Energy-Efficient Multi-cell NOMA Design via Coalition Formation Game

Yanqing Xu[1], Donghong Cai[2(✉)], Shuai Wang[1], Zhicheng Dong[3], and Weixi Zhou[4]

[1] The School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen 518172, China
xuyanqing@cuhk.edu.cn, shuaiwang@link.cuhk.edu.cn

[2] The College of Cyber Security, Jinan University, Guangzhou 510632, China
dhcai@jnu.edu.cn

[3] College of Information Science and Technology, Tibet University, Lhasa, China
dongzc666@163.com

[4] School of Computer Science, Sichuan Normal University, Chengdu, China
zhouweixi@sicnu.edu.cn

Abstract. In this work, we consider a downlink non-orthogonal multiple access (NOMA) network where multiple single-antenna users are served by multiple multi-antenna base stations (BSs). For practical considerations, we assume that only the imperfect channel state information (CSI) of each user is available at the BSs. Based on this model, the problem of joint user grouping and robust beamforming design is formulated to minimize the sum transmission power, and meanwhile, guarantee the quality of service requirements of users. Due to the integer variables of user grouping, coupling effects of beamformers, and infinitely many constraints caused by the imperfect CSI, the formulated problem is challenging to solve. For computational complexity reduction, the original problem is divided into a user grouping subproblem and a robust beamforming design subproblem. First, the user grouping problem is efficiently solved by a coalition formation game based algorithm. Then, for the robust beamforming design problem, a semidefinite relaxation (SDR) based method is proposed to produce a suboptimal solution efficiently. Moreover, we provide a sufficient condition under which the SDR based approach can guarantee to obtain an optimal rank-one solution, which is theoretically analyzed. Simulation results demonstrate the efficacy of the proposed algorithms.

Keywords: User grouping · Robust beamforming design · Coalition formation game · Semidefinite relaxation

1 Introduction

To support the exponentially growing data traffic and the number of devices in future wireless networks, non-orthogonal multiple access (NOMA) was proposed

F. Fang and F. Shu (Eds.): GameNets 2022, LNICST 457, pp. 215–229, 2022.
https://doi.org/10.1007/978-3-031-23141-4_16

to serve multiple users simultaneously on the same resource block [1–3]. The key idea of NOMA is to combine the superposition coding at the transmitter and successive interference cancellation (SIC) at the receiver, as such the spectral efficiency of wireless systems can be significantly improved [4,5]. Due to its capability to provide superior spectral efficiency and massive connectivity, NOMA has been applied to many aspects of wireless communication systems, e.g., the Internet of Things network [6], intelligent reflective surface network [7], and mobile edge computing network [8]. User grouping, also known as user pairing, is one fundamental issue of NOMA. The impacts of user grouping on the system performance have been intensively investigated from both performance analysis and system design perspectives. For example, [9] studied the influences of user pairing on the outage probabilities of users in the NOMA systems. [10] proposed to use a branch-and-bound based algorithm to solve the joint user pairing and power allocation optimally with the worst-case computation complexity of NP-hard. To develop a computation-efficient algorithm for the user grouping problem, matching theory based heuristic algorithms were presented in [10–12].

To benefit from additional spatial degrees of freedom, applying the multi-antenna technique to NOMA is an important way to further improve the energy and spectrum efficiency of the systems. To fully exploit the advantages of the multi-antenna technique, beamforming design relying on the channel state information (CSI) between the BS and users has been widely studied in the literature. For example, For more practical applications, the beamforming designs for multi-cell NOMA systems have been investigated in [13–15]. However, the above works are all based on the assumption that the CSI is perfectly known by the system. However, all the aforementioned works assumed that the CSI of the system can be perfectly acquired. In practice, due to, e.g., imperfect channel estimation and finite feedback or channel aging [16], the system can never have perfect CSI. Therefore, it is interesting to investigate the robust design of communication systems under imperfect CSI assumptions. The robust beamforming design has been widely studied in the orthogonal frequency division multiple access systems, and recently, has been extended to the NOMA system [17]. However, to the best of our knowledge, none of the existing works considered the robust beamforming design in multi-cell NOMA systems with interference channels.

In this paper, we study the joint user grouping and robust beamforming design problem of a downlink multi-cell NOMA network with imperfect CSI assumptions. The formulated problem is a mixed-integer programming problem. Due to the integer variables relevant to user grouping, coupling effects of the downlink beamformers, and the infinitely many constraints brought by the imperfect CSI, the resultant problem is challenging to solve. For computational complexity reduction, the original problem is divided into the user grouping subproblem and robust beamforming design subproblem. For the user grouping subproblem, we present a coalition formation game based algorithm to efficiently determine the user grouping. Under imperfect CSI assumption, the worse-case robust beamforming design problem, with signal-to-interference-noise-ratio (SINR) and SIC constraints is also investigated. The coupling effects of the beamformers, both from inter- and intra-group, make the formulated problem nonconvex and challenging to solve. Moreover, the imperfect CSI assumption makes the considered problem much more complicated, since each of the worst-case SINR or SIC constraints corresponds to an infinite number of nonconvex constraints. The contributions of this work are summarized as follows:

- We formulate the joint user grouping and robust beamforming problem in a multi-cell downlink NOMA system under imperfect CSI assumptions. For computation-efficiency consideration, the original problem is decoupled into two subproblems, i.e., the user grouping problem and the robust beamforming problem. To solve the user grouping problem efficiently, we propose to use a coalition formation game based user grouping algorithm.
- For the robust beamforming design problem. To simplify the corresponding problem, we first use the semidefinite relaxation (SDR) based method to transform the quadratic terms to beamformers into linear ones, and then, the S-lemma is invoked to deal with the infinitely many constraints caused by the imperfect CSI. By omitting the rank-one constraints, the reformulated problem refers to the semidefinite programming (SDP) problem, which is convex and can be efficiently solved by the existing optimization tools. Further, to gain more insights into the proposed SDR-based algorithm, a sufficient condition, under which the rank-one optimality of the obtained solution can be guaranteed, is provided and the rank-one optimality of the obtained solution is theoretically proved.

