Improved Incremental Freezing HARQ Schemes Using Polar Codes over Degraded Compound Channels

Tianze Hu[1], Lei Xie[1,2(✉)], Huifang Chen[1,3], Hongda Duan[1], and Kuang Wang[1,2]

[1] College of Information Science and Electronic Engineering, Zhejiang University, Hangzhou 310027, China
{21631077,xiel,chenhf,wangk}@zju.edu.cn

[2] Zhejiang Provincial Key Laboratory of Information Processing, Communication and Networking, Hangzhou 310027, China

[3] Zhoushan Ocean Research Center, Zhoushan 316021, China

Abstract. The error propagation problem in incremental freezing (IF) hybrid automatic repeat request (HARQ) scheme using Polar codes is studied. We propose two IF HARQ schemes using polar codes, namely the cyclic redundancy check (CRC)-aided IF HARQ scheme and the cumulative-path-metrics-based IF HARQ scheme. In the CRC-aided IF HARQ scheme, several CRC bits are added to each transmitted block. Using these CRC bits, the proposed IF HARQ scheme and the Chase Combining HARQ scheme can be combined to achieve a better error correction performance in the cost of a larger decoding delay. In the cumulative-path-metrics-based IF HARQ scheme, the successive joint decoder maintains multiple possible paths simultaneously, and the cumulative path metrics is used to represent the reliability of each surviving path in the decoding process. Moreover, a modified path splitting reduced successive cancellation list (SCL) decoding algorithm is presented to reduce the computational complexity and the memory requirement of cumulative-path-metrics-based IF HARQ scheme. Simulation results show that, using the Polar code constructed under long block length and high block error rate, the CRC-aided IF HARQ scheme has a higher system throughput. With the Polar code constructed under short block length and low block error rate, the cumulative-path-metrics-based IF HARQ scheme has a higher system throughput. In both situations, the system block error rate of the CRC-aided IF HARQ scheme performs well.

Keywords: Degraded compound channel · Polar codes · Incremental freezing HARQ scheme · Path splitting reduced SCL decoding algorithm

This work was partly supported by the Science and Technology Department of Zhejiang Province (No. LGG18F010005, No. 2018R52046), and National Natural Science Foundation of China (No. 61671410).

H. Gao et al. (Eds.): ChinaCom 2019, LNICST 312, pp. 316–330, 2020.
https://doi.org/10.1007/978-3-030-41114-5_24

1 Introduction

Polar codes are one of capacity-achieving linear block codes first proposed in [1]. In order to construct polar codes, it is necessary to acquire the reliability of each polarized channel. Given the code length, the transmission rate and the signal-to-noise ratio (SNR) of channel, a variety of algorithms can be used to calculate the construction of polar codes effectively. However, in many communication scenarios, the channel state is time-varying and unknown by the transmitter. Hence, how to construct polar codes under such situation is an open problem.

The most conservative transmission strategy is to develop a code structure with a fixed rate in the worst possible channel, but this strategy results in a low transmission efficiency. For another transmission scheme, the transmitter continues to transmit the encoded bits until the receiver can decode them correctly. However, this transmission scheme requires the receiver to send a feedback (ACK/NACK) to indicate whether the decoding is successful or not although the condition can be satisfied easily in most communication scenarios. The transmission strategy with feedback is known as hybrid automatic repeat request (HARQ) which can resolve the error problem caused by the mismatch between the channel state and the code construction.

Recently, some HARQ schemes using polar codes have been proposed. In [2], a chase combining (CC) HARQ scheme using polar code is presented, where each retransmission code block is identical to the original one. The receiver combines the soft information of current received block with all the previous decoding-failed blocks using the maximum ratio combination (MRC) rule, and tries to decode the code block again. Hence, the CC HARQ scheme achieves a diversity gain. In [3], an incremental redundancy (IR) HARQ scheme using polar code is proposed, where each retransmission code block contains different information compared with the CC HARQ scheme. For a set of information bits, multiple sets of coded bits are generated. The retransmission uses a different set of coded bits with different redundancy versions generated by puncturing the encoder output. Hence, with each retransmission, the receiver obtains the extra information. In [4], an incremental freezing (IF) HARQ scheme using polar code is proposed, where the retransmitted bits are the information bits with a lower reliability in previous incorrect decoding blocks. The IF HARQ scheme is proved to be capacity-achieving over a class of channels with degradation relationship which also named as the degraded compound channels.

However, the IF HARQ scheme proposed in [4] adopts a successive joint decoding structure, which results in the problem of error propagation. At the receiver, some information bits in previous received code block are used as the frozen bits in subsequent received blocks' decoding process. Hence, if the decoding result of previous received block is incorrect, all the decoding results of subsequent received blocks should be wrong, which leads to the performance degradation of error correction.

To mitigate the error propagation, we propose two improved IF HARQ schemes using polar codes, the cyclic redundancy check (CRC)-aided IF HARQ scheme and the cumulative-path-metrics-based IF HARQ scheme.

