



Trajectory Optimization for UAV-Aided Data Collections

Hanxin Zhang, Caiyu Zhang, Zixuan Zhang, Changwen Zhou, Yu Zhang,
and Congduan Li^(✉)

Sun Yat-sen University, Shenzhen, Guangdong, China
licongd@mail.sysu.edu.cn

Abstract. The application of the unmanned aerial vehicles (UAV) in future wireless networks is getting more and more popular. This article investigates the flight trajectory optimization problem with minimum energy consumption when the UAVs are communicating with the ground terminals (GT) for data collections. The specific flying speed is determined to minimize the energy consumption of the whole flying process. In addition, the algorithms to find the optimal trajectory are proposed. Experimental results are presented to show the effectiveness of our proposed algorithms.

Keywords: Wireless communication · Energy consumption · Trajectory optimization

1 Introduction

The applications of UAVs have been developed for more than 30 years in the domestic market, gradually expanding from the initial military area to civilian area nowadays [1]. With the wide usage of UAVs, the general public's recognition and demands for UAVs have been significantly increased. It is predicted that the market value of the UAV will reach 70 billion by 2025 [2].

The UAVs began to be used in the military in the 1960s. During the Vietnam War, the United States applied the UAVs to military reconnaissance, air strikes and target destruction. By the end of the 20th century, many countries had developed new UAVs for battlefield [3], intelligence reconnaissance [4], weather forecast, electronic countermeasures and jamming. What's more, in the field of agriculture [5, 6], the UAVs technology in Japan has developed more mature after 30 years of improvement. It can be used to supply medicine and add gasoline with high efficiency [7]. Moreover, the UAVs also play an important role in transportation [8].

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At present, lots of cities in China generally have the road congestion and traffic management problems. With the help of UAVs, the government can conduct area surveillance [9] to ensure the traffic move smoothly. Meanwhile, when emergency happens, it is convenient for relevant departments to carry out urgent rescues [10]. Another common application of UAVs is in data collections, where data from sensors will be collected by a UAV flying by them one by one.

The UAVs have been widely applied in military, agriculture, transportation, photograph, data collections, and so on. However, the energy onboard usually restricts the covering distance and missions of the UAVs. It has been a challenging problem to choose the best trajectory for UAVs to save their energy. Not only the performance of the UAVs should be considered, but also the current surroundings have become a threat to the UAVs. Calculating the optimal trajectory will be beneficial to estimate the prime cost of the flight mission. What's more, it can greatly reduce the flight time, helping the UAVs fulfill those missions more quickly and accurately, and thus speeding up the mission process. At the same time, by designing the most suitable flight speed of the UAVs, the energy consumed by the UAVs can be maximumly saved and the endurance can be greatly improved.

So far, there are not many articles pointing out how to calculate a suitable velocity to get a smaller energy consumption in the flying process. This is one of our research interests. The commonly used model of UAVs' trajectory is that UAVs swerve directly above the GT's center. In this model, the duration of communicating with GT is shortest. In other words, the energy consumption is smaller in communication, but maybe larger in flight, compared to a trajectory that not right above the GT center. In order to reduce energy consumption in flight, one can design that the UAVs change direction while passing through the communication range of GT. This consists of two models: straight or curve trajectory beside GTs. Based on these models, we'd like to go a step further to propose several algorithms to determine the trajectory inside GTs' communication range with different methods to calculate the entry and exit points on the circle of GTs' communication range. By comparing these algorithms, we finally get an optimal algorithm to find the optimal trajectory of UAVs.

This paper mainly addresses the following two issues:

1. The Energy Consumption of the UAVs

It has been a challenge to apply UAVs to communicate with terminals under limited energy has always been a challenge. In order to reduce the consumption of energy for the UAVs, a relatively optimal flight path should be established, and a new energy consumption formula be improved as well. Therefore, in order to minimize energy consumption, variables of easily changeable values should be selected.

