns) of class ‘realRatingMatrix’ with 25 ratings are built in the recommendation system.

3.2 Recommendation

A recommendation based on statistical tendency is a recommendation system with the influence of statistical implicative. It is built with statistical tendency and knowledge discovery techniques to make product recommendations [26].

Recommendation based on user’s preferences by the similarity measures and interestingness measures [24, 27]. Recommendation algorithm with the influence of statistical tendency is presented in Fig. 3 follows.

3.3 Evaluation

To evaluate the recommendation model, we need to build it on the training set and test it on the test set. Therefore, the first step is to prepare the data. In this step, the experimental dataset is divided into two subsets: the training set and the test set [28]. In this paper, we use the k-fold method to divide the dataset to evaluate the recommendation model.

K-fold cross-validation [28]: is a method of cutting experimental data into k subsets of the same size (called k-fold). Then, do k evaluations, with each evaluation using one subset as the test set and the other k – 1 as the training set. Finally, evaluation results are calculated from the results of k tests using the average calculation. This method ensures that all users appear in the test set at least once.

We evaluate the proposed method with the metric of Receiver Operating Characteristic (ROC curve). From the prediction results, we get the predicted variable scores. If we set a cut point for the model, we will have a threshold to evaluate the model that predicts a positive or negative result. The Receiver operating characteristic (ROC) graph exhibits each cut-point corresponding to its sensitivity and False positive rate ratio. The vertical axis corresponds to the Sensitivity rate, and the horizontal axis corresponds to the False positive rate. Based on the ROC curve, one can show whether a model is effective or not. An efficient model has low FPR and high TPR, i.e., a point on the ROC curve close

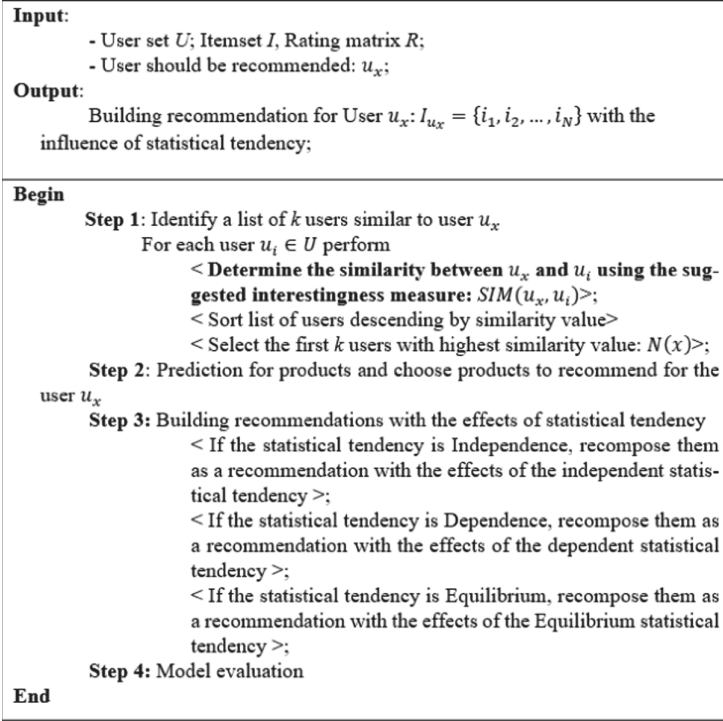


Fig. 3. Subjective tendency recommendation.

to the point with the coordinates (0, 1) on the graph (upper left corner). The closer the curve is, the more efficient the model is.

Also, we select the metrics of Precision, Recall, TPR (True Positive Rate), and FPR (False Positive Rate) for methods comparison.

$$Precision = \frac{\text{Number of recommendation products has been selected}}{\text{Total number of products introduced}}$$

$$Recall = \frac{\text{Number of recommendation products has been selected}}{\text{Total number of products selected}}$$

4 Experiment

4.1 Dataset

The data in this paper is used that is the Jester5k¹ (Jester dataset with 5,000 samples). The data set has 5000 users from the anonymous rating data from the

¹ <https://rdrr.io/cran/recommenderlab/man/Jester5k.html>, accessed on February 01, 2021.

