Analysis and Design of Wireless Distributed Fountain Codes with Multiplicative Network Coding

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Abstract. A novel wireless distributed fountain coding scheme is proposed for wireless distributed networks. In this scheme, a multiplicative network coding method is adopted instead of exclusive-or (XOR) network coding. Thus, the processing complexity of the relay can be reduced, and the error propagation can be avoided. Moreover, the degree distributions of the proposed coding scheme are derived and the performance is analyzed asymptotically using semi-Gaussian approximation analysis technique. Furthermore, an efficient optimization method employing linear program is presented to optimize the degree distributions of the proposed scheme. Simulation results reveal that with optimized degree distributions, the proposed scheme has good performance on additive white Gaussian noise (AWGN) channels and outperforms the scheme using alternate forwarding.

Keywords: Fountain codes · Multiplicative network coding · AWGN channels · Asymptotic analysis · Degree distributions · Linear programming

1 Introduction

Digital fountain codes [1,2] are proposed to achieve large-scale network data distribution and reliable transmission. As a novel forward error correction technology with rateless property, fountain encoders do not need to fix the code rate, and it can generate limitless number of codewords theoretically. Therefore, fountain codes can adapt to the channel state and have strong flexibility. Several examples of fountain codes are proposed and developed, such as Luby Transform (LT) codes [3], Raptor codes [2] and Batched Sparse (BATS) codes [4,5].

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Digital fountain codes are originally proposed and designed for erasure channels. With well-designed degree distributions [6–10], fountain codes can adapt to various erasure channels, and approach the channel capacity of arbitrary erasure rate. Inspired by the good performance on erasure channels, more and more researchers are working to expand the application of fountain codes from application layer to physical layer, and to study their performance on wireless channels [11–19].

Palanki and Yedidia [20] studied the performances of fountain codes on wireless channels, and concluded that there is an obvious "error floor" for LT codes on wireless noisy channels, but not for Raptor codes. Etesami and Shokrollahi [21] studied the performance of Raptor codes on binary memoryless symmetric channels, and proposed a BP decoding algorithm for fountain codes on additive white Gaussian noise (AWGN) channels. It is shown that the LT codes and Raptor codes with well-designed degree distributions for erasure channels are still with good performance on AWGN channels, but cannot approach the capacity of AWGN channels arbitrarily. Castura and Mao [22] proposed a fountain coding scheme on fading channels. The scheme has better performance than a conventional fixed-rate coding scheme while the channel state information is unknown for the receiver. Zhang [23] proposed a joint network-channel coding scheme to optimize the degree distribution on AWGN and fading channels. Nessa [24] proposed a cooperative communication scheme based on fountain codes, and applied it to Long Term Evolution-Advanced (LTE-A) network.

On wireless channels or wireless networks, a decode-and-forward (DF) scheme based on exclusive-or (XOR) network coding is usually performed at the relay nodes [4,23]. Firstly, the relay needs to decode the input symbols. Then, the relay performs XORed network coding on these decoded bits. Finally, the relay needs to re-encode and re-modulate to these bits. However, this scheme increases the coding complexity of the relay node, and brings extra processing delay. Furthermore, there is a certain decoding error probability at the relay nodes. If the error bits are re-encoded through network coding, the decoding error will further spread, which will deteriorate the decoding performance of the destination node.

This paper aims to solve the above problems. We propose a novel fountain coding scheme called wireless distributed fountain codes with multiplicative network coding [25–27] for wireless distributed networks. In the proposed scheme, a multiplicative network coding method is adopted instead of XORed network coding. The relay node can directly perform multiplicative network coding on incoming modulated symbols without the decoding and re-encoding processing, which can reduce the processing complexity greatly. Furthermore, since the relay node does not need to perform decoding, the error propagation caused by incorrect decoding can be avoided. In this paper, we further derive the degree distributions and analyze the performance of the proposed scheme using modified Gaussian approximation method. Finally, we propose an optimal design method of the degree distributions for the relay and source nodes. Simulation results reveal that the proposed fountain coding scheme has good performance on AWGN channels and the benefits are observed in comparison with the separate LT codes.

The rest of this paper is organized as follows. Section 2 gives the system model of the proposed scheme. In Sect. 3, we propose a novel fountain coding scheme with multiplicative network coding called wireless distributed fountain codes. In Sect. 4, we analyze the asymptotic performance of the proposed scheme on AWGN channels. In Sect. 5, an optimal design method of the degree distributions for the relay and source nodes is proposed. Some simulation results are given in Sect. 6 and a conclusion is made in Sect. 7.