2 System Model and Problem Formulation

Consider a downlink multi-cell network which consists of M multi-antenna BSs and K single-antenna users with $K \geq 2M$. Let $\mathcal{M} \triangleq \{1, 2, ..., M\}$ and $\mathcal{K} \triangleq \{1, 2, ..., K\}$ denote the index sets of BSs and users, respectively. The randomly deployed users are grouped into M non-overlapping user groups, i.e., $\mathcal{C}_n \cap \mathcal{C}_{\hat{n}} = \emptyset, \forall n \neq \hat{n}$ where $\mathcal{C}_n \triangleq \{\mathrm{U}_{n1}, \mathrm{U}_{n2}, ..., \mathrm{U}_{nN_n}\}$ denotes the user set of the n-th group and $N_n = |\mathcal{C}_{\hat{n}}|$ is the number of users in the n-th group, satisfying $\sum_{n=1}^{M} N_n = K$. Each group is served by a BS.

Denote by, $\mathbf{h}_{mnk}^H \in \mathbb{C}^{N_t}$, the channel from BS m to U_{nk}. As discussed previously that the BSs inevitably suffer from CSI errors in practical systems. Thus, imperfect CSI model is considered in this work. Let $\tilde{\mathbf{h}}_{mnk} \in \mathbb{C}^{N_t}$ denote the pre-assumed CSI at the BS m for U_{nk}. Then, the real CSI between BS m and U_{nk} is given by

$$\mathbf{h}_{mnk} = \tilde{\mathbf{h}}_{mnk} + \mathbf{e}_{mnk}, \forall m, n \in \mathcal{M}, k \in \mathcal{C}_n, \tag{1}$$

where $\mathbf{e}_{mnk}$ is the bounded CSI error associated with $\mathbf{h}_{mnk}$. In particular, the bounded CSI error can be modelled by

$$\mathbf{e}_{mnk}^H \mathbf{Q}_{mnk} \mathbf{e}_{mnk} \leq 1, \forall m, n \in \mathcal{M}, k \in \mathcal{C}_n, \tag{2}$$

where $\mathbf{Q} \in \mathbb{H}^{N_t}$ determines the range and shape of the CSI error. For instance, $\mathbf{Q} = \frac{1}{\epsilon^2}\mathbf{I}_{N_t}$ characterizes the popular spherical error model $||\mathbf{e}_{mnk}||^2 \leq \epsilon^2$.

For spectral efficiency consideration, the NOMA technique is applied to each user group. Following the rationale of NOMA, the signals of users in group n are combined by using the superposition coding technique, then the users with stronger channel conditions will first remove the signals for the users with weaker channel conditions by invoking the SIC technique. Let $s_{nk} \in \mathbb{C}$ denote the signal for user k in group n with $\mathbb{E}\{|s_{nk}|^2\} = 1$. So, after superposition coding, the transmit signal of BS n is given by

$$\mathbf{s}_n = \sum_{k=1}^{N_n} \mathbf{w}_{nk} s_{nk}, \forall n \in \mathcal{M}, \tag{3}$$

where $\mathbf{w}_{nk}$ is the beamformer for U_{nk}. The received signal at U_{nk} is give by

$$y_{nk} = \mathbf{h}_{nnk}^H \mathbf{w}_{nk} s_{nk} + \mathbf{h}_{nnk}^H \sum_{i \neq k}^{N_n} \mathbf{w}_{ni} s_{ni} + \sum_{m \neq n}^{M} \mathbf{h}_{mnk}^H \sum_{i=1}^{N_m} \mathbf{w}_{mi} s_{mi} + z_{nk}, \tag{4}$$

where $z_{nk} \in \mathbb{C}$ is the received additive white Gaussian noise at U_{nk} with zero mean and variance σ_{nk}^2. The first item in (4) denotes the desired signal of U_{nk}, the second and third items in (4) denote the intra-cell and inter-cell interference, respectively.

Without loss of generality, we assume that the users, in each group, are ordered by their channel gains in a descending manner, i.e., $|\mathbf{h}_{mn1}|^2 \geq |\mathbf{h}_{mn2}|^2 \geq \cdots \geq |\mathbf{h}_{mnN_n}|^2$. Thus, according to the principle of NOMA, U_{nk} would first remove the information, s_{nj}, for U_{nj} for $j > k$ by using SIC, and then decoding its own information. Based on the above model, the SINR for decoding s_{nj}, $\forall j > k$, at U_{nk} in the SIC process is given by

$$\begin{aligned} &\mathrm{SINR}_{nk}^{s_{nj}} \left(\{\mathbf{w}_{nk}\}_{\forall n,k}, \{\mathbf{h}_{mnk}\}_{\forall m,n,k}\right) \\ &= \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_{nj}|^2}{\sum_{i=1}^{j-1} |\mathbf{h}_{nnk}^H \mathbf{w}_{ni}|^2 + \sum_{m \neq n}^{M} \sum_{i=1}^{N_m} |\mathbf{h}_{mnk}^H \mathbf{w}_{mi}|^2 + \sigma_{nk}^2}, \forall \mathbf{e}_{mnk}^H \mathbf{Q}_{mnk} \mathbf{e}_{mnk} \leq 1. \end{aligned} \tag{5}$$

After s_{nj} is removed from the received signal, the SINR at U_{nk} for decoding s_{nk} is given by

$$\begin{aligned} &\mathrm{SINR}_{nk}^{s_{nk}} \left(\{\mathbf{w}_{nk}\}_{\forall n,k}, \{\mathbf{h}_{mnk}\}_{\forall m,n,k}\right) \\ &= \frac{|\mathbf{h}_{nnk}^H \mathbf{w}_{nk}|^2}{\sum_{i=1}^{k-1} |\mathbf{h}_{nnk}^H \mathbf{w}_{ni}|^2 + \sum_{m \neq n}^{M} \sum_{i=1}^{N_m} |\mathbf{h}_{mnk}^H \mathbf{w}_{mi}|^2 + \sigma_{nk}^2}, \forall \mathbf{e}_{mnk}^H \mathbf{Q}_{mnk} \mathbf{e}_{mnk} \leq 1. \end{aligned} \tag{6}$$