The idea of the CRC-aided IF HARQ scheme is straightforward. Several CRC bits are added to each retransmitted block as well as the original transmitted block. With the help of these CRC bits, the IF HARQ scheme can be combined with the CC HARQ scheme to achieve a good error correction performance. The receiver has the information that whether the current transmitted block is decoded correctly or not using the CRC bits. If the decoding result is incorrect, the receiver sends a feedback to the transmitter for requesting a re-transmission until the decoding successes or the retransmission time exceeds the upper limit. Hence, the performance degradation caused by the decoding error propagation problem is mitigated. However, the CRC-aided IF HARQ scheme has a larger decoding delay, and the additional CRC bits in each retransmission block reduces the system throughput.

In the cumulative-path-metrics-based IF HARQ scheme, referring to the idea of the successive cancellation list (SCL) decoding algorithm, we introduce the cumulative path metrics to represent the reliability of each surviving path in the decoding process. The successive joint decoder maintains multiple possible decoding paths at the same time to eliminate the error propagation. To reduce the calculation complexity and space complexity, a path splitting reduced SCL decoding algorithm is used in the proposed cumulative-path-metrics-based IF HARQ scheme.

2 Preliminaries

In this section, we briefly introduce the polar code, channel model and the IF HARQ scheme using polar codes.

2.1 Polar Code

Let $\mathbf{X} = \{0, 1\}$ and $\mathbf{Y} = \{0, 1\}$. $W : \mathbf{X} \to \mathbf{Y}$ denotes a binary-input discrete memoryless channel (B-DMC). The basic channel transform of polar code creates two polarized channels with different capacities as

$$W_2^{(0)}\left(y_0^1|u_0\right) = \frac{1}{2}\sum_{u_1} W(y_0|u_0 \oplus u_1)W(y_1|u_1)\Bigg\} = (W, W)^-, \tag{1}$$

$$W_2^{(1)}\left(y_0^1, u_0|u_1\right) = \frac{1}{2}W(y_0|u_0 \oplus u_1)W(y_1|u_1) = (W, W)^+. \tag{2}$$

where u_0^1 is the input vector uniformly distributed over $\mathbf{X}^2$, and y_0^1 is the corresponding channel output.

For a block of bits, the construction of polar code lays on the polarization effect of matrix $\mathbf{G}_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. Let $\mathbf{G}_2^{\otimes n}$ denote the n^{th} Kronecker power of $\mathbf{G}_2$. For a block of $N = 2^n$ bits, U_0^{N-1}, applying the transform $\mathbf{G}_2^{\otimes n}$ to get the encoded block, $X_0^{N-1} = U_0^{N-1}\mathbf{G}_2^{\otimes n}$, and transmitting each bit through independent copies of B-DMC, W.

Applying the chain rule to mutual information between input U_0^{N-1} and output Y_0^{N-1}, we have

$$I\left(U_0^{N-1}; Y_0^{N-1}\right) = \sum_{i=0}^{N-1} I\left(U_i; Y_0^{N-1}|U_0^{i-1}\right) = \sum_{0}^{N-1} I\left(U_i; Y_0^{N-1}, U_0^{i-1}\right). \tag{3}$$

An important property of polar code is that, except for a negligible fraction, the terms in the right summation of (3) either approach to 0 (bad channel) or 1 (good channel) as n increases. Moreover, the fraction of those terms approaching to 1 converges the symmetric mutual information, $I(W)$. This phenomenon is the channel polarization.

The recursive expressions of polarization channel are

$$W_N^{(2i)} = \left(W_{N/2}^{(i)}, W_{N/2}^{(i)}\right)^-, \tag{4}$$

$$W_N^{(2i+1)} = \left(W_{N/2}^{(i)}, W_{N/2}^{(i)}\right)^+. \tag{5}$$

2.2 Channel Model

Let $\mathbf{W}_c$ denote a compound channel, which is a set of S sub-channels, $\mathbf{W}_c = \{W_1, W_2, \ldots, W_S\}$. This work aims at the flat-fading channel environment, which means each symbol in one block will transmit through the same channel state. The channel state information (CSI) and channel distribution information (CDI) are unknown to the transmitter. The knowledge it only has is the set of channels to which the channel belongs. Every round of transmission (including the original transmitted block and retransmitted blocks) happens on one of sub-channels, W_i. On the other hand, the CSI is known at the receiver. In the real applications, this assumption is correct in most cases since the receiver can use pilot symbols to estimate the channel state.

Let $I(\mathbf{W}_c)$ denote the compound capacity of $\mathbf{W}_c$, which is the rate that can be reliably transmitted irrespective of used channel. The compound capacity is defined as [5]

$$I(\mathbf{W}_c) = \max_P \inf_{W_i \in \mathbf{W}_c} I_P(W_i), \tag{6}$$

where $I_P(W_i)$ denotes the mutual information between the input and output of W_i with the input distribution P.