2 System Model

Consider the wireless distributed network model in Fig. 1, where multiple source nodes transmit information to a destination node through a single relay node, which is similar to a distributed network over lossy channels with erasures. The source-relay and relay-destination channels are wireless noisy channels rather than wired erasure channels. For simplicity, we take the two-source network as an example in Fig. 1. Due to the openness of wireless channel and noise interference, each source must avoid transmitting information to the relay at the same time. Therefore, a simple time-division multiplexing method can be used. The duration of transmission can be divided into several time slots. In each time slot, only one source is allowed to send information. Thus, a complete transmission cycle needs four time slots. In a network coding scheme, less time slots are needed for transmission. Assuming that the source S_1 and S_2 send symbols to the relay R in time slot 1 and 2, respectively. The relay R performs network coding on the incoming symbols and transmits the coded symbols to the destination D in time slot 3. In this scheme, a complete transmission cycle only consumes three time slots, which is less than the above time-multiplexed scheme. Therefore, the network coding of the relay can save time slots and improve the throughput of the networks.

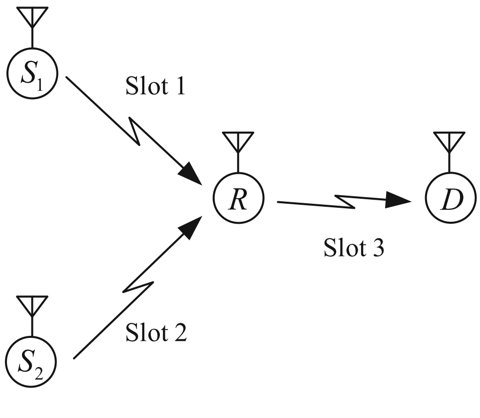

Fig. 1. Wireless distributed network model.

3 Wireless Distributed Fountain Codes with Multiplicative Network Coding

Figure 2 depicts the proposed fountain coding scheme for the wireless distributed network model in Fig. 1, named wireless distributed fountain codes (WDFC). Each source node performs LT encoding on k information bits independently. The information bit streams of source S_1 and S_2 are denoted by $\mathbf{m}_1 = (m_{1,1}, m_{1,2}, \ldots, m_{1,k})$ and $\mathbf{m}_2 = (m_{2,1}, m_{2,2}, \ldots, m_{2,k})$, respectively. The encoded bits are denoted by $\mathbf{x}_1 = (x_{1,1}, x_{1,2}, \ldots)$ and $\mathbf{x}_2 = (x_{2,1}, x_{2,2}, \ldots)$, respectively. Then, a BPSK modulator is employed to each encoded bit streams, and the modulated symbols are denoted by $\tilde{\mathbf{x}}_1 = (\tilde{x}_{1,1}, \tilde{x}_{1,2}, \ldots)$ and $\tilde{\mathbf{x}}_2 = (\tilde{x}_{2,1}, \tilde{x}_{2,2}, \ldots)$, respectively. $\tilde{\mathbf{x}}_1$ and $\tilde{\mathbf{x}}_2$ will be time-multiplexed to the relay through wireless channels. The noises of the channels are denoted by $\mathbf{n}_1$ and $\mathbf{n}_2$, respectively. Assuming that both channels between the sources and relay are AWGN channels with the same variance σ_{sr}^2.

The decoding process of the destination D is also shown in Fig. 2. The symbol stream $\mathbf{r}$ received by D can be expressed as

$$\mathbf{r} = \tilde{\mathbf{z}} + \mathbf{n}_3, \tag{1}$$

where $\mathbf{n}_3$ is Gaussian noise of the relay-destination channel with variance σ_{rd}^2. Finally, the destination D uses BP decoding and soft-decision on $\mathbf{r}$ to recover the bits stream $\hat{\mathbf{m}}$.

At the relay node, a decode-and-forward (DF) scheme is usually used. In this scheme, the relay node cannot perform bit-wise XOR on the received symbols directly. A belief propagation (BP) decoder is needed before XORed network coding. Therefore, it increases the complexity of the relay and leads to the existence of decoding error propagation. In this paper, we propose a novel network coding scheme based on multiplication rather than XORing, which is shown in Fig. 2.

