





Motion Constraint Aided Underwater Integrated Navigation Method Based on Improved Adaptive Filtering

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Abstract. The underwater environment is complicated. Aiming at the problem of sudden changes in measurement information when the submarine encounters submarine trenches, clusters of fish, and strong maneuvering turns during navigation, this paper introduces centripetal acceleration errors on the basis of traditional motion constraints, which effectively restrict the forward speed of the carrier. Meanwhile, this paper proposed the fault judgment of the measurement information and the optimal estimation of the scale factor to improve the Sage-Husa adaptive filtering. Finally, a simulation experiment is carried out and the results show that the improved algorithm can reduce the computation complexity when the measurement information is correct. What's more, it can also suppress the divergence of the system filter effectively when the measurement information has sudden errors, which proves the robustness and reliability of the system.

Keywords: Motion constraint · Centripetal acceleration · Sage-Husa adaptive filtering · Fault judgment · Scale factor

1 Motion Constraint Model Based on Centripetal Acceleration

1.1 Background

With the development of the technology of underwater vehicles, as reliable detection tools, underwater vehicles have received widespread attention at home and abroad. How to improve the accuracy of underwater navigation has also become the focus of attention of scholars in various fields. However, the underwater environment is complicated. Usually, underwater submarines will glide in sawtooth waves on the bottom of the sea [1]. During the gliding process, if it encounters submarine ditches, clusters of fish or strong maneuvering turns [2]. The measurement information provided by the Doppler will be wrong, and the preset measurement noise matrix will not be able to adapt to the disturbed model, which will affect the accuracy of positioning. Meanwhile, the centripetal acceleration will restrict the speed of the underwater vehicle.

In order to solve the above problems, this paper introduces the centripetal acceleration constraint [3] and the improved Sage-Husa adaptive algorithm [4] to assist inertial/Doppler integrated navigation. When the underwater vehicles encounter submarine trenches, fish schools and strong maneuverability, and the Doppler measurement information is wrong. The centripetal acceleration will be introduced to restrict the speed of the underwater vehicle, and the improved Sage-Husa adaptive filtering algorithm will be adapted to reduce positioning error and improve positioning accuracy.

1.2 Motion Constraint Model

The operating environment of the submarine is usually more than ten meters underwater, and the water flow is relatively gentle. It can be considered that the two directions of the submarine perpendicular to the forward speed are only related to the sea water velocity and assuming that the velocity is 0, the constraint conditions [5] can be obtained as:

$$\begin{cases} v_x^b = 0 \\ v_z^b = 0 \end{cases} \tag{1}$$

Cause any motion of the submarine can be decomposed into two planes perpendicular to the y-axis, as shown in Fig. 1, according to the kinematic formula, we can get:

$$a_{rx} = v_y^b w_{nbz}^b \tag{2}$$

$$a_{rz} = v_y^b w_{nbx}^b \tag{3}$$

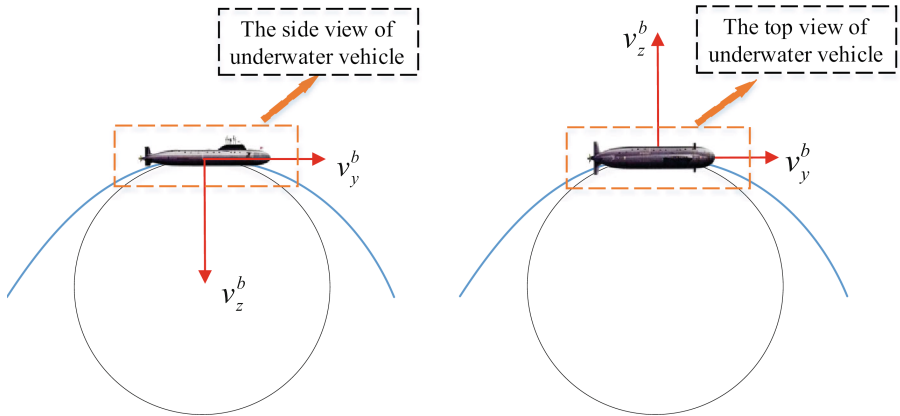


Fig. 1. Motion decomposition diagram

From the inertial device, the formula (4) can be obtained:

$$\begin{cases} a_{rx} = f_x^b + (C_n^b g^n)_1 \\ a_{rz} = -f_z^b + (C_n^b g^n)_3 \\ v_y^b = (C_n^b v^n)_2 \\ w_{nbz}^b = w_{ibz}^b - (C_n^b (w_{ie}^n + w_{en}^n))_3 \\ w_{nbx}^b = w_{ibx}^b - (C_n^b (w_{ie}^n + w_{en}^n))_1 \end{cases} \tag{4}$$

