Dynamical Analysis of a Predator-Prey Economic Model with Impulsive Control Strategy

Airen Zhou(✉) and Jianjun Jiao

School of Mathematics and Statistics, Guizhou University of Finance and Economics, Guiyang 550025, People's Republic of China

Abstract. In the present work, a predator-prey economic model with impulsive stocking immature predator and impulsive harvesting mature predator is studied. First, the method of stroboscopic mapping of the discrete dynamics is used in our demonstrate. After that, it is proved that the relevant solution of the studied system is globally asymptotically stable. Next, based on the principle of comparison of pulsed differential equations, it is also proved that the studied system possesses its persistence. In the end, the numerical experiment is introduced to demonstrate our results. The obtained conclusions provide a methodological guidance for the actual biological economic managements.

Keywords: Control strategy · Extinction · Permanence

1 Introduction and Background

The ultimate goal of scientific management and rational development and application of renewable resources (fisheries, forestry resources, etc.) is to make them an inexhaustible resource. However, if unreasonable utilization of resources damages resources and disrupts their renewal cycle, it will cause resource depletion, which will not only cause economic losses, but also affect the living environment of human beings in severe cases. Therefore, in order to protect the natural environment on which humans depend, the development of renewable resources must be reasonable and appropriate. Under the premise of sustainable resources, it is a feasible way to analyze, evaluate and predict the development and utilization of renewable resources by means of mathematical models [1].

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In recent years, Predator-prey model and Stage-structured model have been received extensively and deeply studied [2,10–15].

2 The Research Model

Once the immature predator population grows into mature predator population, it should be harvested [7]. In order to examine the rational development and sustainable development of resources, we build an economic model as follows:

$$\begin{cases} \left.\begin{array}{l} \dfrac{dp(t)}{dt} = rp(t)(1-\dfrac{p(t)}{K}) - \dfrac{\beta_1 p(t)}{1+c_1p(t)}p_1(t) - \dfrac{\beta_2 p(t)}{1+c_2p(t)}p_2(t), \\ \dfrac{dp_1(t)}{dt} = \dfrac{l_1\beta_1 p(t)}{1+c_1p(t)}p_1(t) - (d_1+\kappa)p_1(t), \\ \dfrac{dp_2(t)}{dt} = \dfrac{l_2\beta_2 p(t)}{1+c_2p(t)}p_2(t) + \kappa p_1(t) - d_2p_2(t), \end{array}\right\} t \neq nU, \\ \left.\begin{array}{l} \Delta p(t) = 0, \\ \Delta p_1(t) = \dfrac{\delta_1 p(t)}{1+\lambda_1 p(t)} + \mu_1, \\ \Delta p_2(t) = -\mu_2 p_2(t), \end{array}\right\} t = nU, n \in Z^+, \end{cases} \tag{1}$$

where $p(t), p_1(t), p_2(t)$ are the population densities of prey, immature predator and mature predator, respectively. $\frac{\delta_1 p(t)}{1+\lambda_1 p(t)}$ is a nonlinear input term, $\delta_1 \geq 0$ is a proportional coefficient (capture rate), $\lambda_1 \geq 0$ is the half saturation constant for the prey due to stocking immature predator. $\Delta p_i(t) = p_i(t^+) - p_i(t)$. $\mu_1 (\mu_1 \geq 0)$ denotes the stocking quantity of immature predator at $t = nU, n \in Z^+$. $\mu_2(1 > \mu_2 \geq 0)$ represents the gathering portion of mature predator population owing to behaviors of biological economic management at $t = nU, n \in Z^+$. The meaning of the other variables can be referred to [6,8,9].

3 Some Lemmas

Obviously, the positivity of solutions of (1) results from the positive initial value conditions. The following Lemma 1° is not difficult to be proved.

Lemma 1°. It is bounded for the solution of system (1). i.e., there exists a positive constant $L > 0$ satisfying $p(t) \leq L/l, p_1(t) \leq L, p_2(t) \leq L$, for each solution $(p(t), p_1(t), p_2(t))$ of (1) for t big enough.