The relay R receives symbols from source S_1 and S_2 alternately. Assume that the received streams of coded symbols are $\mathbf{y}_1 = (y_{1,1}, y_{1,2}, \ldots)$ and $\mathbf{y}_2 = (y_{2,1}, y_{2,2}, \ldots)$, respectively. $\mathbf{y}_i$ can be expressed as

$$\mathbf{y}_i = \tilde{\mathbf{x}}_i + \mathbf{n}_i, \quad i = 1, 2. \tag{2}$$

Unlike $\tilde{\mathbf{x}}_1$ or $\tilde{\mathbf{x}}_2$, the range of $\mathbf{y}_1$ or $\mathbf{y}_2$ is no longer restricted to binary field, but the real number field. Therefore, the network coding with XOR cannot be operated directly. The relay node uses an amplify-and-forward (AF) method, and performs network coding by multiplying two received symbols from both sources. The processing of the relay includes only two steps as follows:

- The relay R multiplies the corresponding symbols of $\mathbf{y}_1$ and $\mathbf{y}_2$, which are received in different time slots alternately. The products form a new symbol stream $\mathbf{y}$;
- The relay R amplifies $\mathbf{y}$, and then send the amplified signal $\tilde{\mathbf{z}}$ to the destination D through wireless channel or link.

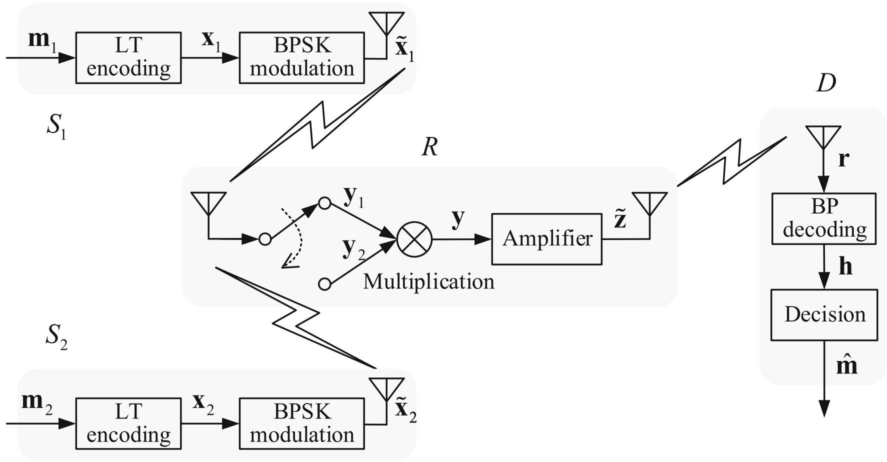

Fig. 2. The proposed coding scheme with multiplicative network coding at the relay.

Compared with XORed network coding, the proposed scheme multiplies the received symbols directly. Therefore, the decoding, re-encoding and re-modulating are not needed. As a result, the complexity of the relay can be reduced greatly. In addition, the error propagation caused by incorrect decoding can be avoided in this scheme.

Assuming that b_1 and b_2 are the information bits to be modulated, x_1 and x_2 are the BPSK-modulated symbols, where $b_1, b_2 \in \{0, 1\}$, $x_1, x_2 \in \{+1, -1\}$. We have $x_i = \exp(j\theta_i),\ i = 1, 2$, where $\theta_i = b_i\pi$ is the phase of x_i. The product of x_1 and x_2 can be calculated as follows

$$z = x_1 x_2 = \exp[j(\theta_1 + \theta_2)] = \exp[j(b_1 + b_2)\pi]. \tag{3}$$

The product z can be seen as a modulated symbol. The corresponding bit of z is denoted by b, and the phase of z is denoted by θ. Obviously, the product z is a BPSK modulated symbol. The phase θ is consistent with the phase of a BPSK modulation, which can be expressed as $\theta = (\theta_1 + \theta_2) \bmod 2\pi$. The bit b is determined by $b_1 + b_2$, which can be expressed as $b = (b_1 + b_2) \bmod 2 = b_1 \oplus b_2$. Therefore, we can conclude that the product of two BPSK modulated symbols is equivalent to the XOR of two corresponding bits. It is indicated that multiplicative network coding is equivalent to XORed network coding. This is an important reason for performing multiplicative network coding instead of XORed network coding.

4 Asymptotic Performance Analysis of Proposed Wireless Distributed Fountain Codes

4.1 Degree Distributions

In this section, we will analyze the degree distribution of the proposed coding scheme WDFC. Assuming that each source performs LT encoding with the same degree distribution $\Phi(x)$. Let $\Phi(x) = \sum_{d=1}^{k} \Phi_d x^d$ and $\Lambda(x) = \sum_{d=1}^{D_i} \Lambda_d x^d$ be the output/input node perspective degree distribution of the source nodes, respectively. The corresponding output/input edge perspective degree distribution can be denoted by $\phi(x) = \Phi'(x)/\Phi'(1)$ and $\lambda(x) = \Lambda'(x)/\Lambda'(1)$, respectively. Let $\Gamma(x) = \Gamma_1 x + \Gamma_2 x^2$ and $\gamma(x) = \Gamma'(x)/\Gamma'(1)$ be the output node/edge perspective degree distribution of the relay node. The overall output node/edge perspective degree distribution of the received symbols by the destination node can be denoted by $\Omega(x)$ and $\omega(x) = \Omega'(x)/\Omega'(1)$. The input degree distribution $\Lambda(x)$ follows a binomial distribution, which can be expressed as