2. The Flight Trajectory Optimization Problem

The UAVs usually fly directly above the center of the ground terminals, but they do not need to fly to the exact terminal center to start the communication. They

can start communicating once they enter the communication range of a ground terminal. In this way, flight trajectory[11] of the UAVs is optimized to reduce energy consumption.

2 System Model and Energy Minimization

2.1 System Model

It is widely known that the information-interacted distance between GT and UAV is corresponded to the current coordinates of the UAV. For simplicity, we assume that the UAV flies horizontally at a fixed altitude H . On this condition, we can realize a reduction of dimensions in our problem, so that our model is built in a two-dimensional plane rather than a three-dimensional space. In reality, the minimum altitude depends on the environment where the UAV flies in. If altitude H is too low, there may be obstacles such as buildings in the trajectory which will cause collision. In addition, the communication radius of a GT is assumed to be $R = 10$ m. Once the UAV enters the circle with center node and radius R , UAV can communicate with the node without distortion, which strikes a balance between maintaining the superior channel condition and minimizing the propulsion energy consumption. The size of the active range of UAV is set $100\text{ m} * 100\text{ m}$ grid. The fixed-wing UAV, as shown in Fig. 1, is considered in this paper. Further, we assume that the UAV's acceleration is taken into account only at the stage where the UAV takes off and lands. The UAV mainly flies with constant velocity for communication between GTs, thus ignoring the acceleration and deceleration stage for ease of analysis and calculation.

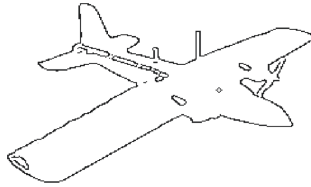


Fig. 1. Structure of a fixed-wing UAV.

The point UAV takes off is marked as point O . For each GT, we choose two points on the circle of radius R centered at GT. Thus, we get two series of points: $A_1, A_2, \dots, A_i, \dots, A_n$ and $B_1, B_2, \dots, B_i, \dots, B_n$. A_i is the point that UAV enters the distortion free range of the i 'th node, while B_i is the point that UAV move out of the range. Considering the specific characteristics about UAVs while changing flying directions, some more assumptions are made as follows.

- The UAV makes a uniform linear motion when flying out of the communication range, where the flight trajectory of uniform linear motion of the i^{th} GT

is the straight line $B_i A_{i+1}$, as $i = 1, 2, \dots, n - 1$. Meanwhile, OA_1 and $B_n O$ are separately the straight line that UAV flies towards the first GT and flies away the n^{th} GT.

- The UAV makes a uniform circular motion when flying within the communication range, where the flight trajectory of uniform circular motion tangent to that of uniform linear motion.
- The UAV makes both two motions with the same speed V .

The demonstration is presented in Fig. 2.

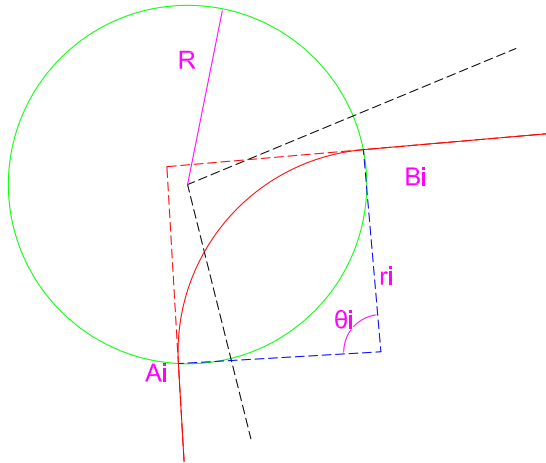


Fig. 2. Demonstration of the flight trajectory at a single GT node.