Jester Online Joke Recommendation System collected between April 1999 and May 2003. Moreover, it is used in many types of research around the world. The format of Jester5k is: Formal class ‘realRatingMatrix’ [package “recommenderlab”].

Jester5k contains a 5000×100 rating matrix (5000 users and 100 jokes) with ratings between -10.00 and $+10.00$. All selected users have rated 36 or more jokes.

4.2 Tool

This model is experimented with by the ‘irlba.2.3.3’², ‘proxy.0.4-26’³, ‘registry.0.5-1’⁴, ‘kernlab.0.9-29’⁵, and ‘rimarules’ tool is developed by ‘arules.1.6-6’⁶. The recommenderLab package develops this tool in the R programming language. Besides, this work also inherited several open-source tools it has researched and has built on the world community. The “rimarules” tool was developed to perform and evaluate the effects of statistical properties according to subjective bias. In addition, this tool can build and run other collaborative filter-based recommendation systems for mutual comparison and evaluation. The fifteen similarity measures were used to build the “rimarules” tool.

Some functions have been built by the system such as: The function displayed the suggestion list for the users and compared the size of the recommendation models with different implications tendencies, the function compared the predicted speeds of the models, the function built data to be evaluated model, and built a recommendation system with statistical implications tendency.

4.3 Scenario 1: Recommendation with Independence Tendency

This scenario discusses the effects of independence tendency obtained from 8 corresponding subjective recommendation models.

Figure 4 shows that the independence tendency positively influences the recommendation model with TRP (true positive rate) and FTP (false positive rate). However, with the Collective Strength measure, the correct predictability is the shortest from about 0.0 to 0.1, opposite the probability of predicting false is height about 0,6. In comparison, Kappa’s ability to predict correctly is the highest, about 0,7 (Fig. 5).

Table 6 reveals the results of 8 measures for Equilibrium. Again, Lerman exhibits the best performance at 0.6916667 in Precision, while Collective Strength, Implication index, Gini_index, Kappa are shown the worst, about 0,1.

² <https://cran.r-project.org/web/packages/irlba/>.

³ <https://cran.r-project.org/web/packages/proxy/index.html>.

⁴ <https://cran.r-project.org/web/packages/registry/index.html>.

⁵ <https://cran.r-project.org/web/packages/kernlab/index.html>.

⁶ <https://cran.r-project.org/web/packages/arules/arules.pdf>.

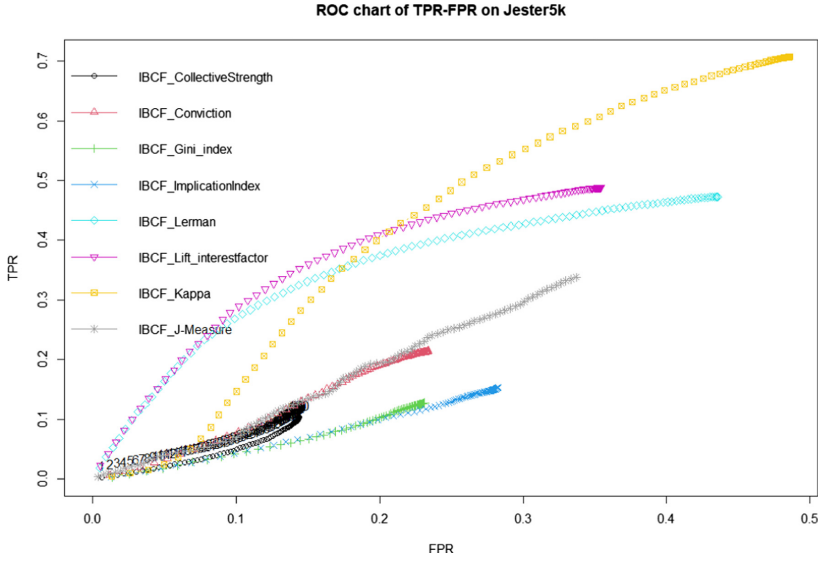


Fig. 4. The ROC chart of TPR-FPR on Jester5k with the eight measures.