The compound capacity of $\mathbf{W}_c$ can be smaller than the infimum of the capacity of individual channels in $\mathbf{W}_c$, because the capacity achieving distribution for the individual channels might be different. If W_i is a symmetric channel, the compound capacity equals to the infimum of individual capacities. If channels in $\mathbf{W}_c$ are degraded, the infimum is obtained by the worst channel, and then the compound capacity equals to the capacity of the worst channel.

2.3 IF HARQ Scheme

Given two channels $Q : \mathbf{X} \to \mathbf{Z}$ and $W : \mathbf{X} \to \mathbf{Y}$, we say that Q is degraded with respect to W if there exists a channel $V : \mathbf{Y} \to \mathbf{Z}$ such that for all $z \in \mathbf{Z}$ and $x \in \mathbf{X}$,

$$Q(z|x) = \sum_{y \in \mathbf{Y}} W(y|x)V(z|y). \tag{7}$$

Let $Q \preccurlyeq W$ denote that Q is degraded with respect to W. It is proved in [6] that the channel polarization preserves degradedness. That is, if $Q \preccurlyeq W$, we have $Q^+ \preccurlyeq W^+$ and $Q^- \preccurlyeq W^-$. According to the recursive expressions (4) and (5), we have $Q_N^{(i)} \preccurlyeq W_N^{(i)}$, which means that the good polarization channel indices set $\mathbf{A}_Q$ for channel Q must be a subset of the good polarization channel indices $\mathbf{A}_W$ for channel W in the polarization process. This is called as the nesting property of polar codes, which leads to the development of the IF HARQ scheme.

For the compound channel $\mathbf{W}_c$, we assume that all the sub-channels are B-DMCs and have a degradation relationship. That is, $W_S \preccurlyeq W_{S-1} \preccurlyeq \ldots \preccurlyeq W_1$. According to the nesting property, we have $\mathbf{A}_{W_s} \subseteq \mathbf{A}_{W_{s-1}} \subseteq \cdots \subseteq \mathbf{A}_{W_1}$. At the transmitter, let $\mathbf{S}_k$ denote the k^{th} transmitted block, polar code construction $PC_{W_i}\left(N, K_{W_i}, \mathbf{A}_{W_i}, u_{\mathbf{A}^c_{W_i}}\right)$ on each sub-channel is calculated offline before the transmission starts, where N denotes the code length, K_{W_i} denotes the length of information bits for sub-channel W_i, $u_{\mathbf{A}^c_{W_i}}$ denotes the frozen bits for sub-channel W_i.

We greedily use the PC_{W_1} to encode the first transmission information bits and get $\mathbf{S}_1$, send it to the receiver, and wait for the feedback. If an ACK is received, the transmitter knows that all the transmitted bits are decoded correctly, and it prepares for the next transmission. However, if a NACK is received, which means a decoding error has happened at the receiver or the receiver cannot decode the information, the transmitter will reduce the transmission rate and transmit some information bits with a lower reliability from the previous transmitted blocks. Generally speaking, if k NACK feedbacks have been received, which means the previous k blocks are not decoded correctly or cannot be decoded, the transmitter will choose the information bits at the polarization channel indices of $\mathbf{A}_{W_k} - \mathbf{A}_{W_{k+1}}$ from previous k transmitted blocks, $\mathbf{S}_1$, $\mathbf{S}_2$, …, $\mathbf{S}_k$, and use $PC_{W_{k+1}}$ to encode the retransmitted bits. The number of the $(k+1)^{\text{th}}$ retransmitted bits is $k\left|\mathbf{A}_{W_k} - \mathbf{A}_{W_{k+1}}\right|$. We can put $\left|\mathbf{A}_{W_{k+1}}\right| - k\left|\mathbf{A}_{W_k} - \mathbf{A}_{W_{k+1}}\right|$ new information bits into the block and get $\mathbf{S}_{k+1}$.

Let $\mathbf{R}_k$ denote the k^{th} received block at the receiver. When $\mathbf{R}_k$ is received, the receiver decides whether to decode it according to the real channel state W_r. If $W_r \preccurlyeq W_k$, the transmission rate $R = I(W_k) > I(W_r)$. Referring to the Shannon theorem, the block $\mathbf{R}_k$ cannot be decoded, and then the receiver sends a NACK. If $W_r \succcurlyeq W_k$, the receiver needs to decode all the information bits from $\mathbf{R}_1, \mathbf{R}_2, \ldots, \mathbf{R}_k$ blocks, which is a successive joint decoding structure. Since block $\mathbf{S}_i$ contains some information bits with a lower reliability in $\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_{i-1}$ blocks, we can take the decoding result of $\mathbf{R}_i$ as the side information of frozen bits and put it into the decoding process of $\mathbf{R}_{i-1}, \mathbf{R}_{i-2}, \ldots, \mathbf{R}_1$, which leads to a lower successive joint decoding rate. It is proved in [4] that the IF HARQ scheme using polar code is capacity-achieving over the degraded compound channel owing to the nesting property.

3 Proposed IF HARQ Schemes Using Polar Codes

Since the IF HARQ scheme adopts a successive joint decoding structure, the problem of error propagation takes place. As the block error rate on polar code PC_{W_i} is e_i, the system block error rate should be $e_s = 1 - \prod_{i=1}^{k} (1 - e_i)$.