2.2 UAV Energy Consumption Model and Energy Optimization

In this paper, we choose to ignore the communication energy consumption. Two main reasons are considered. On the one hand, for fixed-wing UAVs, the energy consumption in communication, [12] is quite small compared to the energy expenditure during flight [13], and thus, can be neglected. On the other hand, if we consider the communication energy consumption, then all the algorithms will add a communication energy consumption with the same value. It is equivalent to adding the same value on both sides of the inequality for us to compare which case has the least energy consumption. From these two perspectives, we do not need to consider the communication energy consumption. Furthermore, the total propulsion energy required is a function of the trajectory, which corresponds to the classic aircraft power consumption model known in aerodynamics theory.

The expression [14] is expressed as:

$$E(v(t), a(t)) = \int_0^T \left[\frac{c_2}{\|v(t)\|} \left(1 + \frac{\|a(t)\|^2 - a^T(t)v(t)^2}{g^2} \right) + c_1 \|v(t)^3\| \right] dt + \frac{1}{2} m (\|v(T)\|^2 - \|v(0)\|^2), \quad (1)$$

where c_1 and c_2 are two parameters related to the aircrafts weight, wing area, air density, etc. Supposing the UAVs fly at the same speed, the expression(1) can be simplified as:

$$E(V, a(t)) = E_{SLF}(V) + \frac{c_2}{Vg^2} \int_0^T a(t)^2 dt \quad (2)$$

The expression has two components. The first part is the energy consumption of uniform linear motion. In [3], it is expressed as:

$$E_{SLF}(V) = T(c_1 V^3 + \frac{c_2}{V}) \quad (3)$$

The variable T is the time UAV takes to complete uniform linear motion:

$$T = \frac{\sum_{i=1}^{n-1} \sqrt{(y_{A_{i+1}} - y_{B_i})^2 + (x_{A_{i+1}} - x_{B_i})^2}}{V} + \frac{\sqrt{(y_{A_1} - y_O)^2 + (x_{A_1} - x_O)^2}}{V} + \frac{\sqrt{(y_{B_n} - y_O)^2 + (x_{B_n} - x_O)^2}}{V} \quad (4)$$

So far, we get:

$$E_{SLF}(V) = (c_1 V^2 + \frac{c_2}{V^2}) \left[\sum_{i=1}^{n-1} \sqrt{(y_{A_{i+1}} - y_{B_i})^2 + (x_{A_{i+1}} - x_{B_i})^2} + \sqrt{(y_{A_1} - y_O)^2 + (x_{A_1} - x_O)^2} + \sqrt{(y_{B_n} - y_O)^2 + (x_{B_n} - x_O)^2} \right] \quad (5)$$

The radius of each communication ranges is:

$$r_i = \frac{VT_0}{\theta_i} \quad (6)$$

With the hypothesis of uniform circular motion, the total function can be expressed as:

$$E(V, a(t)) = E_{SLF}(V) + \frac{c_2 V}{g^2 T_0} \sum_{i=1}^n \theta_i^2 \quad (7)$$

As the arc trajectory inside the distortion free range of the i 'th GT is tangent to both $B_{i-1}A_i$ and B_iA_{i+1} , θ_i is determined by $B_{i-1}A_i$ and B_iA_{i+1} :

$$\theta_i = \pi - \arccos \frac{(y_{A_i} - y_{B_{i-1}})(y_{B_i} - y_{A_{i+1}}) + (x_{A_i} - x_{B_{i-1}})(x_{B_i} - x_{A_{i+1}})}{\sqrt{(y_{A_i} - y_{B_{i-1}})^2 + (x_{A_i} - x_{B_{i-1}})^2} \sqrt{(y_{B_i} - y_{A_{i+1}})^2 + (x_{B_i} - x_{A_{i+1}})^2}} \quad (8)$$