Thus the motion constraint after introducing centripetal acceleration is:

$$\begin{cases} v_x^b = 0 \\ v_z^b = 0 \\ a_{0x} = a_{rx} - v_y^b w_{nbz}^b \\ a_{0z} = a_{rz} - v_z^b w_{nbx}^b \end{cases} \tag{5}$$

The error model is further obtained as follows:

$$\begin{cases} \delta v_x^b = (C_n^b \delta v^n)_1 - (C_n^b \phi^n \times v^n)_1 \\ \delta v_y^b = (C_n^b \delta v^n)_3 - (C_n^b \phi^n \times v^n)_3 \\ \delta a_{0x} = (C_n^b \phi^n \times v^n)_2 \omega_{ibz}^b - (C_n^b \phi^n \times g^n)_1 - (C_n^b \delta v^n)_2 \omega_{ibz}^b + \nabla_x^b - (C_n^b v^n)_2 \varepsilon_z^b \\ \delta a_{0z} = (C_n^b \phi^n \times v^n)_2 \omega_{ibx}^b - (C_n^b \phi^n \times g^n)_3 - (C_n^b \delta v^n)_2 \omega_{ibx}^b - \nabla_z^b - (C_n^b v^n)_2 \varepsilon_x^b \end{cases} \tag{6}$$

where v_x^b, v_y^b, v_z^b are the speed of the submarine in the carrier coordinate system, a_{rx}, a_{rz} are the centripetal acceleration values in the x and z directions under the submarine carrier coordinate system. w_{ibx}^b, w_{ibz}^b are the sensitive angular velocities of the x-axis and z-axis of the inertial device, f_x^b, f_z^b represent the specific force on the x-axis and z-axis of the accelerometer. w_{ie}^n, w_{en}^n are respectively the angular velocity of the earth’s rotation and the angular velocity caused by the movement of the carrier.

2 Improved Sage-Husa Adaptive Algorithm

In traditional Kalman filter [6], the measuring prediction error can be expressed as formula (7):

$$\begin{aligned} \tilde{Z}_{k/k-1} &= Z_k - \hat{Z}_{k/k-1} \\ &= H_k \tilde{X}_{k/k-1} + V_k - H_k \hat{X}_{k/k-1} \\ &= H_k \tilde{X}_{k/k-1} + V_k \end{aligned} \tag{7}$$

Find the variance [7] on both sides at the same time to get:

$$E[\tilde{Z}_{k/k-1} \tilde{Z}_{k/k-1}^T] = H_k P_{k/k-1} H_k^T + R_k \tag{8}$$

Using exponential reduction memory weighted average method, we can get:

$$\hat{R}_k = (1 - \beta_k) \hat{R}_{k-1} + \beta_k (\tilde{Z}_{k/k-1} \tilde{Z}_{k/k-1}^T - H_k P_{k/k-1} H_k^T) \tag{9}$$

In traditional Sage-Husa adaptive filtering [8], β_k can be expressed as formula (10):

$$\beta_k = \frac{1 - b}{1 - b_k} \tag{10}$$

where b is the fading factor, but as the number of filtering k increases, it will approach 0, and the weight of adaptive filtering will approach $1 - b$ and will remain unchanged.

At the same time, the distribution weight of value \hat{R}_0 to value \hat{R}_k gradually decays and approaches the constant value 0. The reasons above reduce the adaptive degree of the noise estimator, and the accuracy of filtering will decrease accordingly.

According to the prediction residual method [9], it is possible to artificially judge whether the filtering is divergent. Filtering divergence criterion [10] can be expressed as formula (11):

$$\tilde{Z}_{k/k-1}^T \tilde{Z}_{k/k-1} > \gamma \cdot \text{tr}[E(\tilde{Z}_{k/k-1} \tilde{Z}_{k/k-1}^T)] \quad (11)$$

When the above formula is established, it represents the filtering divergence, where γ is the reserve coefficient, when $\gamma = 1$, the strictest convergence criterion will be established. Let \hat{R}_k replace R_k , we can get:

$$\begin{aligned} \tilde{Z}_{k/k-1}^T \tilde{Z}_{k/k-1} &= \text{tr}[E(\tilde{Z}_{k/k-1} \tilde{Z}_{k/k-1}^T)] \\ &= \text{tr}(H_k P_{k/k-1} H_k^T + (1 - \beta_k) \hat{R}_{k-1} + \beta_k (\tilde{Z}_{k/k-1} \tilde{Z}_{k/k-1}^T - H_k P_{K/K-1} H_k^T)) \end{aligned} \quad (12)$$