Proof. Denote that $V(t) = lp(t) + p_1(t) + p_2(t)$, $l = \max\{l_1, l_2\}$, $H = \min\{d_1, d_2\}$ and $b = \frac{r}{K}$. When t is not equal to nU,

$$\begin{aligned} D^+V(t) + HV(t) &\leq l[(r+H)p(t) - bp^2(t)] - (d_1 - H)p_1(t) - (d_2 - H)p_2(t) \\ &= -bl\big(p(t) - \frac{r+H}{2b}\big)^2 + \frac{l(r+H)^2}{4b} \\ &\leq \frac{l(r+H)^2}{4b} = L_0. \end{aligned} \tag{2}$$

When t is equal to nU, we have $V(nU^+) < V(nU) + \frac{\delta_1}{\lambda_1} + \mu_1$. Thanks the Lemma in [4], as $t \in (nU, (n+1)U]$, holds

$$\begin{aligned} V(t) &\le V(0)\exp(-Ht) + \frac{L_0}{H}(1-\exp(-Ht)) \\ &\quad + \frac{(\frac{\delta_1}{\lambda_1}+\mu_1)\exp(-H(t-U))}{1-\exp(HU)} + \frac{(\frac{\delta_1}{\lambda_1}+\mu_1)\exp(HU)}{\exp(HU)-1} \\ &\quad \longrightarrow \frac{L_0}{H} + \frac{(\frac{\delta_1}{\lambda_1}+\mu_1)\exp(HU)}{\exp(HU)-1}, \quad t\to\infty, \end{aligned} \tag{3}$$

which implies the consistent boundedness of $V(t)$. Therefore, Lemma 1° holds in terms of the above definition.

Taking the subsystem of (1) into account:

$$\begin{cases} \left.\begin{aligned} \frac{dp_1(t)}{dt} &= -(\kappa+d_1)p_1(t), \\ \frac{dp_2(t)}{dt} &= \kappa p_1(t) - d_2 p_2(t), \end{aligned}\right\} t \ne nU, n \in Z^+, \\ \left.\begin{aligned} \Delta p_1(t) &= \frac{\delta_1 p(t)}{1+\lambda_1 p(t)} + \mu_1, \\ \Delta p_2(t) &= -\mu_2 p_2(t), \end{aligned}\right\} t = nU, n \in Z^+. \end{cases} \tag{4}$$

Lemma 2° can also be obtained:

Lemma 2°. For system (4), there is only one positive U-periodic solution $(\widetilde{p_1(t)}, \widetilde{p_2(t)})$ with globally asymptotical stability, here

$$\begin{cases} \widetilde{p_1(t)} = \dfrac{\mu_1 e^{-(\kappa+d_1)(t-nU)}}{1-e^{-(\kappa+d_1)U}}, nU < t \le (n+1)U, n \in Z^+, \\ \widetilde{p_2(t)} = \dfrac{\mu_1\kappa}{(\kappa+d_1-d_2)(1-e^{-(\kappa+d_1)U})}\left[\dfrac{1-(1-\mu_2)e^{-(\kappa+d_1)U}}{1-(1-\mu_2)e^{-d_2U}}e^{-d_2(t-nU)} \right. \\ \qquad \left. -e^{-(\kappa+d_1)(t-nU)}\right], nU < t \le (n+1)U, n \in Z^+. \end{cases} \tag{5}$$

4 Dynamical Analysis of (1)

Next, we are going to show an important conclusion in this paper, that is, the sufficient condition for the globally asymptotic stability of solution $(0, \widetilde{p_1(t)}, \widetilde{p_2(t)})$ of Eq. (1).