To solve the above equation, we can get some basic formulas:

$$\left\{ \begin{array}{l} (x_{A_1} - x_1)^2 + (y_{A_1} - y_1)^2 = 100 \\ (x_{A_2} - x_2)^2 + (y_{A_2} - y_2)^2 = 100 \\ \vdots \\ (x_{A_n} - x_n)^2 + (y_{A_n} - y_n)^2 = 100 \end{array} \right. \quad (9)$$

$$\left\{ \begin{array}{l} (x_{B_1} - x_1)^2 + (y_{B_1} - y_1)^2 = 100 \\ (x_{B_2} - x_2)^2 + (y_{B_2} - y_2)^2 = 100 \\ \vdots \\ (x_{B_n} - x_n)^2 + (y_{B_n} - y_n)^2 = 100 \end{array} \right. \quad (10)$$

This problem is difficult to be directly solved. Firstly, it requires to put every pair of points into the formula which is related to the initial settings. What's more, the equation is too complex to find closed-form expressions. As a consequence of the above difficulties, we decide to choose only one point on the communication range to simplify our trajectory.

3 Trajectory Design and Optimization

According to the minimum energy consumption formula above, the energy consumption of UAV is related to the path length. The shorter the path length is, the lower the energy consumption of UAV is. Therefore, in order to get the minimum energy consumption, we need to find the shortest flight path of UAV.

In *Section II*, it is hypothesized that an initial communication radius R is $10m$. The target is to determine where the UAV should change its flying direction towards the next GT after entering the current circle with center node and radius R . There exist infinite combinations of flight paths because there are infinite number of points in each circle, leading to that we cannot find the shortest path by exhausting all the possibilities. In order to figure out the shortest path, we optimize the problem into two steps:

- First of all, take each communication range as a point to find out the sequence of UAV passing through GTs. When each circle is regarded as a point, the *GRASP* (*Greedy Randomized Adaptive Search Procedures*) [17, 18] can be used to determine the flight order. *GRASP* is an iterative process with multiple starting points. Each iteration consists of two stages: one is the construction stage of generating feasible solutions. On the basis of greed, some random factors are added to construct the initial feasible solutions. Secondly, in the local search phase of finding the local optimal solution, neighborhood search is carried out for the initial feasible solution constructed above until a local

optimal solution is found. If the local optimal solution is better than the current optimal solution, the original optimal solution is updated. Before iteration, we construct a solution randomly, and then judge whether it is feasible. If it is not feasible, we will enter the repair function to modify it. The feasible solution is locally searched, and the solution is updated in the local search process. It is worthy of note that the limitation of *GRASP* is that it only applies to precise points.

- Next, based on the sequence, we introduces 5 algorithms whose performances of path length vary from the worst to the best. In Algorithm 5, the arcs and radius corresponding to the shortest flight path on each circle are repeatedly quartered to finally obtain the turning points which determines the shortest flight path within the error range.

The above work realizes the transformation from non-realizable problem to realizable problem in mathematical analysis. We state our algorithms as follows.

Algorithm 1 (Shortest Path Based on Communication Center). The UAV flight trajectory is obtained by visiting the circle centers of the GTs based on *GRASP* [15, 16]. Therefore, the shortest flying distance can be calculated by substituting the center of each circle into the formula of the distance between two points.

Algorithm 2 (Shortest Path Based on Minimum Path). It is presumed that the UAV will visit the shortest path points (A pair of shortest path points are the intersection of two arcs and a segment which connects the centers of two circles, abbreviated to SPP) in sequence based on *GRASP* (Algorithm 1) and connect the shortest path points in the same circle with a straight line. As is shown in Fig. 3, where red points are the SPPs and the green lines form the UAV flight trajectory.

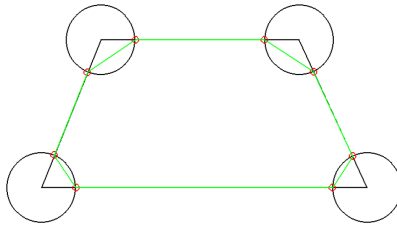


Fig. 3. Rough UAV flight trajectory.