From formula (12), β_k can be expressed as follows:

$$\beta_k = \begin{cases} \frac{\tilde{Z}_{k/k-1}^T \tilde{Z}_{k/k-1} - \text{tr}(H_k P_{k/k-1} H_k^T) - \text{tr}(\hat{R}_{k-1})}{\text{tr}(\tilde{Z}_{k/k-1} \tilde{Z}_{k/k-1}^T) - H_k P_{k/k-1} H_k^T - \text{tr}(\hat{R}_{k-1})} & \tilde{Z}_{k/k-1}^T \tilde{Z}_{k/k-1} > \text{tr}[E(\tilde{Z}_{k/k-1} \tilde{Z}_{k/k-1}^T)] \\ 1 & \tilde{Z}_{k/k-1}^T \tilde{Z}_{k/k-1} \leq \text{tr}[E(\tilde{Z}_{k/k-1} \tilde{Z}_{k/k-1}^T)] \end{cases} \quad (13)$$

3 Simulation Experiment and Comparative Analysis

3.1 Parameter Settings

In order to verify the accuracy of the algorithm, this paper sets up a simulation experiment based on the MATLAB platform, in which the inertial original indicators and the initial navigation parameters are set as follows:

$$\sigma_{\delta_r}^2(0) = (1 \text{ m})^2$$

$$\sigma_{\delta_v}^2(0) = (0.1 \text{ m/s})^2$$

$$\sigma_{\delta_\alpha}^2(0) = \sigma_{\delta_\beta}^2(0) = (10'')^2 \quad \sigma_{\delta_\gamma}^2(0) = (1')^2$$

Gyro bias stability: $eb = 0.2^\circ/\text{h}$

Accelerometer bias stability: $db = 100 \mu\text{g}$

Angle random walk: $web = 0.018^\circ/\sqrt{h}$

DVL velocity error: $\sigma_{\delta_{vd}}^2 = (0.01 \text{ m/s})^2$

DVL deviation angle error: $\sigma_{\delta_\Delta}^2 = (1')^2$.

DVL scale factor error: $\sigma_{\delta_C}^2 = (0.001)^2$.

3.2 Comparison of Simulation Results

Generally, the submarine often glides in sawtooth waves at the bottom of the water. In order to simulate the navigation state of the submarine, this paper first conducts the trajectory simulation of the submarine, assuming that the initial coordinates of the submarine are [34.246048; 108.909664; 380] and the simulated time is 1029 s. The whole simulation includes different states of motion, such as constant speed, uniform acceleration, uniform deceleration, heading angle change, pitch angle change, and roll angle change.

Combining the above parameters, using the motion constraint aided underwater integrated navigation method based on improved Sage-Husa adaptive filtering proposed in this paper, the velocity error and position error of the submarine are shown in Figs. 2 and 3:

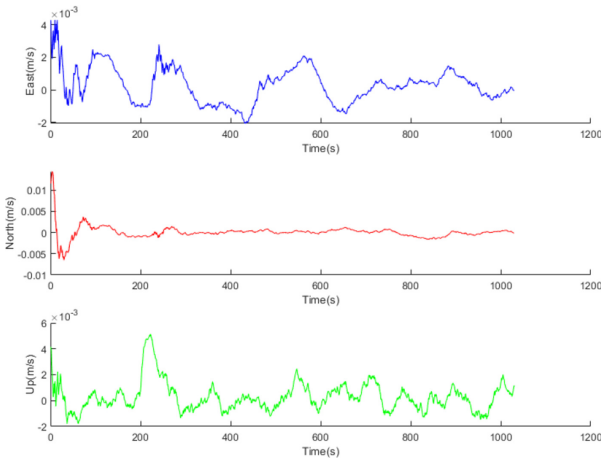


Fig. 2. Velocity errors of the submarine

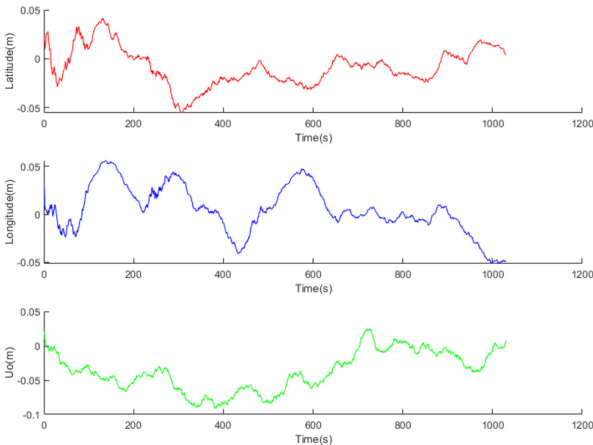


Fig. 3. Position errors of the submarine

Calculate the root mean square errors of the velocity and position errors of the submarine based on the improved algorithm and the traditional Kalman filter method, which are shown in Table 1:

Table 1. Root mean square error

Errors		RMSE	
		Traditional Kalman filter	Motion constrain aided improved Sage-Husa adaptive filtering
Velocity error	East (m/s)	0.0026	1.3907e-04
	North (m/s)	0.0012	3.0682e-04
	Up (m/s)	0.0187	6.9625e-05
Position error	Latitude (°)	1.9037e-07	7.1751e-08
	Longitude (°)	2.0420e-07	1.0497e-08
	Up (m/s)	1.0243	0.0637

It can be seen that the standard Kalman algorithm lacks an understanding of the statistical noise characteristics of the system in the initial stage of the experimental calculation, and the filtering error is relatively large. The improved Sage-Husa adaptive filtering algorithm with additional motion constraints constrains the three-directional speed of the carrier and adapts the measurement noise, which can well suppress the divergence of the combined system error and greatly improve the navigation accuracy of the system.

In order to verify the improved performance of adaptive filtering in the case of filtering divergence, the measurement noise was added ten times from 600 s to 610 s. The final velocity errors and position errors of the submarine are shown in Figs. 4 and 5:

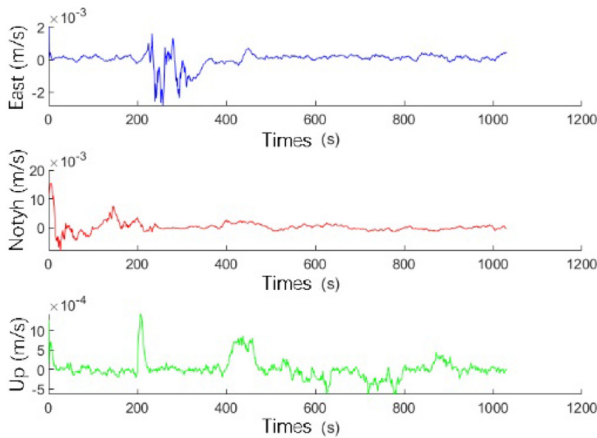


Fig. 4. Velocity errors of the submarine

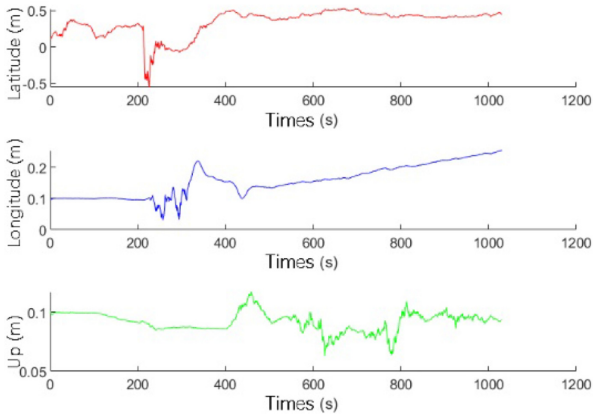


Fig. 5. Position errors of the submarine

Calculate the root mean square errors of the velocity and position of the submarine under the two methods of the traditional Kalman filter and motion constraint aided algorithm based on improved Sage-Husa adaptive filtering, and the results are shown in Table 2:

Table 2. Root mean square errors after adding noise

Errors		RMSE	
		Traditional Kalman filter	Motion constrain aided improved Sage-Husa adaptive filtering
Velocity error	East (m/s)	0.0155	1.2874e-04
	North (m/s)	0.0124	7.4445e-04
	Up (m/s)	0.0103	3.4963e-05
Position error	Latitude (°)	2.5621e-07	4.5233e-08
	Longitude (°)	9.4087e-07	2.4090e-08
	Up (m/s)	3.4723	0.0904

It can be seen from the table that due to the sudden increase of measurement noise and the divergence of the traditional Kalman filter, the trajectory of the submarine cannot be accurately estimated, resulting in the continuous increase of the velocity and position errors of the submarine. But the motion constraint-aided algorithm based on improved Sage-Husa adaptive filtering proposed in this paper can suppress the filtering divergence very well, which has better fault tolerance, and higher accuracy than the traditional Kalman filtering algorithm.

4 Conclusion

The motion constraint-aided adaptive filtering algorithm based on the improved Sage-Husa proposed in this paper is more complete than traditional motion constraints. It can effectively constrain the forward speed of the carrier and avoid the large measurement information caused by strong maneuvering of the carrier, which effectively improves the navigation accuracy of the system. On the basis of the traditional Sage-Husa adaptive filtering, the fault judgment of the measurement information and the optimal estimation of the scale factor are added, which can not only reduce the amount of filtering calculation when the measurement information is correct, but can also reduce the divergence of system filtering when the measurement information has errors, which has better robustness and reliability, and can improve the fault tolerance and navigation accuracy of the entire system.

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