Theorem 1°. The solution $(0, \widetilde{p_1(t)}, \widetilde{p_2(t)})$ of (1) is globally asymptotically stable under the condition

$$\mu_1 > rU$$

$$[\frac{\beta_1}{\kappa+d_1} + \frac{\kappa\beta_2}{(\kappa+d_1-d_2)(1-e^{-(\kappa+d_1)U})}(\frac{1-(1-\mu_2)e^{-(\kappa+d_1)U}(1-e^{-d_2U})}{d_2(1-(1-\mu_2)e^{-d_2U})} + \frac{e^{-(\kappa+d_1)U}-1}{\kappa+d_1})]^{-1}$$

is true.

Proof. According to local stability and global attraction, we can derive global asymptotic stability [2,3]. The first step aims at the local stability. Let $p(t), p_1(t) - \widetilde{p_1(t)}, p_2(t) - \widetilde{p_2(t)}$ relabel as $p(t), q_1(t), q_2(t)$, respectively. We do the following linearization:

$$\begin{pmatrix} p'(t) \\ q_1'(t) \\ q_2'(t) \end{pmatrix} = \begin{pmatrix} r - \beta_1\widetilde{p_1(t)} - \beta_2\widetilde{p_2(t)} & 0 & 0 \\ l_1\beta_1\widetilde{p_1(t)} & -(\kappa+d_1) & 0 \\ l_2\beta_2\widetilde{p_2(t)} & \kappa & -d_2 \end{pmatrix} \begin{pmatrix} p(t) \\ q_1(t) \\ q_2(t) \end{pmatrix}.$$

Assume $\Phi(t)$ is the fundamental matrix above system, so $\Phi(t)$ should meet

$$\Phi(U) = \begin{pmatrix} \exp(\int_0^U (r - \beta_1\widetilde{p_1(s)} - \beta_2\widetilde{p_2(s)})ds) & 0 & 0 \\ \dagger & \exp(-(\kappa+d_1)U) & 0 \\ \star & * & \exp(-d_2U) \end{pmatrix},$$

it is not necessary to calculate the terms $\dagger, \star$ and $*$ with the exact forms for the rest of analysis. The linearization of the 4^{th}, 5^{th} and 6^{th} equations of (1) turns to be

$$\begin{pmatrix} p(nU^+) \\ q_1(nU^+) \\ q_2(nU^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1-\mu_2 \end{pmatrix} \begin{pmatrix} p(nU) \\ q_1(nU) \\ q_2(nU) \end{pmatrix} = A(U) \begin{pmatrix} p(nU) \\ q_1(nU) \\ q_2(nU) \end{pmatrix}.$$

Thus, the absolute values of three eigenvalues of the following matrix will control the local stability of the solution $(0, \widetilde{p_1(t)}, \widetilde{p_2(t)})$ of equations (1)

$$M = A(U)\Phi(U)$$

$$= \begin{pmatrix} \exp[\int_0^U (r - \beta_1\widetilde{p_1(s)} - \beta_2\widetilde{p_2(s)})ds] & 0 & 0 \\ 0 & \exp(-(\kappa+d_1)U) & 0 \\ 0 & 0 & (1-\mu_2)\exp(-d_2U) \end{pmatrix}.$$

The eigenvalues of matrix M satisfy

$$|\Lambda_1| = \exp[\int_0^U (r - \beta_1\widetilde{p_1(s)} - \beta_2\widetilde{p_2(s)})ds],$$

$$|\Lambda_2| = \exp(-(\kappa+d_1)U) < 1, |\Lambda_3| = (1-\mu_2)\exp(-d_2U) < 1.$$

This manifests that the solution $(0, \widetilde{p_1(t)}, \widetilde{p_2(t)})$ is locally stable if and only if

$$rU - \beta_1 \int_0^U \widetilde{p_1(s)} ds - \beta_2 \int_0^U \widetilde{p_2(s)} ds < 0.$$