Algorithm 3 (Greedy Algorithm). Regardless of global optimization, we obtain the optimal solution by using greedy algorithm in each step of the solution. For any node except the starting point, the SPP is selected as the hovering point for the UAV. Therefore, the UAV flight trajectory is the current shortest trajectory. Connect the center of the first circle and the second circle, obtain the intersection

point of the line and the second circle. Then connect the intersection point and the center of the third circle, obtaining the intersection point of the line and the third circle, and so on. Ultimately, with this method, we go through all hovering points of the UAV. The sum of piecewise paths is the total trajectory length.

Algorithm 4 (Sub-Optimal Algorithm). As is widely known, the limitation of *GRASP* is that there must be exact points. Instead of exact points, a series of circles are identified, so the shortest flying distance calculated by *GRASP* brings some errors. To improve it in Algorithm 4, the circle is divided equally along the arc direction and the radius direction by points (A1, A2, A3,) (B1, B2,) to find the shortest path (Fig. 4).

Specific steps are as follows:

Step 1: Read in the SPPs according to Algorithm 2 to determine the arcs. Quarter each arc to attain more SPPs.

Step 2: Traverse all possible paths formed by SPPs.

Step 3: Find the shortest 2 paths. SPPs of each circle on these 2 paths determine new arcs.

Step 4: After repeating n times, the simulation points of high precision are regarded as the final SPPs along the arc.

Step 5: Connect each point in step 4 with the center of the corresponding circle. Quarter each segment to attain more SPPs along the radius. Use the same traversing method of the above steps to find the shortest path. Repeat n times.

Step 6: Connect the points in step 5 to attain the final trajectory.

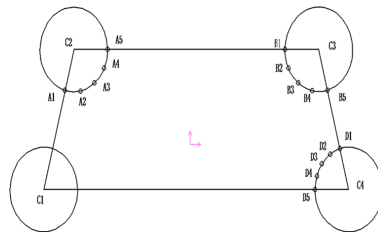


Fig. 4. Segmentation method of Algorithm 4.

Algorithm5 (Newton Iteration). It is out of the question to apply Algorithm 4 to the situations where there are 10 GTs or above. Revisiting the optimization problem, we can first work out the expression of total path length related to the SPP of each circle. Then let the partial derivative of the expression be 0 to figure out the SPPs.

We adopt Newton iteration as a replacement for calculating the analytical solution since the derivative is too sophisticated to compute.

The specific steps are as follow.

Step 1: Express each coordinate with variable θ (relative to the corresponding center). The coordinates of moving points can be determined by θ uniquely since the radius is set.

Step 2: Express the function of the total path length related to coordinates gained in Step 1 with the formula of the distance between two points, say $f(\theta)$.

Step 3: Choose the initial value for each circle. The difference between the initial value and SPP is constrained in $\frac{\pi}{2}$ to ensure that the result of iteration is the shortest path but not the longest.

Step 4: Update θ according to Newton iteration:

$$\theta_{k+1} = \theta_k - \frac{f(\theta_k)}{f'(\theta_k)} \quad (11)$$

Step 5: If the absolute error is within range, break out from the iteration, then gain the shortest path.

Algorithm 5. Newton iteration

Require

a:x-coordinate of center of circles

b:y-coordinate of center of circles

n:number of GTs

r:communication radius of GTs

theta:the initial value of the point on the circle

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1:  $x \leftarrow a + r * \cos(theta)$ 
2:  $y \leftarrow b + r * \sin(theta)$ 
3: //the length of the path calculate by x and y
4:  $f \leftarrow dis(x, y)$ 
5:  $f_\theta \leftarrow diff(f, 1, theta)$ 
6:  $f_{\theta_2} \leftarrow diff(f, 2, theta)$ 
7: for  $i = 1$  to  $iteration\_times$  do
8:   if  $max(f_\theta/f_{\theta_2} < 0.01)$  then break
9:   else
10:      $theta \leftarrow theta - f_\theta/f_{\theta_2}$ 
11:     Update  $f_\theta$  and  $f_{\theta_2}$  by new  $theta$ 
12:   end if
13: end for
14:  $X \leftarrow a + r * \cos(theta)$ 
15:  $Y \leftarrow b + r * \sin(theta)$ 
16: return  $X, Y$ 

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4 Numerical Results

Simulations of Algorithms 1–3 and Algorithm 5 are performed on 10, 20 and 30 communication nodes.