3.1 CRC-Aided IF HARQ Scheme Using Polar Code

To mitigate the error propagation problem of the IF HARQ scheme using polar code, a straightforward method is to add the extra CRC bits in the original transmission block and retransmission block(s). Using these CRC bits, the decoder can know the block is decoded correctly or not. Moreover, with CRC bits, the IF HARQ scheme and the CC HARQ scheme can be combined to achieve a better error correction performance.

In order to combine the CC HARQ scheme, we extend the NACK message to IF NACK and CC NACK.

If receiving an IF NACK message, the transmitter knows the previous transmission rate exceeds the channel capacity, and the receiver cannot decode the received block. Thus, the transmitter reduces the transmission rate, and sends some redundant information bits with a lower reliability in the previous transmitted blocks, as the IF HARQ scheme in the Subsect. 2.3.

If the transmitter receives a CC NACK message, it means that the previous transmission rate is smaller than the channel capacity. A successive joint decoding process is executed at the receiver, but a decoding error occurred in one of received blocks. Thus, the transmitter retransmits the corresponding block according to the index information carried in the CC NACK message.

At the receiver, the condition of whether sending ACK message or IF NACK message is the same as the IF HARQ scheme. It is assumed that the receiver has received k blocks. If $W_{\mathrm{r}} \succcurlyeq W_k$, the receiver should decode all the information bits from $\mathbf{R}_1$, $\mathbf{R}_2$, …, $\mathbf{R}_k$ blocks. In the successive joint decoding structure, if block $\mathbf{R}_i$ fails to decode, the receiver sends a CC NACK message including the index of $\mathbf{R}_i$, i.

Hence, a better error correction performance can be achieved at the price of a higher decoding delay, and a slight reduction system throughput.

It is analyzed in [7] that the computation complexity of the SCL decoding algorithm is $O(LN\log N)$, and the space complexity is $O(LN)$. Hence, the computation complexity of the CRC-aided IF HARQ scheme is $O(kLN\log N)$, and the space complexity is $O(kLN)$.

3.2 Cumulative-Path-Metrics-Based IF HARQ Scheme Using Polar Code

As we know, the successive joint decoding structure is similar to the SC decoding algorithm. For the SC decoding algorithm, the decoding results of all the previous bits $\hat{u}_1^{i-1}$ are needed to decode the current bit $\hat{u}_i$. For the successive joint decoding structure of IF HARQ scheme, if the receiver has received k blocks, the decoding results of $\mathbf{R}_{i+1}$, $\mathbf{R}_{i+2}$, …, $\mathbf{R}_k$ are needed to decode $\mathbf{R}_i$ block.

For the SCL decoding algorithm, several SC decoders operate in parallel and maintain multiple possible decoding paths at the same time to improve the block error correction performance. Referring to the idea of the SCL decoding algorithm, the successive joint decoder can also maintain multiple possible paths at the same time to improve the error correction performance.

In the successive joint decoding structure, the decoding results of received blocks are needed to form a decoding path. As the path metrics in the SCL algorithm, we introduce the cumulative path metrics to indicate the reliability of a decoding path. Let $\hat{\mathbf{c}}_i$ denote the SCL decoding results of block $\mathbf{R}_i$, and the list size of $\hat{\mathbf{c}}_i$ is L. Let $\hat{\mathbf{c}}_{i,l}$ be one of the decoding results in list, $1 \leqslant l \leqslant I$. Let $\mathbf{p}_i$ be the decoding path set in successive joint decoding structure after $\mathbf{R}_i$ is decoded, the cardinality of $\mathbf{p}_i$ is M, $M \leqslant L$. Each decoding path $\mathbf{p}_{i,m}$ contains the decoding results of $\mathbf{R}_i$, $\mathbf{R}_{i+1}$, …, $\mathbf{R}_k$. That is,

$$\mathbf{p}_{i,m} = \left\{\hat{\mathbf{c}}_{i,l_{i,m}}, \hat{\mathbf{c}}_{i+1}, l_{i+1,m}, \ldots, \hat{\mathbf{c}}_{k,l_{k,m}}\right\}, \; 1 \leqslant m \leqslant M, \tag{8}$$

where $l_{i,m}$ means that for different decoding path and block, the decoding result chosen by the SCL decoder may be different.

Let $\mathrm{pm}_{i,l}$ be the path metrics of decoding result $\hat{\mathbf{c}}_{i,l}$, $\mathbf{PM}_i$ is the cumulative path metrics set after $\mathbf{R}_i$ is decoded, the cardinality of set $\mathbf{PM}_i$ is M. We have

$$\mathbf{PM}_{i,m} = \sum_{j=i}^{k} \mathrm{pm}_{j,l_{j,m}}, \; 1 \leqslant m \leqslant M. \tag{9}$$

Let $\mathbf{D}_i$ denote the decoding output of $\mathbf{R}_i$ including the decoding path and the cumulative path metrics of this path. That is,

$$\mathbf{D}_i = \left\{\left(\mathbf{p}_{i,1}, \mathbf{PM}_{i,1}\right), \left(\mathbf{p}_{i,2}, \mathbf{PM}_{i,2}\right), \ldots, \left(\mathbf{p}_{i,M}, \mathbf{PM}_{i,M}\right)\right\}. \tag{10}$$

Let $\mathbf{B}_i$ be the decoding buffer for block $\mathbf{R}_i$, and its size is ML.