Therefore, the local asymptotical stability of solution $(0, \widetilde{p_1(t)}, \widetilde{p_2(t)})$ of (1) is dominated by

$$\mu_1 > rU$$

$$[\frac{\beta_1}{\kappa + d_1} + \frac{\kappa\beta_2}{(\kappa + d_1 - d_2)(1 - e^{-(\kappa+d_1)U})}(\frac{1 - (1-\mu_2)e^{-(\kappa+d_1)U}(1 - e^{-d_2U})}{d_2(1 - (1-\mu_2)e^{-d_2U})} + \frac{e^{-(\kappa+d_1)U} - 1}{\kappa + d_1})]^{-1}.$$

The second step, we will go to prove the global attraction of the solution $(0, \widetilde{p_1(t)}, \widetilde{p_2(t)})$. Let $(0, p_1(t), p_2(t))$ be any prey-extinction boundary periodic solution of equaions (1), for this goal, we just have to prove $(0, p_1(t), p_2(t))$ converges to $(0, \widetilde{p_1(t)}, \widetilde{p_2(t)})$ when $t \to \infty$. To pick up a positive number ε, which is little enough, such that

$$\eta = \exp[\int_0^U (r - \beta_1(\widetilde{p_1(t)} - \varepsilon) - \beta_2(\widetilde{p_2(t)} - \varepsilon))dt] < 1.$$

From system (1), we have

$$\begin{cases} x_1'(t) = -(\kappa + d_1)x_1(t), t \neq nU, \\ x_2'(t) = \kappa x_1(t) - d_2 x_2(t), t \neq nU, \\ \Delta x_1(t) = \dfrac{\delta_1 x(t)}{1 + \lambda_1 x(t)} + \mu_1, \quad t = nU, \\ \Delta x_2(t) = -\mu_2 x_2(t), \quad t = nU. \end{cases} \tag{6}$$

With the helps of Lemma 2° and the principle of comparison of differential equations [4], to get

$$\begin{cases} \widetilde{p_1(t)} - \varepsilon \leq x_1(t) \leq p_1(t), \\ \widetilde{p_2(t)} - \varepsilon \leq x_2(t) \leq p_2(t), \end{cases} \tag{7}$$

for t large enough. For simplicity and clarity, we make the admission that (7) be true for all $t \geq 0$. Using (1) and (7), yields

$$p'(t) \leq p(t)[r - \beta_1(\widetilde{p_1(t)} - \varepsilon) - \beta_2(\widetilde{p_2(t)} - \varepsilon)]. \tag{8}$$

Consequently $p(t) \leq p(0^+) \exp[\int_0^t (r - \beta_1(\widetilde{p_1(s)} - \varepsilon) - \beta_2(\widetilde{p_2(s)} - \varepsilon))ds]$, hence $p((n+1)U) \leq p(nU^+) \exp[\int_{nU}^{(n+1)U} (r - \beta_1(\widetilde{p_1(s)} - \varepsilon) - \beta_1(\widetilde{p_2(s)} - \varepsilon))ds]$. As a result, $p(nU) \leq p(0^+)\eta^n$ and $p(nU) \to 0$ as $n \to \infty$. In consequence, $p(t) \to 0$, when t goes to infinity.

Further, need to prove when t goes to ∞, $p_1(t) \to \widetilde{p_1(t)}$, $p_2(t) \to \widetilde{p_2(t)}$. Without loss of generality, making the following assumption: for all $t \geq 0$, $0 < p(t) < \varepsilon$. From the above assumption, for equations (1), yield

$$-(\kappa + d_1)p_1(t) \leq \frac{dp_1(t)}{dt} \leq [-(\kappa + d_1) + l_1\beta_1\varepsilon]p_1(t), \tag{9}$$

and

$$\kappa p_1(t) - d_2p_2(t) \leq \frac{dp_2(t)}{dt} \leq \kappa p_1(t) - (d_2 - l_2\beta_2\varepsilon)p_2(t). \tag{10}$$