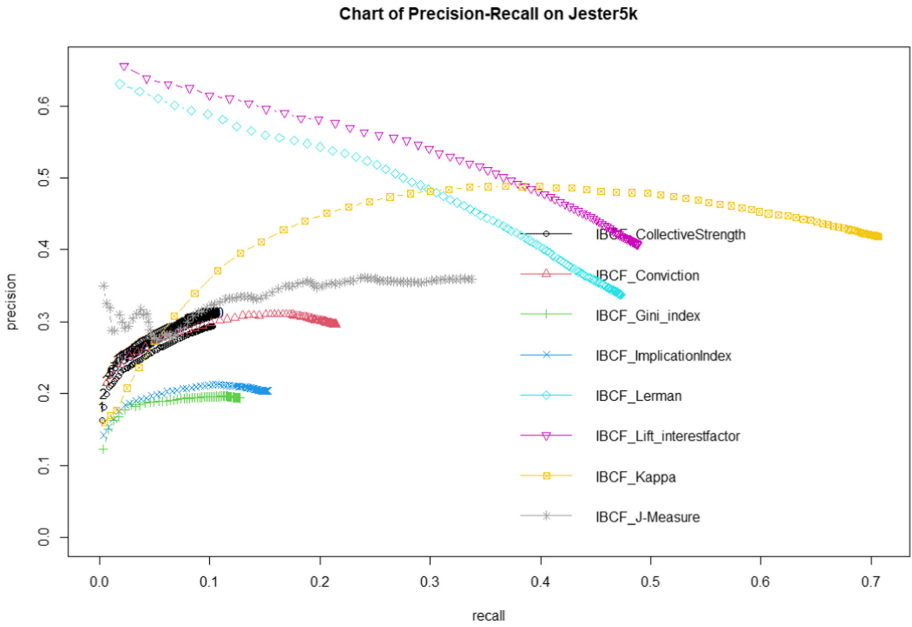


Fig. 5. The chart of Precision-Recall on Jester5k with the eight measures.

Table 6. Recommendation with independence tendency

	Precision	Recall	TPR	FPR
Collective strength	0.145454500	0.001346645	0.001346645	0.005664693
Conviction	0.250000000	0.006371114	0.006371114	0.011342510
Gini_Index	0.138888900	0.004103637	0.004103637	0.014531220
Implication index	0.141844000	0.003823744	0.003823744	0.013962940
Lerman	0.691666700	0.018817320	0.018817320	0.003983070
Lift/interest factor	0.685714300	0.021462830	0.021462830	0.004726987
Kappa	0.151724100	0.003671391	0.003671391	0.013456700
J-measure	0.366987100	0.003461492	0.003461492	0.003691004

4.4 Scenario 2: Recommendation with Dependence Tendency

This scenario presents the effects of the Dependence attribute into the recommendation system (Fig. 7).

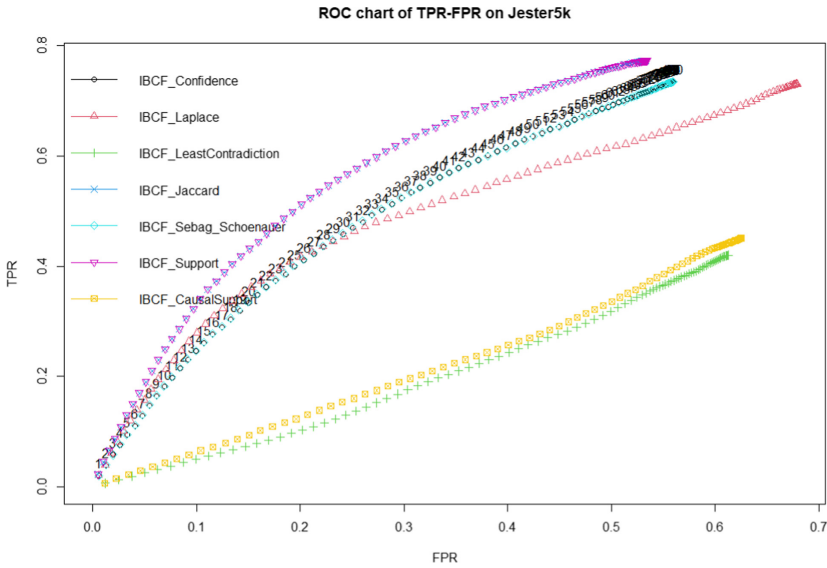


Fig. 6. The ROC of TPR-FPR with the dependence tendency.