Without CRC bits in retransmission block, the decoders in successive joint decoding structure do not output a decoding result but maintain M possible paths to participate in the subsequent decoding process. Based on this idea, we propose the cumulative-path-metrics-based IF HARQ scheme using polar code. The logic at transmitter is the same as the IF HARQ scheme using polar code. The decoding algorithm at the receiver is summarized in Algorithm 1 as follows.

Algorithm 1: The decoding algorithm of cumulative-path-metrics-based IF HARQ scheme using polar code

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Input: the received block **R**, the channel state W_{r}, the maximum retransmission time T.
Output: feedback = union {ACK, NACK}.
Decoding Procedure

(1) Calculate polar code construction PC_{W_i} on each sub-channel offline. Let **RB** denote the receiver buffer, c be the received block number. $c = 0$.
(2) **while true**
(3) Wait for the block to arrive. Receive the block, $c = c + 1$.
(4) **if** $c > T + 1$
(5) Maximum retransmission time is exceeded. Current received block is the first block of a new round transmission. Empty **RB** and set $c = 1$.
(6) **end if**
(7) Set **RB**[c] = **R**, received block goes into the buffer and waits to be decoded.
(8) **if** $W_{\mathrm{r}} \preccurlyeq W_c$
(9) feedback = NACK.
(10) **else**
(11) Initialize $\mathbf{D}_{c+1} = \underbrace{\{(\varnothing, 0), \ldots, (\varnothing, 0)\}}_{M \text{ pairs}}$, the decoding paths are initialized as empty, the cumulative path metrics of each decoding path is initialized to be 0.
(12) **for** $i = c : 1$
(13) **for** $m = 1 : M$
(14) Choose the decoding path $\mathbf{D}_{i+1}[m]$, select correspond information bits in $\mathbf{p}_{i+1,m}$ and set them as frozen bits for **RB**[i] block. Decode **RB**[i] block.
(15) **for** $l = 1 : L$
(16) $\mathbf{curPath} = \mathbf{p}_{i+1,m} \cup \{\hat{\mathbf{c}}_{i,l}\}$.
(17) $curPM = \mathbf{PM}_{i+1,m} + pm_{i,l}$.
(18) $\mathbf{B}_i[m][l] = (\mathbf{curPath}, curPM)$.
(19) **end for**
(20) **end for**
(21) If $i = 1$, choose the path with the smallest cumulative path metrics from $\mathbf{B}_i$ as the decoding result, feedback = ACK. Otherwise, choose M paths with small cumulative path metrics as the decoding results of **RB**[i] block, and $\mathbf{D}_i = \{(\mathbf{p}_{i,1}, \mathbf{PM}_{i,1}), \ldots, (\mathbf{p}_{i,M}, \mathbf{PM}_{i,M})\}$.
(22) **end for**
(23) **end if**
(24) **end while**

As shown in the Algorithm 1, the performance degradation is mitigated by maintaining multiple decoding paths through the successive joint decoding structure.

The computation complexity of cumulative-path-metrics-based IF HARQ scheme using polar code is $O(kMLN\log N)$, and the space complexity is $O(kMLN)$. Hence, the cumulative-path-metrics-based IF HARQ scheme is too complex to use.

3.3 Modified Path Splitting Reduced SCL Algorithm

To reduce the complexity of the cumulative-path-metrics-based IF HARQ scheme, the system should adaptively adjust the number of maintained paths according to the decoding likelihood ratio, and only maintain those paths with a higher reliability.

In the SCL decoding algorithm, each path will split into two after one information bit is decoded. When the decoding confidence of an information bit is high enough, the path does not need to split. Although the hard decision according to the likelihood ratio is used, the error correction performance of the decoder will not degrade too much. In [8], a rule for controlling the path splitting in the SCL decoding algorithm is proposed.

Let $P_e(u_i)$ denote the decoding error rate of information bit u_i. We have

$$\begin{aligned} P_e(u_i) &= P\big(\hat{u}_i = u_i \oplus 1\big) \\ &= \sum_{u_1^{i=1} \in \mathbf{X}} \sum_{y_1^N \in \mathbf{Y}} P\Big((1-2u_i)L(u_i) < 0 | \hat{u}_1^{i-1} = u_1^{i-1}, u_i, y_1^N\Big) \end{aligned} \tag{11}$$

Therefore, $1 - P_e(u_i)$ can be used as a criterion to determine whether the path is split or not. The value of $P_e(u_i)$ can be estimated by Monte-Carlo simulation or Gaussian approximation algorithm [9]. The rule of path splitting is defined as

$$\hat{u}_i = \begin{cases} 0, & L_l(u_i) > \log \frac{1-P_e(u_i)}{P_e(u_i)}, \\ 1, & L_l(u_i) < -\log \frac{1-P_e(u_i)}{P_e(u_i)}, \\ \text{split, otherwise,} \end{cases} \tag{12}$$

where $L_l(u_i)$ denotes the decoding log-likelihood ratio of u_i in the l^{th} path.