Therefore, we can take the following systems of equations into account:

$$\begin{cases} y_1'(t) = -(\kappa + d_1)y_1(t), \quad t \neq nU, \\ y_2'(t) = \kappa y_1(t) - d_2y_2(t), \quad t \neq nU, \\ \Delta y_1(t) = \dfrac{\delta_1 y(t)}{1 + \lambda_1 y(t)} + \mu_1, \quad t = nU, \\ \Delta y_2(t) = -\mu_2 y_2(t), \quad t = nU, \end{cases} \tag{11}$$

and

$$\begin{cases} m_1'(t) = -[(\kappa + d_1) - l_1\beta_1\varepsilon]m_1(t), \quad t \neq nU, \\ m_2'(t) = \kappa m_1(t) - (d_2 - l_2\beta_2\varepsilon)m_2(t), \quad t \neq nU, \\ \Delta m_1(t) = \dfrac{\delta_1 m(t)}{1 + \lambda_1 m(t)} + \mu_1, \quad t = nU, \\ \Delta m_2(t) = -\mu_2 m_2(t), \quad t = nU, \end{cases} \tag{12}$$

according to the principle of comparison of pulsed differential equations, we get $m_1(t) \geq p_1(t) \geq y_1(t)$, $m_2(t) \geq p_2(t) \geq y_2(t)$, together with $y_1(t) \to \widetilde{p_1(t)}, y_2(t) \to \widetilde{p_2(t)}, m_1(t) \to \widetilde{m_1(t)}, m_2(t) \to \widetilde{m_2(t)}$ when t goes to ∞. Here

$$\begin{cases} \widetilde{m_1(t)} = \dfrac{\mu_1 e^{-(\kappa + d_1 - l_1\beta_1\varepsilon)(t-nU)}}{1 - e^{-(\kappa + d_1 - l_1\beta_1\varepsilon)U}}, \quad nU < t \leq (n+1)U, n \in Z^+, \\ \widetilde{m_2(t)} = \dfrac{\mu_1\kappa}{(\kappa + d_1 - l_1\beta_1\varepsilon - d_2)(1 - e^{-(\kappa + d_1 - l_1\beta_1\varepsilon)U})} \cdot \\ \left[\dfrac{1 - (1-\mu_2)e^{-(\kappa + d_1 - l_1\beta_1\varepsilon)U}}{1 - (1-\mu_2)e^{-(d_2 - l_2\beta_2\varepsilon)U}} e^{-(d_2 - l_2\beta_2\varepsilon)(t-nU)} - e^{-(\kappa + d_1 - l_1\beta_1\varepsilon)(t-nU)}\right], \\ \qquad\qquad nU < t \leq (n+1)U, n \in Z^+. \end{cases} \tag{13}$$

Consequently, for any sufficiently small positive number ε_1, there being a t_1 , such that $t \geq t_1 > 0$, holds

$$\widetilde{m_1(t)} + \varepsilon_1 > p_1(t) > \widetilde{p_1(t)} - \varepsilon_1,$$

and

$$\widetilde{m_2(t)} + \varepsilon_1 > p_2(t) > \widetilde{p_2(t)} - \varepsilon_1.$$

Here $\varepsilon \to 0$ should be allowed, generates

$$\widetilde{p_1(t)} + \varepsilon_1 > p_1(t) > \widetilde{p_1(t)} - \varepsilon_1,$$

and

$$\widetilde{p_2(t)} + \varepsilon_1 > p_2(t) > \widetilde{p_2(t)} - \varepsilon_1.$$

So, $p_1(t) \to \widetilde{p_1(t)}$ and $p_2(t) \to \widetilde{p_2(t)}$ when t goes to ∞. The proof is completed.

Next, we set out to study the persistence of equations (1).

Theorem 2°. Equations (1) is persistent under the condition

$$\mu_1 < rU$$

$$[\frac{\beta_1}{\kappa + d_1} + \frac{\kappa\beta_2}{(\kappa + d_1 - d_2)(1 - e^{-(\kappa+d_1)U})}(\frac{1 - (1 - \mu_2)e^{-(\kappa+d_1)U}(1 - e^{-d_2U})}{d_2(1 - (1 - \mu_2)e^{-d_2U})} + \frac{e^{-(\kappa+d_1)U} - 1}{\kappa + d_1})]^{-1}$$

is true.