Figure 6 shows the dependent attributes on the recommendation system with both TRP and FTP. The ROC chart presents a steady increase between TPR and FPR. However, for two measures (Least Contradiction and Causal Support), the period 0.1 to 0.5 decreased slightly for the value of FPR. In which the value of TPR is relatively increased from 0.0 to 0.3. A comparison between interestingness

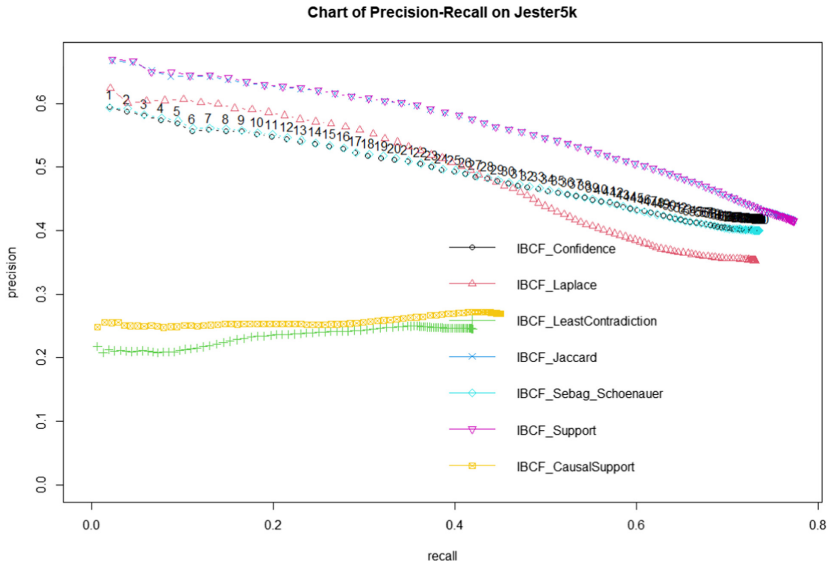


Fig. 7. The Precision-Recall with the dependence tendency.

measures is illustrated in Table 7. As observed, Least Contradiction and Causal Support reveal not better performance in all considered performance measures.

Table 7. Recommendation with dependence tendency.

	Precision	Recall	TPR	FPR
Confidence	0.648275900	0.021474840	0.021474840	0.005511594
Laplace	0.641379300	0.021140840	0.021140840	0.005477616
Least contradiction	0.234482800	0.006757109	0.006757109	0.012850560
Jaccard	0.675862100	0.022506450	0.022506450	0.005006642
Sebag & Schoenauer	0.653793100	0.021644860	0.021644860	0.005422663
Support	0.675862100	0.022455960	0.022455960	0.004997525
Causal support	0.259310300	0.007014131	0.007014131	0.011293410

4.5 Scenario 3: Recommendation with Equilibrium Tendency

In this experimental part, the four measures are used including: Confidence; Laplace; Least Contradiction; Sebag & Schoenauer to describe the influences of equilibrium properties on recommendation model (Fig. 9).

Figure 8 presents Equilibrium attributes on the recommended model. The values of both TPR and FPR increase in the ROC chart. However, the period

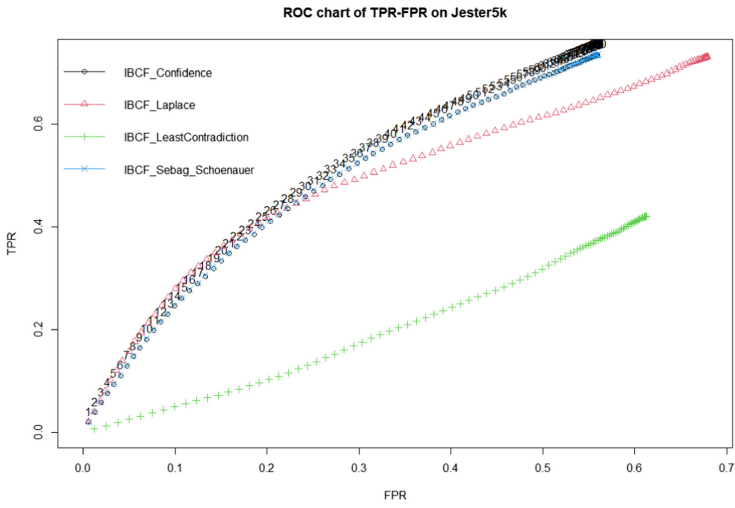


Fig. 8. The ROC of TPR-FPR with the Equilibrium tendency.