The splitting features of the correct decoding path and wrong decoding path are studied in [8] as following:

(i) If all the decoding results of previous information bits $\hat{u}_1^{i-1}$ are correct, after decoding u_i, this path tends not to split and be retained by the decoder.
(ii) For any error decoding path reached at u_i, it tends to split at $\{i + 1, i + 2, \ldots, N\}$ indices.

We modify the path splitting reduced SCL algorithm in [8], where a threshold for the number of continuous splits to remove the paths be more possible wrong is introduced. The modified algorithm is summarized in Algorithm 2 as following.

Algorithm 2: Modified path splitting reduced SCL algorithm

Decoding Procedure

(1) Decode the first bit u_1 of the block. Initialize continuous non-splitting counter $\omega_l[i]=0$, continuous splitting counter $\theta_l[i]=0$.

(2) For the l^{th} decoding path and current decoding bit u_i, if u_i is a frozen bit, set $\hat{u}_i = u_i$. If u_i is an information bit, and the rule of path splitting is not satisfied, directly using hard decision according to the likelihood ratio to decode bit u_i, update the counter, $\omega_l[i]=\omega_l[i-1]+1$, $\theta_l[i]=0$; Otherwise, split the current path l into two paths l' and l'', set $\omega_{l'}[i]=\omega_{l''}[i]=0$, $\theta_{l'}[i]=\theta_{l''}[i]=\theta_l[i-1]+1$.

(3) If the number of surviving paths exceeds the upper limit L, delete those paths with continuous non-splitting counter less than a threshold value ω. If there is no such path, delete those paths with continuous splitting counter greater than another threshold value θ. If the continuous splitting counter of all the surviving paths are greater than θ or less than θ, choose L paths with the highest reliability according to the path metrics.

(4) Decode the next bit. If $i < N$, set $i = i + 1$, go to step (2). Otherwise, the surviving paths are the decoding results.

Combining the modified path splitting reduced SCL algorithm to the successive joint decoding structure, not only the computation complexity of each decoding process can be reduced, but also the error decoding paths of previous decoded block can be filtered according to the path splitting feature.

For the cumulative-path-metrics-based IF HARQ scheme, if the receiver has decoded block $\mathbf{R}_i$ using the modified path splitting reduced SCL algorithm to get p possible decoding paths, the decoding process for next block $\mathbf{R}_{i-1}$ is performed by the decoder. If the number of surviving paths exceeds the upper limit pL, those paths with continuous non-splitting counter less than a threshold value ω are removed. If there is no path with continuous non-splitting counter less than a threshold value ω, those paths with continuous splitting counter greater than another threshold value θ are deleted. If the continuous splitting counter of all the surviving paths are greater than θ or less than θ, pL paths with the highest reliability according to the path metrics are selected.

After the last bit is decoded and the number of surviving paths exceeds M, M paths with small cumulative path metrics are chosen as the decoding results of $\mathbf{R}_{i-1}$.

In the best condition, the decoding confidence of each bit is very high, the path splitting reduced SCL algorithm turns to the SC decoding algorithm. Hence, the computation complexity is $O(kN\log N)$, and the space complexity is $O(kN)$.

In the worst condition, each path splits after the decoding of information bits, the path splitting reduced SCL algorithm degenerates to the SCL decoding algorithm. Hence, the computation complexity is $O(kMLN\log N)$, and the space complexity is $O(kMLN)$.

4 Simulation Results

Figure 1 shows the comparison of the error correction performance of the SCL algorithm, the CRC-aided SCL algorithm, the path splitting reduced SCL algorithm and the modified path splitting reduced SCL algorithm in terms of the block error rate, where $N = 256$, $K = 128$, $L = 8$, the generator polynomial of CRC bits is $g(D) = D^{24} + D^{23} + D^6 + D^5 + D + 1$. From Fig. 1, we observe that given the set value of continuous non-splitting time threshold ω, as continuous splitting time threshold θ gets larger, the block error rate of modified path splitting reduced SCL algorithm decreases, and finally converges to the performance of path splitting reduced SCL algorithm.

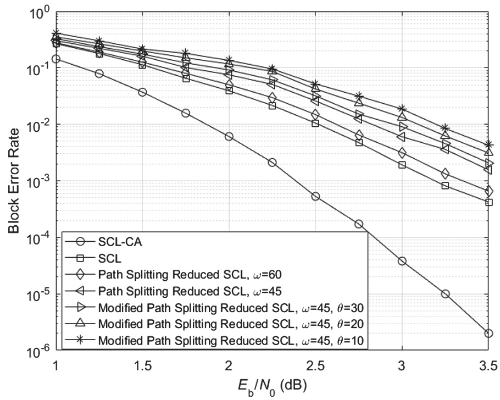


Fig. 1. The error correction performance of different decoding algorithms.