Proof. From Lemma 1°, for convenience, it should be able to assume $p(t) \leq L/l, p_1(t) \leq L, p_2(t) \leq L$ and $L > \frac{r}{\beta_1}$ for all $t > 0$. System (7) indicates that $p_1(t) > \widetilde{p_1(t)} - \varepsilon_2$ and $p_2(t) > \widetilde{p_2(t)} - \varepsilon_2$ for sufficiently large t and some $\varepsilon_2 > 0$, so it is easy to deduce that $p_1(t) \geq \zeta_2$ and $p_2(t) \geq \zeta_2'$ for sufficiently large t. As a result, just need to find $\zeta_1 > 0$ such that $\zeta_1 \leq p(t)$ for sufficiently large t. We intend to take two steps to implement it.

The first step: Takeing sufficiently small positive numbers ζ_3 and ε_1, such that

$$\min\left\{\frac{\kappa + d_1}{l_1\beta_1}, \frac{d_2}{l_2\beta_2}\right\} > \zeta_3, \quad \delta = \max\{l_1\beta_1\zeta_3, l_2\beta_2\zeta_3\} < \min\{\kappa + d_1, d_2\}$$

and

$$\begin{aligned}\sigma = {} & rU - \frac{r}{K}\zeta_3 U - \beta_1\varepsilon_1 U - \beta_2\varepsilon_1 U - \frac{\beta_1\mu_1}{\kappa + d_1 - \delta} \\ & - \frac{\mu_1\kappa\beta_2(1 - (1 - \mu_2)e^{-(\kappa+d_1-\delta)U})(1 - e^{-(d_2-\delta)U})}{(\kappa + d_1 - d_2)(1 - e^{-(\kappa+d_1-\delta)U})(d_2 - \delta)(1 - (1 - \mu_2)e^{-(d_2-\delta)U})} \\ & + \frac{\beta_2\mu_1\kappa}{(c + d_1 - d_2)(\kappa + d_1 - \delta)} \\ & > 0.\end{aligned}$$

Presume for all $t \geq 0$, $\zeta_3 < p(t)$, yields

$$p_1'(t) \leq [-(\kappa + d_1) + \delta]p_1(t),$$

and

$$p_2'(t) \leq \kappa p_1(t) - (d_2 - \delta)p_2(t).$$

According to Lemmas 2°, we obtain $x_1(t) \geq p_1(t), x_2(t) \geq p_2(t)$, $(x_1(t), x_2(t)) \to (\overline{x_1(t)}, \overline{x_2(t)})$, when t goes to infinity, here $(x_1(t), x_2(t))$ is the solution to

$$\begin{cases} \left.\begin{aligned} x_1'(t) &= -[(\kappa + d_1) - \delta]x_1(t), \\ x_2'(t) &= \kappa x_1(t) - (d_2 - \delta)x_2(t), \end{aligned}\right\} t \neq nU, \quad n \in Z^+, \\ \left.\begin{aligned} \Delta x_1(t) &= \frac{\delta_1 x(t)}{1 + \lambda_1 x(t)} + \mu_1, \\ \Delta x_2(t) &= -\mu_2 x_2(t), \end{aligned}\right\} t = nU, \quad n \in Z^+, \end{cases} \tag{14}$$

and

$$\begin{cases} \overline{x_1(t)} = \dfrac{\mu_1 e^{-(\kappa+d_1-\delta)(t-nU)}}{1 - e^{-(\kappa+d_1-\delta)U}}, nU < t \leq (n+1)U, n \in Z^+, \\ \overline{x_2(t)} = \dfrac{\mu_1 \kappa}{(\kappa + d_1 - d_2)(1 - e^{-(\kappa+d_1-\delta)U})} \Big[\dfrac{1 - (1 - \mu_2)e^{-(\kappa+d_1-\delta)U}}{1 - (1 - \mu_2)e^{-(d_2-\delta)U}} e^{-(d_2-\delta)(t-nU)} \\ \qquad\qquad -e^{-(\kappa+d_1-\delta)(t-nU)} \Big], nU < t \leq (n+1)U, n \in Z^+. \end{cases} \tag{15}$$