Based on the above model, the energy-efficient joint user-grouping and robust beamforming design problem can be formulated as

$$\min_{\{\mathcal{C}_n\}_{n=1}^M, \{\mathbf{w}_{nk}\}_{n,k}} \quad \sum_{n=1}^{M} \sum_{k=1}^{N_n} ||\mathbf{w}_{nk}||^2 \tag{7a}$$

$$\text{s.t. } \mathcal{C}_n \cap \mathcal{C}_{n'} = \emptyset, \forall n \neq n' \tag{7b}$$

$$\mathrm{SINR}_{nk}^{s_{nj}}\left(\{\mathbf{w}_{nk}\}_{n,k}, \{\mathbf{h}_{mnk}\}_{m,n,k}\right) \geq \gamma_{nj},$$
$$\forall \mathbf{e}_{mnk}^H \mathbf{Q}_{mnk}\mathbf{e}_{mnk} \leq 1, m, n \in \mathcal{M}, k \in \mathcal{C}_n, j > k, \tag{7c}$$

$$\mathrm{SINR}_{nk}^{s_{nk}}\left(\{\mathbf{w}_{nk}\}_{n,k}, \{\mathbf{h}_{mnk}\}_{m,n,k}\right) \geq \gamma_{nk},$$
$$\forall \mathbf{e}_{mnk}^H \mathbf{Q}_{mnk}\mathbf{e}_{mnk} \leq 1, m, n \in \mathcal{M}, k \in \mathcal{C}_n, \tag{7d}$$

$$\sum_{k=1}^{N_n} ||\mathbf{w}_{nk}||^2 \leq P_{\max}, \forall n \in \mathcal{M}, \tag{7e}$$

where $P_{\max}$ is the transmission power budget of BS, (7b) represents that each user will be uniquely assigned into one group, (7c) guarantees the success of SIC procedure at each user, and (7e) signifies the Quality-of-Service requirement for each user.

It is not difficult to verify that problem (7) is a nonconvex optimization problem due to the coupling of the quadratic beamforming vectors and also the channel uncertainty. To efficiently produce a high-quality solution to problem (7), similar to [11,12,18], we will decouple it into two subproblems, i.e., the user grouping problem and the robust beamforming design problem. In the following sections, the user paring problem and the robust beamforming design problem will be solved alternatively. Specifically, the user paring problem is solved by a coalition formation game based algorithm and the robust beamforming design problem is handled by the semidefinite relaxation based algorithm.

3 Coalition Formation Game Based Algorithm for User Grouping

In this section, we solve the user paring problem via the coalition formation game based algorithm. To this end, we first introduce the definition of a coalition formation game.

Definition 1 (Coalition Formation Game [19]*).* A coalition formation game is given by a pair $(\mathcal{K}, \mathcal{P})$, where $\mathcal{K}$ is the set of players, and $\mathcal{P} = [\succ_1, \succ_2, ..., \succ_K]$ denotes the preference profiles, specifying for each player $k \in \mathcal{K}$ its preference relation $\succ_k$.

In this work, the mobile users are viewed as players, and BSs are viewed as coalitions. The set of players is $\mathcal{K} = \{U_1, U_2, ..., U_K\}$. For the sake of distinction, denote $\mathcal{C} = \{C_1, C_2, ..., C_M\}$ as the set of coalitions. Users who are associated with the same BS are referred to as players who join the same coalition. We assume that players are more willing to join the coalition yielding higher social welfare. In other words, they are cooperative rather than rational (selfish).

At the beginning of coalition formation, each player proposes to joint a coalition according to its preference list which is built based on the consumed transmit power and can be described as follows:

$$\mathrm{U}_{PF}(k) = [\mathrm{C}_{\mathcal{C}}(1), \mathrm{C}_{\mathcal{C}}(2), \ldots, \mathrm{C}_{\mathcal{C}}(j), \ldots, \mathrm{C}_{\mathcal{C}}(K)], \forall k \in \mathcal{K}, \tag{8}$$

where the lists are sorted in an ascending order of the consumed power. The preference lists satisfy the condition that the BS who can provide the minimum consumed power in the coalition set will be ranked as the first one. This can be described as

$$\mathrm{U}(k)_j \succ_k \mathrm{U}(k)_{j'},\ \forall k \in \mathcal{K} \tag{9}$$

for all k, j in $\mathcal{K}$ and $\mathcal{C}$ respectively. This means that player k prefers to form coalition with BS j to j' if the total consumed power of user pair $\langle \mathrm{U}_i, \mathrm{C}_j \rangle$ is less than that of the user pair $\langle \mathrm{U}_i, \mathrm{C}_{j'} \rangle$.

To better handle the minimization of the total transmit power, we introduce the swap operation between two players, e.g., U_i and $\mathrm{U}_{i'}$, in $\mathcal{K}$ and the corresponding two matched coalitions, e.g., $\mathcal{F}(\mathrm{U}_{i'})$ and $\mathcal{F}(\mathrm{U}_{i'})$, in $\mathcal{C}$ in the coalition formation process. The swap operation can be defined by

$$\begin{aligned}\mathcal{F}_i^{i'} = &\mathcal{F} \backslash \{\langle \mathrm{U}_i, \mathcal{F}(\mathrm{U}_i)\rangle, \langle \mathrm{U}_{i'}, \mathcal{F}(\mathrm{U}_{i'})\rangle\} \\ &\cup \{\langle \mathrm{U}_i, \mathcal{F}(\mathrm{U}_{i'})\rangle, \langle \mathrm{U}_{i'}, \mathcal{F}(\mathrm{U}_i)\rangle\},\end{aligned} \tag{10}$$

where U_i and $\mathrm{U}_{i'}$ switch the matched BSs while keeping other coalitions invariant. Based on the swap operation, we define the swap blocking pair as below:

Definition 2 (swap blocking pair). Given a matching $\mathcal{F}$ and two user pairs $\langle \mathrm{U}_i, \mathrm{C}_j \rangle$ and $\langle \mathrm{U}_{i'}, \mathrm{C}_{j'} \rangle$. If there exists a new coalition formation $\mathcal{F}_i^{i'}$ such that the total transmit power of the new user pairs gets a decrease, then the swap operation is approved, and $\langle \mathrm{U}_i, \mathrm{C}_{j'} \rangle$, $\langle \mathrm{U}_{i'}, \mathrm{C}_j \rangle$ are swap blocking pairs under the coalition formation $\mathcal{F}_i^{i'}$.