$$\Lambda_d = \binom{n\beta}{d} p^d (1-p)^{n\beta-d}. \tag{4}$$

Where n is the number of output coded symbols, $\beta = \Phi'(1)$ is the average degree of output symbols, and $p = 1/k$ is the probability that an edge is connected to a particular input node of Tanner graph. When n goes to infinity ($n \to \infty$), the input degree distribution approaches to a Poisson distribution asymptotically as follows

$$\Lambda(x) = \lambda(x) = \exp(-\alpha(1-x)). \tag{5}$$

where $\alpha = (1+\varepsilon)\beta$ is the average degree of input symbols, ε is coding overhead.

Reviewing that b_1 and b_2 are the information bits to be modulated of source S_1 and S_2, respectively. x_1 and x_2 are the corresponding BPSK-modulated symbols. b_1 and b_2 can be regarded as independent random variables with the same probability generating function $\Phi(x)$. According to probability theory, $b_1 \oplus b_2$ is a random variable with probability generating function $\Omega(x) = \Phi^2(x)$. As mentioned in Sect. 3, the proposed multiplicative network coding is essentially equivalent to XORed network coding. Therefore, we can conclude that the overall degree distribution after multiplicative network coding is still $\Omega(x) = \Phi^2(x)$.

In Fig. 2, the relay performs network coding on all the received symbols, that is, the probability of network coding is 1. Thus the output degree distribution of the relay is $\Gamma(x) = x^2$. In practice, selective network coding is often used in order to ensure that the degree one symbols are still exist among the received symbols at destination node. The relay randomly selects one symbol from two received symbols with probability Γ_1, and multiplies two received symbols with probability Γ_2, where $\Gamma(x) = \Gamma_1 x + \Gamma_2 x^2$ is the output degree distribution of the relay. Therefore, the overall degree distribution $\Omega(x)$ can be expressed as a composition of $\Gamma(x)$ and $\Phi(x)$, i.e., $\Omega(x) = \Gamma(\Phi(x))$.

4.2 Asymptotic Performance Analysis

The BP decoding algorithm for wireless channel is still applicable to the proposed WDFC. According to (2), it is assumed that the relay node performs multiplicative network coding on two received symbols $y_1 = \tilde{x}_1 + n_1$ and $y_2 = \tilde{x}_2 + n_2$. The coded symbol can be expressed as

$$y = y_1 y_2 = \tilde{x}_1\tilde{x}_2 + \tilde{x}_1 n_2 + \tilde{x}_2 n_1 + n_1 n_2. \tag{6}$$

The destination node receives the symbol after the transmission through the relay-destination channel. The received symbol can be expressed as

$$r = y + n_3 = \tilde{x}_1\tilde{x}_2 + \underbrace{\tilde{x}_1 n_2 + \tilde{x}_2 n_1 + n_3}_{n} + n_1 n_2. \tag{7}$$

Where $\tilde{x}_1\tilde{x}_2$ is the useful signal, $\tilde{x}_1 n_2 + \tilde{x}_2 n_1 + n_3$ can be regarded as a noise term and denoted by the equivalent noise n, $n_1 n_2$ is a small product term that can be negligible, n_1, n_2 and n_3 are independent Gaussian variables. Assuming that both n_1 and n_2 are Gaussian variables with mean 0 and variance σ_{sr}^2, n_3 is a Gaussian variable with mean 0 and variance σ_{rd}^2. Therefore, we can conclude that the overall equivalent noise n is a Gaussian variable with variance $\sigma_n^2 = 2\sigma_{sr}^2 + \sigma_{rd}^2$. The log-likelihood ratio (LLR) of the channel can be expressed as

$$\begin{aligned} Z_o &= \ln \frac{\Pr(r|\tilde{x}_1\tilde{x}_2 = +1)}{\Pr(r|\tilde{x}_1\tilde{x}_2 = -1)} = \ln \frac{\Pr(n = r-1)}{\Pr(n = r+1)} \\ &= \ln \frac{\exp(-(r-1)^2/2(2\sigma_{sr}^2 + \sigma_{rd}^2))}{\exp(-(r+1)^2/2(2\sigma_{sr}^2 + \sigma_{rd}^2))} \\ &= \frac{2r}{2\sigma_{sr}^2 + \sigma_{rd}^2}. \end{aligned} \tag{8}$$

Z_o can be used as the initial value for LLR iteration of BP decoding algorithm.