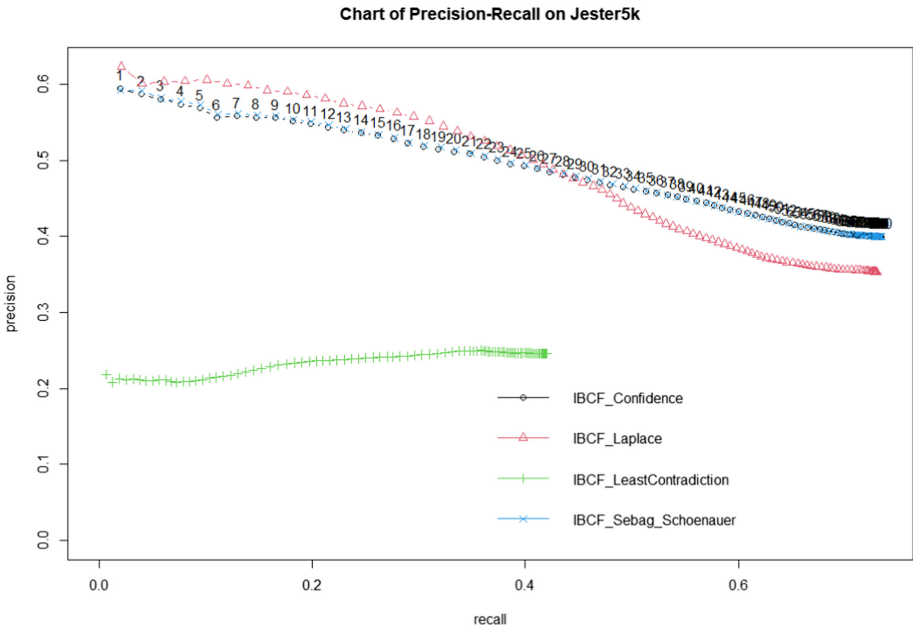


Fig. 9. The Precision-Recall with the Equilibrium tendency.

0.22 to 0.48 is the slow increase rate for Confidence, Laplace, and Sebag & Schoenauer. However, these three measures still offer higher predictability than Least Contradiction.

Table 8. Recommendation with equilibrium tendency.

	Precision	Recall	TPR	FPR
Confidence	0.648275900	0.021474840	0.021474840	0.005511594
Laplace	0.641379300	0.021140840	0.021140840	0.005477616
Least contradiction	0.234482800	0.006757109	0.006757109	0.012850560
Sebag & Schoenauer	0.653793100	0.021644860	0.021644860	0.005422663

Table 8 illustrates the results in detail for four considered interestingness measures. The four measures of the Equilibrium attribute have the positive effects on the recommendation system. As we can see, the gives the lowest predicted value in TPR while Confidence and Sebag & Schoenauer share near the same pattern.

5 Conclusion

In this work, we compared various recommendation models and considered the effects of association law on recommendation systems. Also, we evaluate and compare some tendency including Independence, Dependence, Equilibrium. We investigated the relationships between the user and the user, between the item and the item, and the user with the item with three approaches using three various subjective tendencies (Independence, Dependence, and Equilibrium) along with fifteen popular similarity measures (Collective Strength, Confidence, Conviction, Gini_index, Implication Index, Laplace, Least Contradiction, Lerman, Sebag & Schoenauer, Jaccard, Support, Kappa, j-Measure, CausalSupport) for the comparison.