Note that the above definition implies that there is a benefit by exchanging the coalition formation of $\langle \mathrm{U}_i, \mathrm{C}_j \rangle$, $\langle \mathrm{U}_{i'}, \mathrm{C}_{j'} \rangle$ and this operation will not hurt the benefit of other coalitions. Thus, with this new coalition formation, the total transmission power of the system can be decreased. Based on the above definitions, the coalition formation procedure can be described as follows. Firstly, each user can be associated with a BS to form a potential swap blocking pair. Then the system will check whether these two user pairs can get a benefit by exchanging their current coalition formation. The users will keep performing approved swap operations until they reach a stable status, which is known as *Nash stable*. Its definition is described below.

Definition 3 (Nash-stable [20]). A coalition partition is Nash-stable if and only if the following formula holds:

$$\mathrm{U}(k)_i \succ_k \mathrm{U}_{\mathcal{K}}(k)_j, \forall k \in \mathcal{K}, i, j \in \mathcal{C}, i \neq j, \tag{11}$$

which implies that, in a Nash-stable partition, any player is more (at least as well) willing to stay in its current coalition than to join other coalitions. In other words, there does not exist such a coalition that any player strictly prefers over its current one.

As per the above definitions, we can develop a coalition formation game based algorithm to solve the user pairing problem. The detailed procedure is summarized in Algorithm 1. It is not difficult to verify that the coalition formation game based algorithm can converge to a Nash-stable partition.

Algorithm 1. Coalition formation game based algorithm for user groping

```
Initialize preference lists for players as U_PF(i), i ∈ 𝒦.
Stage I : initial coalition formation
for i = 1 to K do
    Each player U_i requests to its preferred BS j according to U_PF(i).
    if The number of joined players does not exceed the maximum of C_j then
        Player U_i joins the coalition formation of C_j.
    else
        U_i will join the other coalition according to U_PF(i).
    end if
end for
Stage II : swap operation
repeat
    Search the coalition formation to check whether there exists swap blocking pair.
    if there is swap blocking pair then
        Swap the user pair and update the current coalition formation.
    end if
until There is no swap blocking pair in the coalition formation.
```

The computational complexity of Algorithm 1 consists of two parts. One is due to the initial coalition formation phase, which has a complexity order of $\mathcal{O}(K)$. The other is due to the swap operations in the second phase. For each user, there exist $M-1$ possible BSs to do swapping, thus the complexity order is given by $\mathcal{O}(K(M-1))$. Therefore, the total complexity is $\mathcal{O}(MK)$. Compared to the optimal strategy using exhaustive search, which has a complexity order of $\mathcal{O}(M^K)$, the computational complexity of the proposed coalition formation game based algorithm is dramatically decreased.

4 Solve the Robust Beamforming Design Problem

Once the user grouping is determined, problem (7) boils down to the robust beamforming design problem. In this section, we will propose to use the SDR method to solve the beamforming design problem, and the optimality of the obtained solution is analyzed.

4.1 SDR-Based Algorithm Design

To apply the SDR method, we first introduce a set of rank-one matrix $\mathbf{W}_{n,k} = \mathbf{w}_{nk}\mathbf{w}_{nk}^H, \forall n, k$. Then, by ignoring the rank-one constraint on the matrix, the robust beamforming problem can be relaxed as

$$\min_{\{\mathbf{W}_{nk}\}_{\forall n,k}} \sum_{n=1}^{M}\sum_{k=1}^{N_n} \mathrm{Tr}(\mathbf{W}_{nk}) \tag{12a}$$

$$\text{s.t.}\quad \mathbf{h}_{nnk}^H\left(\frac{1}{\gamma_{nj}}\mathbf{W}_{nj} - \sum_{i=1}^{j-1}\mathbf{W}_{nk}\right)\mathbf{h}_{nnk} \geq \sum_{m\neq n}^{M}\mathbf{h}_{mnk}^H\left(\sum_{i=1}^{N_m}\mathbf{W}_{mi}\right)\mathbf{h}_{mnk} + \sigma_{nk}^2,$$

$$\forall \mathbf{e}_{mnk}^H\mathbf{Q}_{mnk}\mathbf{e}_{mnk} \leq 1, m, n, j > k, \tag{12b}$$

$$\mathbf{h}_{nnk}^H\left(\frac{1}{\gamma_{nk}}\mathbf{W}_{nk} - \sum_{i=1}^{k-1}\mathbf{W}_{ni}\right)\mathbf{h}_{nnk} \geq \sum_{m\neq n}^{M}\mathbf{h}_{mnk}^H\left(\sum_{i=1}^{N_m}\mathbf{W}_{mi}\right)\mathbf{h}_{mnk} + \sigma_{nk}^2,$$

$$\forall \mathbf{e}_{mnk}^H\mathbf{Q}_{mnk}\mathbf{e}_{mnk} \leq 1, m, n, k, \tag{12c}$$

$$\sum_{k=1}^{N_n}\mathrm{Tr}(\mathbf{W}_{nk}) \leq P_{\max}, \forall n, \tag{12d}$$

$$\mathbf{W}_{nk} \succeq \mathbf{0}, \forall n, k, \tag{12e}$$

which is a convex problem as the objective function and constraints are linear. However, it is still computationally intractable due to the infinite number of constraints. Next, to make problem (12) tractable, we propose the following lemma.

Lemma 1. *The infinitely many constraints in* (12b) *and* (12c) *can be equivalently recast into the following finite number of linear matrix inequalities:*

$$\mathbf{\Phi}_{nj}\left(\{\mathbf{W}_{ni}\}_{i=1}^{j}, \{\theta_{mnk}\}_m, \lambda_{mnk}\right)$$
$$\triangleq \begin{bmatrix} \mathbf{A}_{nj} + \lambda_{nnk}\mathbf{Q}_{nnk} & \mathbf{A}_{nj}\hat{\mathbf{h}}_{nnk} \\ \hat{\mathbf{h}}_{nnk}^H\mathbf{A}_{nj} & \hat{\mathbf{h}}_{nnk}^H\mathbf{A}_{nj}\hat{\mathbf{h}}_{nnk} - \sum_{m\neq n}^{M}\theta_{mnk} - \sigma_{nk}^2 - \lambda_{nnk} \end{bmatrix} \succeq \mathbf{0}, \forall m, n, j > k, \tag{13a}$$