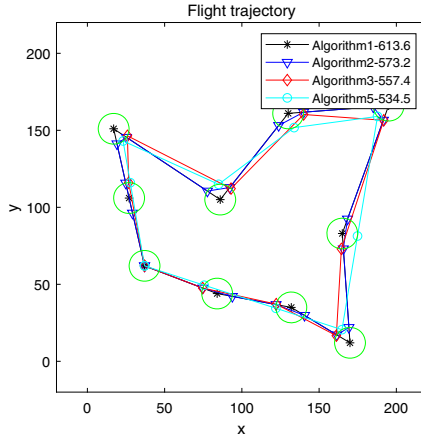


Fig. 5. Flight trajectories of 10 GTs

For 10 GTs (Fig. 5), the path lengths of Algorithm 1–3 and Algorithm 5 are, in order, 613.6, 573.2, 557.4, 534.5. Algorithm 5 can save 12.89%, 6.75%, 4.11% compared with Algorithm 1–3.

For 20 GTs (Fig. 6), the path lengths of Algorithm 1–3 and Algorithm 5 are, in order, 851.3, 769.1, 736.0, 693.9. Algorithm 5 can save 18.49%, 9.78%, 5.72% compared with Algorithm 1–3.

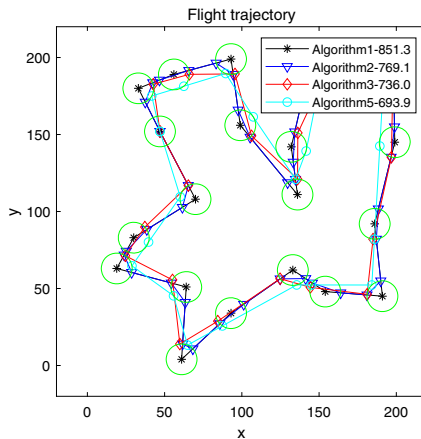


Fig. 6. Flight trajectories of 20 GTs

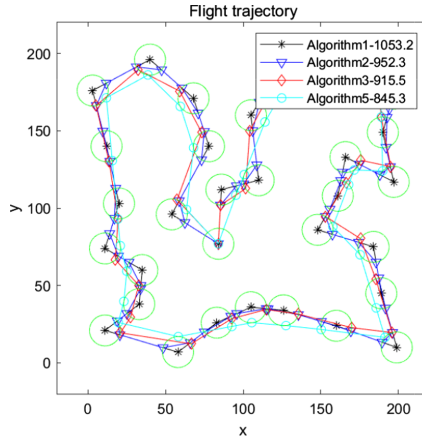


Fig. 7. Flight trajectories of 30 GTs

For 30 GTs (Fig. 7), the path lengths of Algorithm 1–3 and Algorithm 5 are, in order, 1053.2, 952.3, 915.5, 845.3. Algorithm 5 can save 19.74%, 11.24%, 7.67% compared with Algorithm 1–3.

It is evident that the property of Algorithm 5 improves as the number of GTs increases.

As is shown from the figures, Algorithm 5 performs the best, Algorithm 3 is the second, and Algorithm 1 and Algorithm 2 have the worst performance.

5 Conclusion

This paper considers the problem of the UAV energy consumption in data collections. The minimum energy consumption formula is derived with some assumptions. Also, we propose the shortest path algorithm for UAVs trajectory. Effective solutions about reducing the energy consumption of the flight are also proposed. Moreover, compared to the traditional shortest UAV flight path, analogue simulation shows that the solution can reduce about 10% drone flight paths.

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