Experiments were run on Jester5k data set with three scenarios using implications nature-Independence, nature-Dependence, and nature-Equilibrium. The obtained results show that the influence of the attributes on the recommender system is apparent. Each recommender model is affected by private tendency. In particular, Kappa has a tremendous influence on the recommender model of Independence tendency. The recommendation systems with Dependence tendency shows Support and Jaccard are feasible compared to other measures, and confidence presents good influence with Equilibrium tendency. While Least Contradiction shows bad recommendations for both the Dependence and Equilibrium for the recommendation models, Gini_index is also bad for the consulting model with Independence. Most of the cases with various similarity measures of Independence tendency archives better than the others.

References

1. Felfernig, A., Jeran, M., Ninaus, G., Reinfrank, F., Reiterer, S., Stettinger, M.: Basic approaches in recommendation systems. In: Robillard, M.P., Maalej, W., Walker, R.J., Zimmermann, T. (eds.) *Recommendation Systems in Software Engineering*, pp. 15–37. Springer, Heidelberg (2014). https://doi.org/10.1007/978-3-642-45135-5_2
2. Aggarwal, C.C.: *Recommender Systems*. Springer, Cham (2016). <https://doi.org/10.1007/978-3-319-29659-3>
3. Ricci, F., Rokach, L., Shapira, B. (eds.): *Recommender Systems Handbook*. Springer, New York (2015). <https://doi.org/10.1007/978-1-4899-7637-6>
4. Adomavicius, G., Tuzhilin, A.: Toward the next generation of recommender systems: a survey of the state-of-the-art and possible extensions. *IEEE Trans. Knowl. Data Eng.* **17**(6), 734–749 (2005)
5. Saqlain, M., Riaz, M., Saleem, M.A., Yang, M.: Distance and similarity measures for neutrosophic hypersoft set (NHSS) with construction of NHSS-TOPSIS and applications. *IEEE Access* **9**, 30803–30816 (2021). <https://doi.org/10.1109/ACCESS.2021.3059712>
6. Yan, H., Tang, Y.: Collaborative filtering based on Gaussian mixture model and improved Jaccard similarity. *IEEE Access* **7**, 118690–118701 (2019)
7. Huynh, H.X., et al.: Context-similarity collaborative filtering recommendation. *IEEE Access* **8**, 33342–33351 (2020)
8. Mpela, M.D., Zuva, T.: A mobile proximity job employment recommender system. In: *2020 International Conference on Artificial Intelligence, Big Data, Computing and Data Communication Systems (icABCD)*, pp. 1–6 (2020)
9. Phan, L.P., Huynh, H.H., Huynh, H.X.: Hybrid recommendation based on implicative rating measures. In: *International Conference on Machine Learning and Soft Computing, ICMLSC 2018, New York, NY, USA*, pp. 50–56. Association for Computing Machinery (2018). <https://doi.org/10.1145/3184066.3184076>
10. Chirico, R., et al.: Guidelines for reporting of phase equilibrium measurements (IUPAC recommendations 2012). *Pure Appl. Chem.* **84**, 1785–1813 (2012)
11. Goldberg, D., Nichols, D., Oki, B.M., Terry, D.: Using collaborative filtering to weave an information tapestry. *Commun. ACM* **35**(12), 61–70 (1992). <https://doi.org/10.1145/138859.138867>
12. Huynh, H.X., Phan, N.Q., Duong-Trung, N., Nguyen, H.T.T.: Collaborative filtering recommendation based on statistical implicative analysis. In: Hernes, M., Wojtkiewicz, K., Szczerbicki, E. (eds.) *ICCCI 2020. CCIS*, vol. 1287, pp. 224–235. Springer, Cham (2020). https://doi.org/10.1007/978-3-030-63119-2_19
13. Banda, L., et al.: Recommender systems using collaborative tagging. *Int. J. Data Wareh. Min.* **16**(3), 183–200 (2020)
14. Nguyen, H.T., Huynh, H.H., Phan, L.P., Huynh, H.X.: Improved collaborative filtering recommendations using quantitative implication rules mining in implication field. In: *Proceedings of the 3rd International Conference on Machine Learning and Soft Computing, ICMLSC 2019, New York, NY, USA*, pp. 110–116. Association for Computing Machinery (2019). <https://doi.org/10.1145/3310986.3310996>
15. Huynh, H.X., Cu, G.N., Huynh, T.M., Luong, H.H., et al.: Recommender systems based on resonance relationship of criteria with Choquet operation. *Int. J. Data Wareh. Min. (IJDWM)* **16**(4), 44–62 (2020)
16. Berkani, L., Betit, L., Belarif, L.: A multi-view clustering approach for the recommendation of items in social networking context. In: Senouci, M.R., Boudaren,