$$\mathbf{\Psi}_{nk}\left(\{\mathbf{W}_{ni}\}_{i=1}^{k}, \{\theta_{mnk}\}_m, \lambda_{mnk}\right)$$
$$\triangleq \begin{bmatrix} \mathbf{B}_{nk} + \lambda_{nnk}\mathbf{Q}_{nnk} & \mathbf{B}_{nk}\hat{\mathbf{h}}_{nnk} \\ \hat{\mathbf{h}}_{nnk}^H\mathbf{B}_{nk} & \hat{\mathbf{h}}_{nnk}^H\mathbf{B}_{nk}\hat{\mathbf{h}}_{nnk} - \sum_{m\neq n}^{M}\theta_{mnk} - \sigma_{nk}^2 - \lambda_{nnk} \end{bmatrix} \succeq \mathbf{0}, \forall m, n, k, \tag{13b}$$

$$\mathbf{\Omega}_{mnk}\left(\{\mathbf{W}_{ni}\}_{i=1}^{N_n}, \theta_{mnk}, \lambda_{mnk}\right)$$
$$\triangleq \begin{bmatrix} -\mathbf{C}_m + \lambda_{mnk}\mathbf{Q}_{mnk} & -\mathbf{C}_m\hat{\mathbf{h}}_{mnk} \\ -\hat{\mathbf{h}}_{mnk}^H\mathbf{C}_m & -\hat{\mathbf{h}}_{mnk}^H\mathbf{C}_m\hat{\mathbf{h}}_{mnk} + \theta_{mnk} + \lambda_{mnk} \end{bmatrix} \succeq \mathbf{0}, \forall m, n, k, \tag{13c}$$

where $\lambda_{mnk} > 0, \forall m, n \in \mathcal{M}, k \in \mathcal{C}_n$ *are auxiliary variables and*

$$\mathbf{A}_{nj} = \frac{1}{\gamma_{nj}}\mathbf{W}_{nj} - \sum_{i=1}^{j-1}\mathbf{W}_{nk}, \quad \mathbf{B}_{nk} = \frac{1}{\gamma_{nk}}\mathbf{W}_{nk} - \sum_{i=1}^{k-1}\mathbf{W}_{ni},$$

$$\mathbf{C}_m = \sum_{i=1}^{N_m}\mathbf{W}_{mi},$$

$$\theta_{mnk} = \max_{\forall \mathbf{e}_{mnk}^H \mathbf{Q}_{mnk}\mathbf{e}_{mnk} \le 1,} \mathbf{h}_{mnk}^H \left(\sum_{i=1}^{N_m}\mathbf{W}_{mi}\right)\mathbf{h}_{mnk}.$$

Proof. The key idea to prove Lemma 1 is using S-lemma to handle the infinitely many constraints. We omit it here due to the space limitation.

Based on Lemma 1, the SDP problem (12) can be equivalently reformulated as

$$\min_{\{\mathbf{W}_{nk}\},\{\theta_{mnk}\},\{\lambda_{mnk}\}} \sum_{n=1}^{M}\sum_{k=1}^{N_n}\mathrm{Tr}(\mathbf{W}_{nk}) \tag{14a}$$

$$\text{s.t.} \quad \mathbf{\Phi}_{nj}\left(\{\mathbf{W}_{ni}\}_{i=1}^{j}, \{\theta_{mnk}\}_m, \lambda_{nnk}\right) \succeq \mathbf{0}, \forall m, n, j > k, \tag{14b}$$

$$\mathbf{\Psi}_{nk}\left(\{\mathbf{W}_{ni}\}_{i=1}^{k}, \{\theta_{mnk}\}_m, \lambda_{nnk}\right) \succeq \mathbf{0}, \forall m, n, k, \tag{14c}$$

$$\mathbf{\Omega}_{mnk}\left(\{\mathbf{W}_{ni}\}_{i=1}^{N_n}, \theta_{mnk}, \lambda_{mnk}\right) \succeq \mathbf{0}, \forall m, n, k, \tag{14d}$$

$$\sum_{k=1}^{N_n}\mathrm{Tr}(\mathbf{W}_{nk}) \le P_{\max}, \forall n, \tag{14e}$$

$$\mathbf{W}_{nk} \succeq \mathbf{0}, \lambda_{mnk} \ge 0, \forall n, k, \tag{14f}$$

which is an SDP and thus can be efficiently solved by `CVX`.

Remind that problem (12) is a relaxed version of the original robust beamforming design problem by ignoring the rank-one constraints. Thus, one important issue in solving problem (12) is to verify whether the obtained matrices from solving problem (14) are rank-one. If it is true, then the optimal beamforming vectors can be obtained by simply applying singular value decomposition to the obtained matrices. Hence, it is interesting to explore the conditions under which solving problem (14) can produce rank-one solutions.

4.2 Rank-One Optimality Analysis

The following Lemma provides a condition that can guarantee the rank optimality of problem (14).

Lemma 2. *Suppose that problem (14) is feasible, the rank-one optimality can be guaranteed if* $\mathbf{Q}_{nnk} = \infty\mathbf{I}_{N_t}$ *for all* n, k*, i.e., no intra-cell CSI error.*

Proof. As perfect intra-cell CSI errors are available at the BSs, problem (12) degrades to

$$\min_{\{\mathbf{W}_{nk}\}\forall n,k} \sum_{n=1}^{M}\sum_{k=1}^{N_n} \mathrm{Tr}(\mathbf{W}_{nk}) \tag{15a}$$

$$\text{s.t. } \hat{\mathbf{h}}_{nnk}^H \left(\frac{1}{\gamma_{nj}}\mathbf{W}_{nj} - \sum_{i=1}^{j-1}\mathbf{W}_{nk}\right)\hat{\mathbf{h}}_{nnk}$$

$$\geq \max_{\forall \mathbf{e}_{mnk}^H \mathbf{Q}_{mnk}\mathbf{e}_{mnk}\leq 1,} \left\{\sum_{m\neq n}^{M} \hat{\mathbf{h}}_{mnk}^H \left(\sum_{i=1}^{N_m}\mathbf{W}_{mi}\right)\hat{\mathbf{h}}_{mnk}\right\} + \sigma_{nk}^2, \forall n,k, \tag{15b}$$

$$\hat{\mathbf{h}}_{nnk}^H \left(\frac{1}{\gamma_{nk}}\mathbf{W}_{nk} - \sum_{i=1}^{k-1}\mathbf{W}_{ni}\right)\hat{\mathbf{h}}_{nnk}$$