Thereupon, there being a positive number T_1, for all $t \geq T_1$, satisfies

$$\overline{x_1(t)} + \varepsilon_1 \geq x_1(t) \geq p_1(t), \quad \overline{x_2(t)} + \varepsilon_1 \geq x_2(t) \geq p_2(t),$$

and

$$p'(t) \geq p(t)\Big[r - bm_3 - \beta_1(\overline{x_1(t)} + \varepsilon_1) - \beta_2(\overline{x_2(t)} + \varepsilon_1)\Big]. \tag{16}$$

Suppose $N_1 \in Z^+$ and $N_1 U > T_1$, let's integrate (16) on both sides with $t \in (nU, (n+1)U), n \geq N_1$, yields

$$\begin{aligned} p((n+1)U) &\geq p(nU^+) \exp(\int_{nU}^{(n+1)U} [r - bm_3 - \beta_1(\overline{x_1(t)} + \varepsilon_1) - \beta_2(\overline{x_2(t)} + \varepsilon_1)]dt) \\ &= p(nU)e^{\sigma}, \end{aligned}$$

so $p((N_1 + k)U) \geq p(N_1 U^+)e^{k\sigma} \to \infty$, when k goes to ∞. Boundedness and unboundedness of $p(t)$ contradict each other. Therefore, it has got a positive number t_1, satisfying $p(t_1) \geq \zeta_3$.

The second step: The proof of the second step can be carried out by referring to [8,9], which is omitted here. So we've done the proof of the Theorem 2°.

5 Discussion

In the present section, we will do some numerical simulations. Taking a group of variables as shown in the table below:

$p(0)$	$p_1(0)$	$p_2(0)$	r	K	β_1	β_2	l_1	l_2	c_1	c_2	d_1	κ	d_2	U	μ_1	μ_2
1.1	1.1	1.1	1.1	2.1	2.1	3.1	0.7	0.9	2.1	2.1	0.4	2.1	0.4	1.1	2.1	0.8

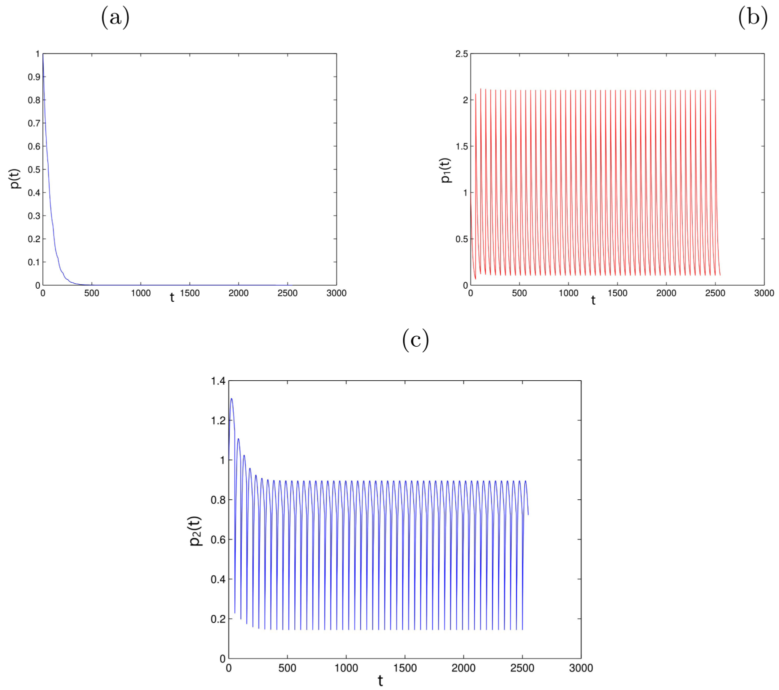

Fig. 1. The kinetic graphics for verifying Theorems 1°.