Figure 2 shows the comparison of the average number of surviving paths after each bit is decoded in the SCL algorithm, the path splitting reduced SCL algorithm and the modified path splitting reduced SCL algorithm, where $N = 256$, $K = 128$, $L = 8$, and $E_b/N_0 = 2$ dB. The area between each curve and x axis denotes the computation complexity and space complexity of each algorithm. From Fig. 2, we observe that as the continuous splitting time threshold θ goes smaller, the average number of surviving paths decreases. Hence, the computation complexity and space complexity reduce.

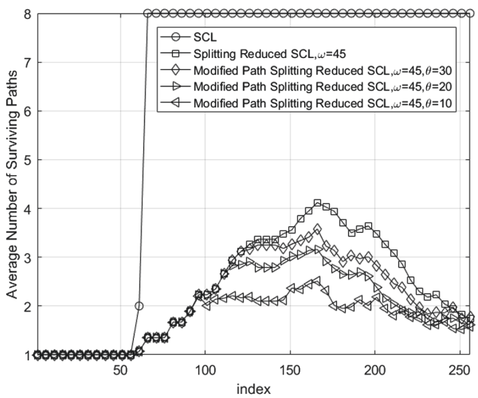


Fig. 2. The complexity of different decoding algorithms.

Two performance metrics are considered for a HARQ scheme, the system throughput and the block error rate. The throughput is the ratio of the expected number of information bits be decoded correctly and the expected number of all the bits be transmitted. For the IF HARQ scheme, if the receiver decodes the information bits out of k blocks, $\mathbf{R}_k$, $\mathbf{R}_{k-1}$, …, $\mathbf{R}_1$, the system throughput can be calculated as

$$\eta_{\mathrm{t}} = \frac{\sum_{i=1}^{k} \left|\mathcal{A}_{W_i}\right| - (i-1)\left|\mathcal{A}_{W_{i-1}} - \mathcal{A}_{W_i}\right|}{kN} = \frac{\sum_{i=1}^{k} \left|\mathcal{A}_{w_i}\right| - (i-1)\left(\left|\mathcal{A}_{w_{i-1}}\right| - \left|\mathcal{A}_{w_i}\right|\right)}{kN} = \frac{k\left|\mathcal{A}_{w_{k+1}}\right|}{kN} = \frac{\left|\mathcal{A}_{w_k}\right|}{N}, \tag{13}$$

which is capacity-achieving.

Figures 3 and 4 show the comparison of the system throughput and the block error rate of the CC HARQ scheme, the IF HARQ scheme, the CRC-aided IF HARQ scheme and the cumulative-path-metrics-based IF HARQ scheme, where the simulations are performed over degraded compound channel $\mathbf{W}_{\mathrm{c}} = \{W_1, W_2, \ldots W_7\}$, the sub-channels are binary input additive white Gaussian noise (BIAWGN), $L = 8$, and polar code is constructed under given block error rate and code length, e_{b} and N.

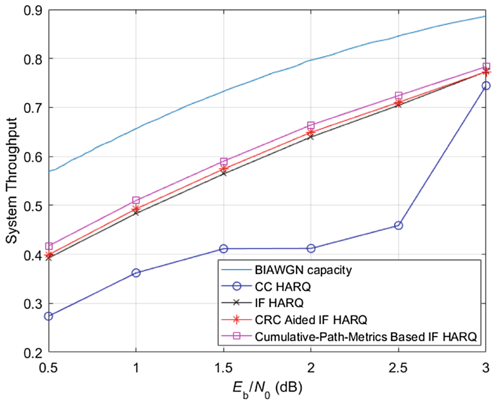
For the CC HARQ scheme and the IF HARQ scheme, the maximum retransmission time is 6. For the CRC-aided IF HARQ scheme, the generator polynomial of CRC is $g(D) = D^{24} + D^{23} + D^6 + D^5 + D + 1$, the maximum retransmission time for incremental freezing bits is 6, and the maximum retransmission time for CC bits is 1. For the cumulative-path-metrics-based IF HARQ scheme, the maximum retransmission time is 6, $M = 8$, $\omega = 60$, and $\theta = 40$.

The transmission rate of polar codes over sub-channels is listed in Table 1. Here, the SNR is calculated by $E_b/N_0 = 1/\left(2C_{\mathrm{BIAWGN}}\sigma^2\right)$. Hence, the real SNR for polar encoded blocks should be lower than the calculated value.