$$\geq \max_{\forall \mathbf{e}_{mnk}^H \mathbf{Q}_{mnk}\mathbf{e}_{mnk}\leq 1,} \left\{\sum_{m\neq n}^{M} \hat{\mathbf{h}}_{mnk}^H \left(\sum_{i=1}^{N_m}\mathbf{W}_{mi}\right)\hat{\mathbf{h}}_{mnk}\right\} + \sigma_{nk}^2, \forall n,k, \tag{15c}$$

$$\sum_{k=1}^{N_n}\mathrm{Tr}(\mathbf{W}_{nk}) \leq P_{\max}, \forall n,k, \tag{15d}$$

$$\mathbf{W}_{nk} \succeq \mathbf{0}, \forall n,k. \tag{15e}$$

Again, by applying *S-Lemma* to the right-hand-sides of (15b) and (15c), problem (15) can be equivalently reformulated as:

$$\min_{\{\mathbf{W}_{nk}\},v_{mnk},\lambda_{mnk}} \sum_{n=1}^{M}\sum_{k=1}^{N_n} \mathrm{Tr}(\mathbf{W}_{nk}) \tag{16a}$$

$$\text{s.t. } \hat{\mathbf{h}}_{nnk}^H \left(\frac{1}{\gamma_{nj}}\mathbf{W}_{nj} - \sum_{i=1}^{j-1}\mathbf{W}_{nk}\right)\hat{\mathbf{h}}_{nnk} \geq \sum_{m=1,m\neq n} v_{mnk} + \sigma_{nk}^2, \forall n,k, \tag{16b}$$

$$\hat{\mathbf{h}}_{nnk}^H \left(\frac{1}{\gamma_{nk}}\mathbf{W}_{nk} - \sum_{i=1}^{k-1}\mathbf{W}_{ni}\right)\hat{\mathbf{h}}_{nnk} \geq \sum_{m=1,m\neq n} v_{mnk} + \sigma_{nk}^2, \forall n,k, \tag{16c}$$

$$\begin{bmatrix} -\sum_{i=1}^{N_m}\mathbf{W}_{mi}+\lambda_{mnk}\mathbf{Q}_{mnk} & -\sum_{i=1}^{N_m}\mathbf{W}_{mi}\hat{\mathbf{h}}_{mnk} \\ -\hat{\mathbf{h}}_{mnk}^H\sum_{i=1}^{N_m}\mathbf{W}_{mi} & -\hat{\mathbf{h}}_{mnk}^H\sum_{i=1}^{N_m}\mathbf{W}_{mi}\hat{\mathbf{h}}_{mnk}+v_{mnk}+\lambda_{mnk} \end{bmatrix} \succeq \mathbf{0}, \forall n,k, \tag{16d}$$

$$\sum_{k=1}^{N_n}\mathrm{Tr}(\mathbf{W}_{nk}) \leq P_{\max}, \forall n,k, \tag{16e}$$

$$\mathbf{W}_{nk} \succeq \mathbf{0}, \forall n,k. \tag{16f}$$

We prove Lemma 2 by using the KKT conditions of (16). In particular, let $\{\delta_{nj}^\star\}$, $\{\epsilon_{nj}^\star\}$, $\mathbf{Y}_{mi}^\star$ and $\mathbf{Z}_{nk}^\star$ denote the optimal dual variables associated with

(16b), (16c), (16d) and (16f), respectively. The KKT conditions related to $\mathbf{W}_{nk}^{\star}$ are as follows:

$$\mathbf{Z}_{nk}^{\star}\mathbf{W}_{nk}^{\star} = \mathbf{0}, \tag{17a}$$

$$\mathbf{Z}_{nk}^{\star} = \mathbf{I}_{N_t} - \left(\frac{\delta_{nk}^{\star}}{\gamma_{nj}} + \frac{\epsilon_{nk}^{\star}}{\gamma_{nk}}\right)\hat{\mathbf{h}}_{nnk}\hat{\mathbf{h}}_{nnk}^{H}, \tag{17b}$$

$$\mathbf{Z}_{nk}^{\star} \succeq \mathbf{0}, \mathbf{W}_{nk}^{\star} \succeq \mathbf{0} \tag{17c}$$

First, note that, from (16c), we can conclude that $\mathbf{W}_{nk}^{\star} \succ \mathbf{0}$. Otherwise, we have

$$-\hat{\mathbf{h}}_{nnk}^{H}\sum_{i=1}^{k-1}\mathbf{W}_{ni}\hat{\mathbf{h}}_{nnk} \geq \sum_{m=1,m\neq n} \upsilon_{mnk} + \sigma_{nk}^{2},$$

which violates the fact that $\upsilon_{mnk} \geq 0,\ \sigma_{nk}^{2} > 0$ and $\mathbf{W}_{ni}$ are positive semidefinite. Then, also note that

$$0 = \operatorname{rank}\left(\mathbf{Z}_{nk}^{\star}\mathbf{W}_{nk}^{\star}\right) \geq \operatorname{rank}\left(\mathbf{Z}_{nk}^{\star}\right) + \operatorname{rank}\left(\mathbf{W}_{nk}^{\star}\right) - N_t. \tag{18}$$

So, we have $\operatorname{rank}\left(\mathbf{W}_{nk}^{\star}\right) \leq N_t - \operatorname{rank}\left(\mathbf{Z}_{nk}^{\star}\right)$. Therefore, to prove that $\mathbf{W}_{nk}$ is rank-one, it suffices to prove that

$$\begin{aligned}\operatorname{rank}\left(\mathbf{Z}_{nk}^{\star}\right) &= \operatorname{rank}\left(\mathbf{I}_{N_t} - \left(\frac{\delta_{nk}^{\star}}{\gamma_{nj}} + \frac{\epsilon_{nk}^{\star}}{\gamma_{nk}}\right)\hat{\mathbf{h}}_{nnk}\hat{\mathbf{h}}_{nnk}^{H}\right) \\ &= N_t - 1.\end{aligned} \tag{19}$$

Notice that $\hat{\mathbf{h}}_{nnk}$ is a non-zero vector, thus $\operatorname{rank}\left(\hat{\mathbf{h}}_{nnk}\hat{\mathbf{h}}_{nnk}^{H}\right) = 1$. So, we have $\operatorname{rank}\left(\mathbf{Z}_{nk}^{\star}\right) \geq N_t - 1$. Finally, based on (18) and $\mathbf{W}_{nk}^{\star} \succ \mathbf{0}$, we can conclude that $\operatorname{rank}\left(\mathbf{Z}_{nk}^{\star}\right) = N_t - 1$ and $\operatorname{rank}\left(\mathbf{W}_{nk}^{\star}\right) = 1$. This completes the proof.