A Gaussian approximation method is usually used to analyze the asymptotic performance of LDPC codes or fountain codes on noisy channels [21,28]. It requires that all the LLRs of the input and output nodes passed at each iteration of the BP algorithm are Gaussian. This requirement is very strong. The LLRs of the output nodes of small degree are far from being Gaussian. Thus, the theoretical results of asymptotic performance analysis based on Gaussian approximation are far away from simulation results. In fact, according to the BP algorithm, the LLR of an input node is the sum of a set of independent random variables with the same distribution. By the central limit theorem, the LLR of an input node is close to a Gaussian variable when the number of the additions is large enough. However, the LLR of an output node does not satisfy this condition. In [21,28], this assumption is called semi-Gaussian approximation, and is applied to analyze the performance of LDPC codes and Raptor codes. In this paper, we can improve the analytical method for WDFC by using this assumption.

Assuming that the LLRs of all input nodes are Gaussian, while the LLRs of all output nodes are not Gaussian. Let $Y^{(l)}$ be the LLR of an output node at the lth iteration. $\{X_1, X_2, \ldots, X_j, \ldots, X_d - 1\}$ represents $d-1$ independent identically distributed random variables with symmetric Gaussian distribution, and they are corresponding to the LLRs of input nodes. $\mu^{(l)}$ is the mean of X_j at the lth iteration. According to BP decoding algorithm [21], the message sent from output node o of degree d to an input node at the lth iteration has an expectation as follows

$$\mathrm{E}\left[Y^{(l)}|\deg(o)=d\right] = 2\mathrm{E}\left[\operatorname{atanh}\left(\tanh\left(\frac{Z_o}{2}\right)\prod_{j=1}^{d-1}\tanh\left(\frac{X_j}{2}\right)\right)\right]. \quad (9)$$

Where $\operatorname{atanh}(\cdot)$ represents the inverse hyperbolic tangent function. It is shown in (9) that the mean of LLR of an output node with degree d can be expressed as a function of $\mu^{(l)}$. Assuming that the function is denoted by $f_d(\cdot)$, we have

$$f_d(\mu^{(l)}) \triangleq 2\mathrm{E}\left[\operatorname{atanh}\left(\tanh\left(\frac{Z_o}{2}\right)\prod_{j=1}^{d-1}\tanh\left(\frac{X_j}{2}\right)\right)\right]. \quad (10)$$

The right side of the equation in (10) can be wrote as the expectation of a random variable U as follows

$$U \triangleq 2\operatorname{atanh}\left(\tanh\left(\frac{Z_o}{2}\right)\prod_{j=1}^{d-1}\tanh\left(\frac{X_j}{2}\right)\right). \quad (11)$$

In practice, $f_d(\mu)$ can be obtained by calculating an empirical mean of U as follows:

(1) Calculating the LLR of the channel $Z_o = 2/\sigma_n^2$;
(2) Let X be a symmetric Gaussian variable with mean μ. Sampling from X for $d-1$ times and getting $d-1$ samples x_i, where $i = 1, 2, \ldots, d-1$;
(3) Calculating the sample $2\operatorname{atanh}\left(\tanh(Z_o/2)\prod_{i=1}^{d-1}\tanh(x_i/2)\right)$ of U;
(4) Repeat steps (2) and (3) for q times to obtain q samples of U. Calculating the average value of q samples as the empirical mean of U. As the number of repetitions increases, the empirical mean is closer to the expectation.

The mean of the LLR of an output node at the lth iteration can be expressed as

$$\mathrm{E}\left[Y^{(l)}\right] = \sum_{d=1}^{D_\omega}\omega_d \mathrm{E}\left[Y^{(l)}|\deg(o)=d\right] = \sum_{d=1}^{D_\omega}\omega_d f_d(\mu^{(l)}). \quad (12)$$

Where D_ω is the maximum degree of $\omega(x)$. Let α be the average degree of an input node, we have

$$\mu^{(l+1)} = \alpha \mathrm{E}\left[Y^{(l)}\right] = \alpha\sum_{d=1}^{D_\omega}\omega_d f_d(\mu^{(l)}). \quad (13)$$

We can call (13) the density evolution equation based on semi-Gaussian approximation. In each iteration, we only need to calculate the mean μ of the LLR of an input node. At the beginning of the iterations, the initial value of μ is set to $\mu^{(0)} = 0$, and the LLR of the channel is $Z_o = 2/\sigma_n^2$.

5 Optimization Design of Degree Distributions for Wireless Distributed Fountain Codes

The output degree distribution is an important parameter in determining the decoding performance of fountain codes. In this section, we will discuss the design of degree distributions for the proposed WDFC. Based on the density evolution analysis, we can jointly optimize the output degree distributions of the source and relay nodes.