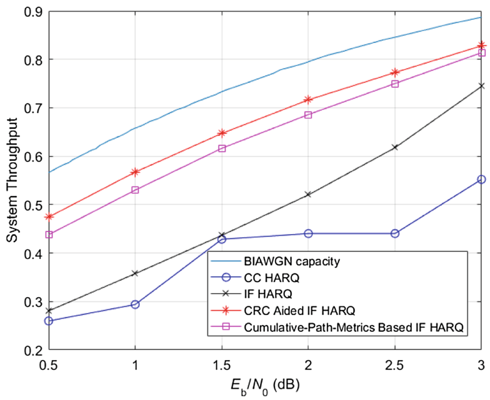
Table 1. The transmission rate of polar codes

SNR (dB)	$e_b = 0.01, N = 1024$	$e_b = 0.1, N = 4096$
W_7: 0.5	0.4229	0.4878
W_6: 1.0	0.5166	0.5815
W_5: 1.5	0.5977	0.6614
W_4: 2.0	0.6729	0.7312
W_3: 2.5	0.7344	0.7878
W_2: 3.0	0.7969	0.8428
W_1: 3.5	0.8467	0.8862

From Figs. 3 and 4, we observe that the CC HARQ scheme does not apply to degraded compound channel, the system throughput is lower than that of the IF HARQ scheme, and it has the discontinuous feature. This is because the retransmission blocks in the CC HARQ scheme is the same as the original transmission block. That is, after the k^{th} retransmission, the system throughput is $\frac{|\mathcal{A}_{W_1}|}{(k+1)N}$. The advantages of the CC HARQ are simple design and low retransmission times.



(a) $N = 1024$, e_b=0.01

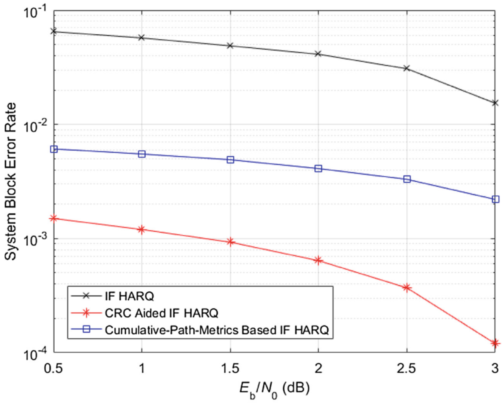


(b) $N = 4096$, e_b=0.1

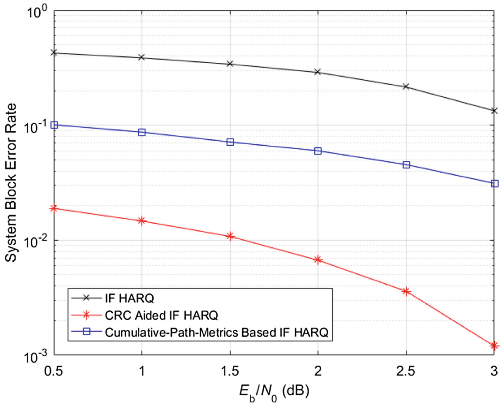
Fig. 3. The system throughput of different HARQ schemes using polar codes.

The IF HARQ scheme performs better than the CC HARQ using polar codes construction in a low the block error rate, e_b. However, the performance of the IF HARQ scheme degrades as the block error rate goes larger, which is caused by error propagation.

For the CRC-aided IF HARQ scheme, the performance in terms of the error correction performs the best. It works well under polar code with long block length and high block error rate. This is because the error propagation problem is resolved by adding



(a) $N = 1024$, e_b=0.01



(b) $N = 4096$, e_b=0.1

Fig. 4. The system block error rate of different HARQ schemes using polar codes.

extra CRC bits. However, under polar code with short block length and low block error rate, the reduction of system throughput caused by CRC bits appended in each transmission block cannot be negligible. Moreover, the CRC-aided IF HARQ scheme has a large decoding delay.

For the cumulative-path-metrics-based IF HARQ scheme, the error propagation problem is resolved by maintaining several decoding paths among the successive joint decoding structure. This HARQ scheme performs well under polar code with short block length and low block error rate. Under polar code with long block length and high block error rate, the gap of the error correction between the path splitting reduced SCL algorithm and the CA-SCL algorithm becomes larger. Hence, the system throughput of cumulative-path-metrics-based IF HARQ scheme is lower than that of CRC-aided IF HARQ scheme.

5 Conclusions

In this paper, we investigated the problem of error propagation in the IF HARQ scheme using polar codes, which results in the performance degradation. We proposed two improved IF HARQ schemes using polar codes, the CRC-aided IF HARQ scheme and the cumulative-path-metrics-based IF HARQ scheme. In the CRC-aided IF HARQ scheme, the CC HARQ scheme is combined, and the error propagation problem is mitigated by appending extra CRC bits to each transmission block. In the cumulative-path-metrics-based IF HARQ scheme, the error propagation problem is solved by maintaining multiple decoding paths among the successive joint decoding structure. Simulation results show that, for the polar code constructed under long block length and high block error rate, the CRC-aided IF HARQ scheme achieves a higher system throughput. For the polar code constructed under short block length and low block error rate, the cumulative-path-metrics-based IF HARQ scheme obtains a higher system throughput. In addition, the system block error rate of CRC-aided IF HARQ scheme outperforms the cumulative-path-metrics-based IF HARQ scheme.

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