We note that, for a general setup, the solutions of problem (14) *may not* be rank-one. In these cases, one can resort to the Gaussian randomization method to produce approximated beamforming vectors based on the obtained non-rank-one solutions [21].

5 Simulation Results

In this section, numerical simulations are present to verify the performance of the proposed transmission schemes and algorithms. The large-scale path loss is set to be $PL_{mnk} = 128.1 + 37.6\log_{10}(d_{mnk})$ with d_{mnk} (in Km) denoting the distance between the BS m and U_{nk}, while the small-scale fading is modelled by the standard Rayleigh fading. The White noise power density is -174 dBm and the bandwidth is 10 MHz. The power budget of each BS is set as $P_{\max} = 36$ dBm. The users are randomly deployed in a 500 m $\times$ 500 m square area. Without loss of generality, the QoS requirements of the users are the same, i.e., $\gamma_{mnk} = \gamma, \forall m, n \in \mathcal{M}, k \in \mathcal{C}_n$. The spherical error model

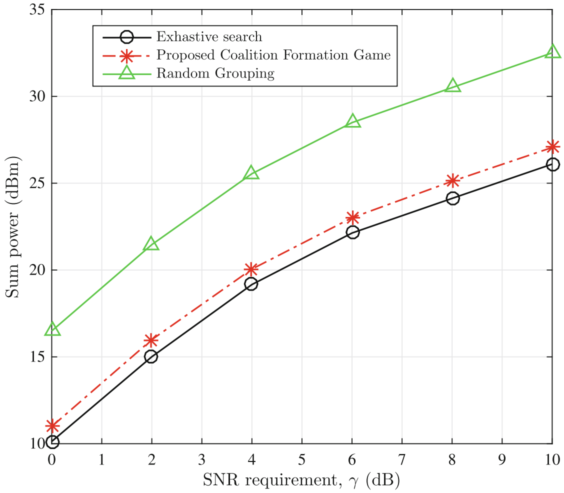

Fig. 1. The performance comparison of the centralized and decentralized algorithm with $M = 4$, $N = 3$, $N_t = 4$, $\epsilon = 0.05$ and $\gamma = 3$ dB.

is used and we assume that the error bounds of all users are the same, i.e., $\mathbf{Q}_{mnk} = \mathbf{Q} = \frac{1}{\epsilon^2}\mathbf{I}_{N_t}, \forall m, n \in \mathcal{M}, \forall k \in \mathcal{C}_n$. For comparison, we introduce some other user grouping approaches and transmission schemes, namely, the exhaustive search based user grouping approach, OMA transmission scheme, and non-robust NOMA transmission scheme, which are described as follows:

- **Exhaustive Search Based User grouping Approach:** In this approach, all the possible combinations of the user group are considered, the optimal user grouping is the one yielding the smallest transmit power.
- **Non-robust NOMA transmission scheme:** In this scheme, the perfect CSI of each user is assumed to be available at the BSs. It is not difficult to verify that the non-robust beamforming problem can be formulated as a SOCP and thus can be solved optimally.

We first evaluate the performance of the proposed coalition formation game based user grouping approach in Fig. 1. The exhaustive search based optimal user grouping algorithm is used as the performance benchmark. As it can be seen that the proposed coalition formation game based approach can achieve near-optimal performance under the system settings. While the random grouping based approach, in which the users are randomly selected to form a group, yields the worst performance.

In Fig. 2, we compare the performance of robust and non-robust designs versus different QoS requirements of users. It can be observed that as a price for a worst-case performance guarantee, the robust designs require higher transmission power than the non-robust design. From Fig. 3, we can also find that, with different CSI error bounds, the non-robust design would underestimate the required power for reliable transmissions. Meanwhile, the sum power increases

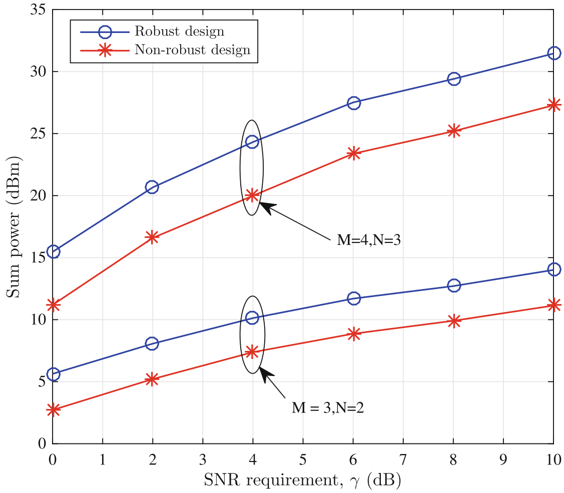

Fig. 2. The performance comparison of robust and non-robust designs versus different γ with $N_t = 4$, and $\epsilon = 0.05$.

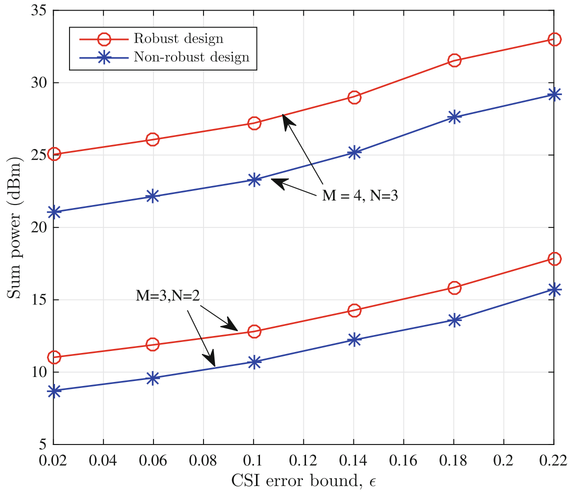

Fig. 3. The performance comparison of robust and non-robust designs versus different ϵ with $N_t = 4$, and $\gamma = 3$ dB.

with the increase of the CSI error bound. The reason is that a larger CSI error requires more transmission power to guarantee the QoS requirements of users.