According to the semi-Gaussian analysis in Sect. 4, the iteration must proceed in the direction of increasing the mean $\mu^{(l)}$ of an input node to guarantee the success of density evolution. Thus, the following inequality must be satisfied

$$\mu^{(l+1)} = \alpha \sum_{d=1}^{D_\omega} \omega_d f_d(\mu^{(l)}) > \mu^{(l)}. \tag{14}$$

Assuming that the expected maximum value of $\mu^{(l)}$ is $\mu_{\max}$, then the inequality (14) holds for $\mu^{(l)} \in (0, \mu_{\max}]$. We can use (14) as the main constraint for the optimization.

When the variance σ_n^2 of the overall equivalent channel noise n is given, the optimization objective is to maximize the code rate. Assuming that the input/output average degree is α and $\Omega'(1)$, respectively. The code rate can be expressed as $R = \Omega'(1)/\alpha$. Since $1/\Omega'(1) = \sum_d \omega_d/d$, maximizing the code rate R is equivalent to minimizing $\sum_d \omega_d/d$. The optimized degree distribution is denoted by $\omega_{\text{opt}}(x) = \arg\min_{\omega} \sum_d \omega_d/d$.

According the above objective and constraints, we can get the following linear program (LP) to optimize the degree distribution $\omega(x)$:

$$\begin{gathered} \min_{\omega} \alpha \sum_{d=1}^{D_\omega} \frac{\omega_d}{d} \\ s.t. \begin{cases} \alpha \sum_{d=1}^{D_\omega} \omega_d f_d(u_i) > u_i, \quad i = 1, 2, \ldots, M \\ \sum_{d=1}^{D_\omega} \omega_d = 1 \\ \omega_d \geq 0, \quad d = 1, 2, \ldots, D_\omega. \end{cases} \end{gathered} \tag{15}$$

The interval $(0, \mu_{\max}]$ is divided into M equal parts and $0 < u_1 < u_2 < \cdots < u_M = \mu_{\max}$ are M equidistant points in this interval. The overall output node perspective degree distribution $\Omega(x)$ can be determined from $\omega(x)$ as follows

$$\Omega(x) = \frac{\int_0^x \omega(u) du}{\int_0^1 \omega(u) du}. \tag{16}$$

The overall output degree distribution $\Omega(x)$ can be expressed as a composition of $\Gamma(x)$ and $\Phi(x)$, i.e., $\Omega(x) = \Gamma(\Phi(x))$. So the degree distributions of the sources and the relay can be further calculated after obtaining $\Omega(x)$. Assuming that the output degree distribution of the relay is $\Gamma(x) = \Gamma_1 x + \Gamma_2 x^2$, and that of the source is $\Phi(x) = \sum_{d=1}^{D_\Phi} \Phi_d x^d$, where D_Φ is the maximum degree of $\Phi(x)$. Then, we have $\Omega(x) = \Gamma_1 \Phi(x) + \Gamma_2 (\Phi(x))^2$ and $\Omega_d = \Gamma_1 \Phi_d + \Gamma_2 (\Phi * \Phi)_d$, where $(\Phi * \Phi)_d$ denotes the dth value of the convolution of two identical sequences $\{\Phi_1, \Phi_2, \dots, \Phi_{D_\Phi}\}$.

Considering the deviation between $\Omega(x)$ and $\Gamma(\Phi(x))$, we denote by $h_d = \Omega_d - \Gamma_1 \Phi_d - \Gamma_2 (\Phi * \Phi)_d$ the dth component of the deviation. The degree distributions $\Gamma(x)$ and $\Phi(x)$ can be optimized by the following optimization problem:

$$\min_{\omega} \sum_{d=1}^{D_\omega} h_d^2$$
$$s.t. \begin{cases} h_d = 0, \quad d = 1, 2, \dots, D_\omega \\ \sum_{i=1}^{2} \Gamma_i = 1 \\ \sum_{i=1}^{D_\Phi} \Phi_i = 1 \\ 0 \le \Gamma_i \le 1, \quad i = 1, 2 \\ 0 \le \Phi_i \le 1, \quad i = 1, 2, \dots, D_\Phi. \end{cases} \tag{17}$$

The objective is to minimize the sum of squared errors. Given $\Omega(x)$, the optimization problem in (17) is a least squares optimization problem with nonlinear constraints.