- M.E.Y., Sebbak, F., Mataoui, M. (eds.) CSA 2020. LNNS, vol. 199, pp. 241–251. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-69418-0_22
17. Nilashi, M., Bagherifard, K., Ibrahim, O., Alizadeh, H., Nojeem, L., Roozegar, N.: Collaborative filtering recommender systems. *Res. J. Appl. Sci. Eng. Technol.* **5**, 4168–4182 (2013)
 18. Osadchiy, T., Poliakov, I., Olivier, P., Rowland, M., Foster, E.: Recommender system based on pairwise association rules. *Expert Syst. Appl.* **115**, 535–542 (2019). <https://www.sciencedirect.com/science/article/pii/S095741741830441X>
 19. Sarwar, B., Karypis, G., Konstan, J., Riedl, J.: Application of dimensionality reduction in recommender system - a case study (2000)
 20. Amatriain, X., Jaimes, A., Oliver, N., Pujol, J.M.: Data mining methods for recommender systems. In: Ricci, F., Rokach, L., Shapira, B., Kantor, P. (eds.) *Recommender Systems Handbook*, pp. 39–71. Springer, Boston (2011). https://doi.org/10.1007/978-0-387-85820-3_2
 21. Gras, R., Kuntz, P.: An overview of the statistical implicative analysis (SIA) development. In: Gras, R., Suzuki, E., Guillet, F., Spagnolo, F. (eds.) *Statistical Implicative Analysis. Studies in Computational Intelligence*, vol. 127, pp. 11–40. Springer, Heidelberg (2008). https://doi.org/10.1007/978-3-540-78983-3_1
 22. Nguyen, H.T., Phan, L.P., Huynh, H.H., Huynh, H.X.: Recommendation with quantitative implication rules. *EAI Endorsed Trans. Context-Aware Syst. Appl.* **6**(16), e2 (2019)
 23. Hills, J., Davis, L.M., Bagnall, A.: Interestingness measures for fixed consequent rules. In: Yin, H., Costa, J.A.F., Barreto, G. (eds.) *IDEAL 2012. LNCS*, vol. 7435, pp. 68–75. Springer, Heidelberg (2012). https://doi.org/10.1007/978-3-642-32639-4_9
 24. Phan, L.P., Phan, N.Q., Phan, V.C., Huynh, H.H., Huynh, H.X., Guillet, F.: Classification of objective interestingness measures. *EAI Endorsed Trans. Context-Aware Syst. Appl.* **3**(10), e4 (2016)
 25. Hills, J., Davis, L.M., Bagnall, A.: Preprint: Interestingness measures for fixed consequent rules (2012)
 26. Sarwar, B., Karypis, G., Konstan, J., Riedl, J.: Analysis of recommendation algorithms for e-commerce. In: *Proceedings of the 2nd ACM Conference on Electronic Commerce*, New York, NY, USA, EC 2000. pp. 158–167. Association for Computing Machinery (2000). <https://doi.org/10.1145/352871.352887>
 27. Mild, A., Reutterer, T.: Collaborative filtering methods for binary market basket data analysis. In: Liu, J., Yuen, P.C., Li, C., Ng, J., Ishida, T. (eds.) *AMT 2001. LNCS*, vol. 2252, pp. 302–313. Springer, Heidelberg (2001). https://doi.org/10.1007/3-540-45336-9_35
 28. Schafer, J.B., Frankowski, D., Herlocker, J., Sen, S.: Collaborative filtering recommender systems. In: Brusilovsky, P., Kobsa, A., Nejdl, W. (eds.) *The Adaptive Web. LNCS*, vol. 4321, pp. 291–324. Springer, Heidelberg (2007). https://doi.org/10.1007/978-3-540-72079-9_9