6 Conclusions

In this work, we have investigated the joint user grouping and robust beamforming design problem in a downlink multi-cell NOMA network under imperfect

CSI assumptions. The formulated sum power minimization problem was shown to be challenging to solve due to the integer variables of user grouping, coupling effects of beamformers, and infinitely many constraints brought by the imperfect CSI. For computational complexity reduction, the original problem was decoupled into a user grouping subproblem and robust beamforming subproblem. A coalition formation game based algorithm has been proposed to solve the user grouping problem. While for the robust beamforming design problem, we proposed an SDR-based suboptimal algorithm was presented and a sufficient condition under which the SDR-based approach could guarantee to produce an optimal solution was presented. Simulation results have provided some interesting results. For example, the proposed coalition game based algorithm can perform close to the exhaustive search based scheme; The robust design would require more transmission power compared to the non-robust design.

Acknowledgements. The work of Y. Xu was supported by the China Postdoctoral Science Foundation under Grants No. 2021M693100 and Guangdong Basic and Applied Basic Research Foundation under Grants No. 2021A1515110018. The work of D. Cai was supported by National Natural Science Foundation of China (NSFC) under Grant No. 62001190 and the China Postdoctoral Science Foundation under Grants No. 2021M691249. The work of Z. Dong was supported by National Science Foundation of China under Grant 61561046 and the Key Research & Development and Transformation Plan of Science and Technology Program for Tibet Autonomous Region (No. XZ201901-GB-16).

References

1. Forecast, C.V.: Cisco visual networking index: global mobile data traffic forecast update, 2016–2017. White Paper, Cisco San Jose, CA, USA (2017)
2. Elbayoumi, M., Kamel, M., Hamouda, W., Youssef, A.: NOMA-assisted machine-type communications in UDN: state-of-the-art and challenges. IEEE Commun. Surv. Tutor. **22**, 1 (2020)
3. Ding, Z., Lei, X., Karagiannidis, G.K., Schober, R., Yuan, J., Bhargava, V.K.: A survey on non-orthogonal multiple access for 5G networks: research challenges and future trends. IEEE J. Sel. Areas Commun. **35**(10), 2181–2195 (2017)
4. Fang, F., Zhang, H., Cheng, J., Leung, V.C.M.: Energy-efficient resource allocation for downlink non-orthogonal multiple access network. IEEE Trans. Commun. **64**(9), 3722–3732 (2016)
5. Xu, Y., et al.: Joint beamforming and power-splitting control in downlink cooperative SWIPT NOMA systems. IEEE Trans. Signal Process. **15**(18), 4874–4886 (2017)
6. Wang, J., Kang, X., Sun, S., Liang, Y.: Throughput maximization for peer-assisted wireless powered IoT NOMA networks. IEEE Trans. Wireless Commun. **19**, 1 (2020)
7. Fang, F., Xu, Y., Pham, Q.-V., Ding, Z.: Energy-efficient design of IRS-NOMA networks. IEEE Trans. Veh. Techn. **69**(11), 14 088–14 092 (2020)
8. Fang, F., Xu, Y., Ding, Z., Shen, C., Peng, M., Karagiannidis, G.K.: Optimal resource allocation for delay minimization in NOMA-MEC networks. IEEE Trans. Commun. **68**(12), 7867–7881 (2020)

9. Ding, Z., Fan, P., Poor, H.V.: Impact of user pairing on 5G nonorthogonal multiple-access downlink transmissions. IEEE Trans. Veh. Techn. **65**(8), 6010–6023 (2016)
10. Cui, J., Liu, Y., Ding, Z., Fan, P., Nallanathan, A.: Optimal user scheduling and power allocation for millimeter wave NOMA systems. IEEE Trans. Wireless Commun. **17**(3), 1502–1517 (2018)
11. Di, B., Song, L., Li, Y.: Sub-channel assignment, power allocation, and user scheduling for non-orthogonal multiple access networks. IEEE Trans. Wireless Commun. **15**(11), 7686–7698 (2016)
12. Xu, Y., Cai, D., Fang, F., Ding, Z., Shen, C., Zhu, G.: Outage constrained power efficient design for downlink NOMA systems with partial HARQ. IEEE Trans. Commun. **68**(8), 5188–5201 (2020)
13. Ni, W., Liu, X., Liu, Y., Tian, H., Chen, Y.: Resource allocation for Multi-Cell IRS-Aided NOMA networks. IEEE Trans. Wireless Commun. **20**(7), 4253–4268 (2021)
14. Chen, G., Qiu, L., Ren, C.: On the performance of cluster-based MIMO-NOMA in Multi-Cell dense networks. IEEE Trans. Commun. **68**(8), 4773–4787 (2020)
15. Ding, J., Cai, J.: Two-side coalitional matching approach for joint MIMO-NOMA clustering and BS selection in Multi-Cell MIMO-NOMA systems. IEEE Trans. Wireless Commun. **19**(3), 2006–2021 (2020)
16. Cai, D., Xu, Y., Fang, F., Ding, Z., Fan, P.: On the impact of time-correlated fading for downlink NOMA. IEEE Trans. Commun. **67**(6), 4491–4504 (2019)
17. Wang, Z., Ng, D.W.K., Wong, V.W.S., Schober, R.: Robust beamforming design in C-RAN with sigmoidal utility and capacity-limited backhaul. IEEE Trans. Wireless Commun. **16**(9), 5583–5598 (2017)
18. Liang, W., Ding, Z., Li, Y., Song, L.: User pairing for downlink non-orthogonal multiple access networks using matching algorithm. IEEE Trans. Commun. **65**(12), 5319–5332 (2017)
19. Saad, W., Han, Z., Debbah, M., Hjorungnes, A., Basar, T.: Coalitional game theory for communication networks. IEEE Signal Process. Mag. **26**(5), 77–97 (2009)
20. Bogomolnaia, A., Jackson, M.O.: The stability of hedonic coalition structures. Games Econ. Behav. **38**(2), 201–230 (2002)
21. Luo, Z.Q., Ma, W.K., So, A.M.C., Ye, Y., Zhang, S.: Semidefinite relaxation of quadratic optimization problems. IEEE Signal Process. Mag. **27**(3), 20–34 (2010)