6 Simulation Results

In this section, we will perform simulations for the proposed WDFC and verify the asymptotic analysis. The degree distributions $\Phi(x)$ and $\Gamma(x)$ are optimized using the proposed optimization method in Sect. 5. Setting $\mu_{\max} = 50$, $D_\omega = 100$ and $M = 1000$, the optimized degree distributions for $\sigma_n = 0.6$ are as follows.

$$\Phi(x) = 0.0065x + 0.8127x^2 + 0.0068x^{14} + 0.1648x^{16} + 0.0052x^{25} + 0.0034x^{39} \tag{18}$$

$$\Gamma(x) = 0.9713x + 0.0287x^2. \tag{19}$$

The overall input degree distribution $\Lambda(x)$ and the overall output degree distribution $\Omega(x)$ are shown in Fig. 3 and Fig. 4, respectively. Figure 3 shows that the input distribution $\Lambda(x)$ of the proposed WDFC approaches to a Poisson distribution with average input degree of 12.66. Figure 4 compares $\Omega(x)$ with $\Gamma(\Phi(x))$. It is shown that the values of $\Gamma(\Phi(x))$ are very close to the values of $\Omega(x)$ with small deviations. In addition, Fig. 4 compare the simulated overall output degree distribution with the corresponding theoretical degree distribution $\Omega(x)$. Simulation results show that the simulated distribution is consistent with

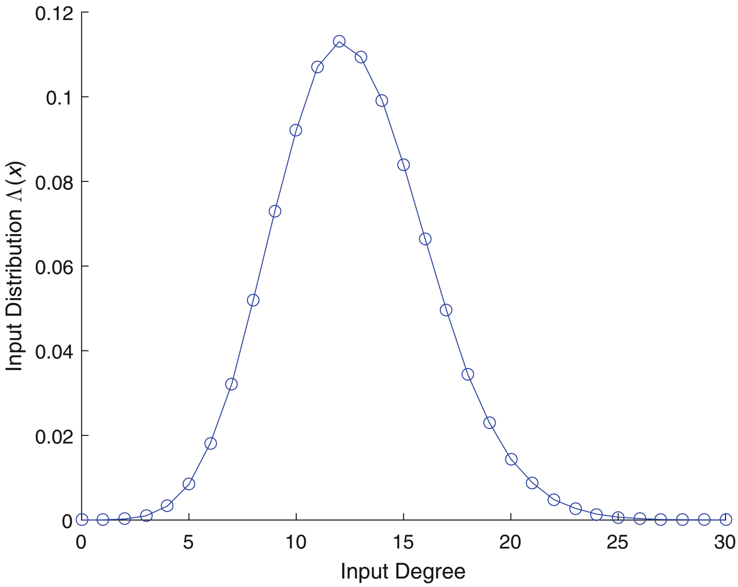

Fig. 3. The overall input degree distribution when $\sigma_n = 0.6$.

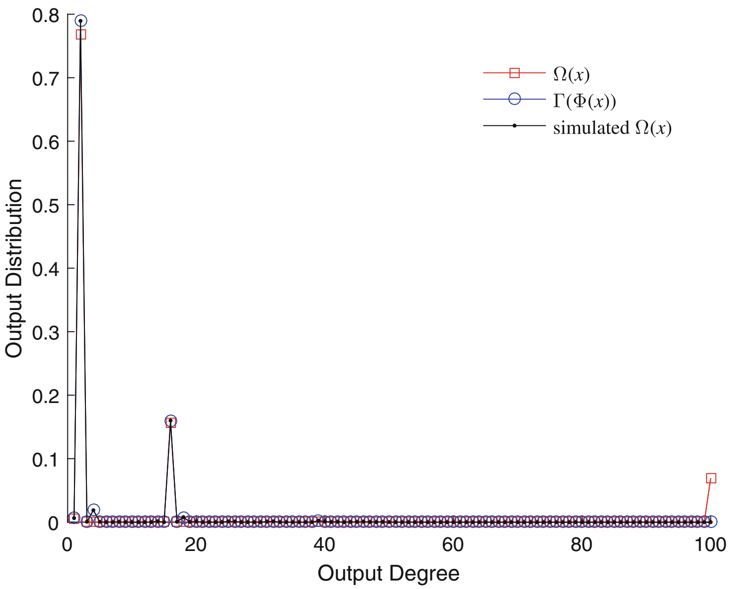

Fig. 4. The overall output degree distribution when $\sigma_n = 0.6$.