Clearly, the condition for Theorem 1° is satisfied, wherefore, the solution $(0, \widetilde{p_1(t)}, \widetilde{p_2(t)})$ of (1) possesses global asymptotic stability (see Fig. 1.).

Taking another group of variables as shown in the table below:

$p(0)$	$p_1(0)$	$p_2(0)$	r	K	β_1	β_2	l_1	l_2	c_1	c_2	d_1	κ	d_2	U	μ_1	μ_2
0.9	0.9	0.9	0.9	1.9	1.9	2.9	0.5	0.8	1.9	1.9	0.3	1.9	0.3	0.9	0.9	0.8

Evidently, the condition for Theorem 2° is also satisfied, thereupon, system (1) can preserve its permanence (see Fig. 2.).

According to Theorem 1° and Theorem 2°, there must being a threshold $\Omega = rU[\frac{\beta_1}{\kappa+d_1} + \frac{\kappa\beta_2}{(\kappa+d_1-d_2)(1-e^{-(\kappa+d_1)U})}(\frac{1-(1-\mu_2)e^{-(\kappa+d_1)U}(1-e^{-d_2U})}{d_2(1-(1-\mu_2)e^{-d_2U})} + \frac{e^{-(\kappa+d_1)U}-1}{\kappa+d_1})]^{-1}$. The solution $(0, \widetilde{p_1(t)}, \widetilde{p_2(t)})$ of (1) possesses global asymptotic stability provided that $\mu_1 > \Omega$; On the other hand, system (1) can preserve its permanence provided that $\mu_1 < \Omega$. The results of numerical experiments verify the validity of our obtained theorems.

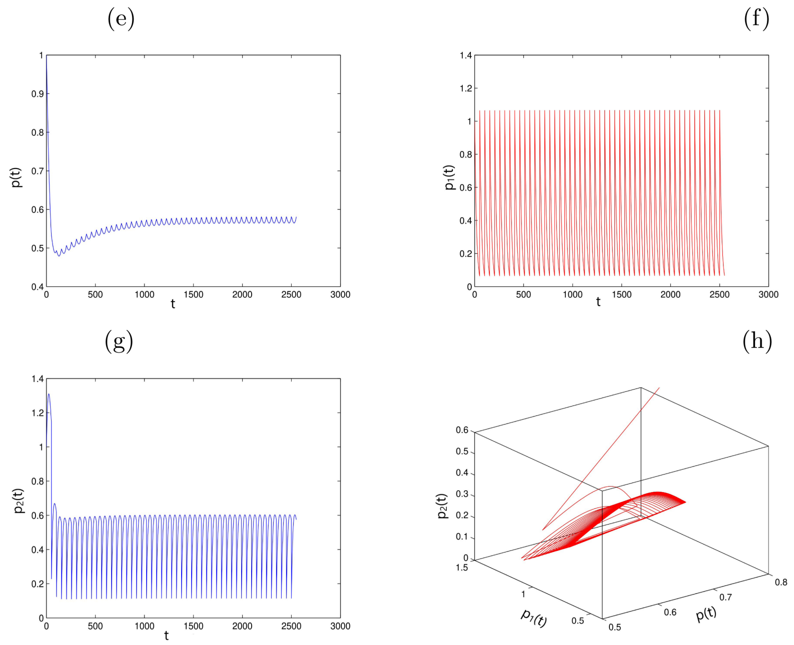

Fig. 2. The kinetic graphics for verifying Theorems 2°.

6 Summary

In the present work, we study a predator-prey economic model with impulsive control strategy. Some threshold conditions (Such as possessing global asymptotic stability, preserving its permanence) have been proved in terms of theory of dynamical system. Further, simulation experiments demonstrate our theorems. The obtained results provide a methodological guidance for the actual biological economic managements.

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