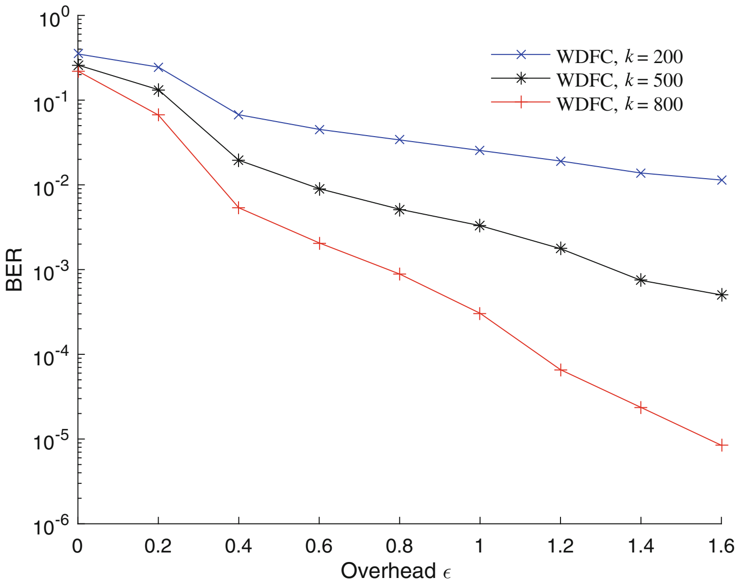

Fig. 5. The BER performance comparison of WDFC for different values of k.

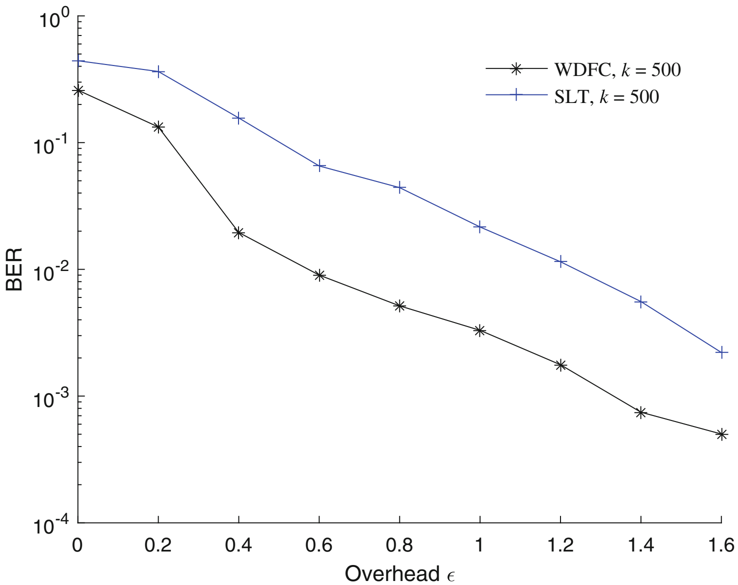

Fig. 6. The BER performance comparison of WDFC with SLT codes when $k = 500$.

the theoretical distribution. It is indicated that the overall degree distribution of the proposed WDFC based on multiplicative network coding can be expressed as a composition of $\Gamma(x)$ and $\Phi(x)$, and the correctness of the analysis for degree distributions in Sect. 4 is verified.

Figure 5 depicts the BER performance of the proposed WDFC for different lengths of information bits with optimized degree distributions in (18) and (19). The length of information bits k takes 200, 500 and 800, respectively. Simulation results show that the proposed WDFC have good performance on wireless AWGN channels. The BER of WDFC decreases as the coding overhead ε increases, which verifies the theoretical analysis. For good LT codes on AWGN channels, we can get the corresponding good degree distributions for the proposed WDFC by using the optimization method in Sect. 5. It is also shown that the BER performance of WDFC can be improved greatly by increasing the length k.

In Fig. 6, we further compare the BER performance of the proposed WDFC with other codes when $k = 500$. The optimized degree distributions in (18) and (19) are still used. A reference coding scheme is provided for comparison purpose. In this reference coding scheme, two source nodes perform LT encoding respectively, and the relay directly and alternately forwards the receiving two separate LT codes to the destination and constructs a stream of encoded symbols. We name this coding scheme as separate LT (SLT) codes. For a fair comparison, the length of information bits, the degree distribution of each source, the overall coding overhead and the channel condition of each coding scheme are with the same settings. It is shown that the BER performance of WDFC is superior to SLT codes with the same length of information bits k. This is because the overall degree distribution of WDFC has been optimized, and the multiplicative network coding is adopted at the relay. Therefore, the performance improvement can be obtained compared with SLT codes.

7 Conclusions

In this paper, we have proposed a novel wireless distributed fountain coding scheme based on multiplicative network coding and have analyzed its decoding performance asymptotically using semi-Gaussian approximation method. Based on the asymptotic performance analysis, we further proposed an optimization method for the design of degree distributions of the proposed WDFC. Due to the adoption of multiplicative network coding, the complexity of the relay node can be reduced greatly and the error propagation can be avoided. Simulation results have showed that the proposed scheme have good performance on AWGN channels, and the theoretical analysis has been verified. It also has revealed that the proposed WDFC outperform SLT codes, and its performance can be further improved by the optimization of degree distributions.

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