Equilibrium Strategies and Social Welfare in Cognitive Radio Networks with Imperfect Spectrum Sensing

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Abstract. Spectrum sensing is essential in opportunistic cognitive radio (CR) systems for detecting primary users' (PUs') activities and protecting the PUs against harmful interference. By extending the perfect spectrum sensing in the literature to more realistic situations, a new random sensing error framework of secondary users (SUs) is proposed to study the SUs' behavior so as to make the best benefit for the CR system in this paper. A novel queueing-game theoretical model is formulated first and then various system stationary performance measures are procured. Furthermore, the SU's equilibrium joining strategies are obtained, the throughput of SUs is derived, and the CR system's social welfare's monotonous in terms of the sensing error and the SU's request frequency are characterized. Particularly, three interesting but counter-intuitive results are observed as below: (i) the expected delay for joining SUs can be non-monotone in their effective arrival rate; (ii) multiple equilibrium joining strategies of SUs can always exist; and (iii) the spectrum sensing error does not necessarily worsen the CS system's social welfare, i.e., some sensing error in the system may possibly lead to more efficient outcomes in terms of throughput and social welfare. The results and observations offered in this paper are expected to extend spectrum sensing research in a more efficient way to better provide CR system services to various users.

Keywords: Cognitive radio · Queueing game · Sensing error · Equilibrium strategy · Social welfare

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1 Introduction

Cognitive radio (CR) technology has been shown to be prevalent in utilizing the wireless spectrum resources [6] and [20]. In CR systems, SUs are allowed to use the unoccupied spectrum in an opportunistic way without affecting the primary user (PU), which can significantly improve the efficiency of networks. In principle, CR systems are mainly classified as two classes: underlay CR and opportunistic (or interweave) CR systems. The former does not require spectrum sensing to detect PUs' activities, they need coordination between PUs and SUs to obtain channel state information (CSI). While opportunistic CR systems do not require coordination between PUs and SUs to acquire CSI corresponding to SU-PU link, they necessitate spectrum sensing to monitor and detect PUs' activities.

In opportunistic CR systems, SUs decide to access the spectrum band if they sense the spectrum is idle or the system is not so congested to some extent. Otherwise, when the system is sensed to be congested, SUs may choose to balk upon arrival. That is, the spectrum sensing for SUs plays an important role in the CR networks. In most previous works investigating the spectrum sharing problem, the SUs are believed to be capable in distinguishing the exact state of spectrum and a perfect sensing assumption is made, in which no collision happens. Nevertheless, owing to the inherent feedback delays, estimation errors, quantization errors caused by the unreliable channel or signal interferences, there are inevitable sensing errors in practice, which lead to heavy interference to the PUs and negatively impact the performance of CR networks. It is for these indispensable reasons, there has been a great deal of related work in the research of CR system so far. We summarize several recent publications that are closely related to our research in this paper as follows:

Devarajan et al. [1] studied the strategic behaviour of users in an Internet of Things (IoT) system that is impeded by an unreliable server, and equilibrium joining probabilities for individual strategies and optimum joining probabilities for social welfare were obtained. Ni et al. [7] investigated a general situation wherein a datacenter can determine the cost of using resources and a Cloud service user can decide whether it will pay the price for the resource or not for an incoming task, and verified that the optimal policy for admitting tasks is a threshold policy. Toma et al. [11] provided a detailed analysis of a broad range of primary channel statistics under Imperfect Spectrum Sensing (ISS) and found a set of closed-form expressions for the calculated statistics under ISS as a function of the original primary channel statistics, probability of error, and the employed sensing period. Wang et al. [13] carried out an extensive analysis of the non-cooperative and cooperative joining behaviors of SUs in a CR system with a single PU band, which is the first time that the strategic behavior of SUs has been studied with a retrial phenomenon in the system from an economics point of view. Furthermore, Wang et al. [12] considered the SUs' joining behavior in cognitive radio network with a single bandwidth under imperfect spectrum sensing by using retrial queueing system formulation with server breakdowns. Wang et al. [14] conducted the game-theoretic analysis of the behavior of SUs in a cognitive radio system with a single primary user band and sensing failures, and the equilibrium behavior of SUs as well as the socially optimal strategies of SUs were obtained. Wang et al. [16] investigated joining strategies and admission control policy for SUs with retrial behavior in a CR system where a single PU coexists with multiple SUs. Wang et al. [15] discussed the decentralized and centralized behavior of SUs with retrial phenomenon under two situations in which SUs have no information or partially observable information. Wang et al. [18] investigated a CR system with two classes of service requests (PU and SU requests). Equilibrium probability strategies of PUs and SUs in no queue length information, and equilibrium Nash balance thresholds for PUs and SUs in partial queue length information and full queue length information cases are obtained, respectively. Yazdani et al. [19] considered an opportunistic cognitive radio system in which secondary transmitter is equipped with a reconfigurable antenna (RA), and established a lower bound on the achievable rates of opportunistic cognitive radio systems using RAs With imperfect sensing and channel estimation. Zhang et al. [21] investigated the price-based spectrum access control policy that characterizes the network operator's provision to heterogeneous and delay-sensitive SUs through pricing strategies. Under different information levels, Zhu et al. [23] investigated the equilibrium strategic behaviors of PU and SUs and attained the optimal service rates of PU under different information structures from the perspective of the service provider. Zhu et al. [22] employed a two-server preemptive priority queue to study CR systems with multiple spectrums, and they obtained a joint optimal pricing strategy to maximize profit for the service provider as well as an optimal pricing strategy to maximize social welfare from the social administrator's viewpoint. For more details on the topic, we refer to the survey paper by PalunčIć et al. [8] and the references therein.

To characterize the underlying data transmission mechanism of CR systems, various queueing models have been established extensively which are beneficial for analyzing the CR systems. In particular, queueing models are recently applied to study the performance of CR systems with interruptions when the system congestion is considered, see [4,5]. In the single spectrum setting, the effect of PU on SUs was investigated in [2], in which SUs make a decision based on the average delay measurement that takes into account the presence of PU. In that work, the visiting of PU is regarded as a kind of breakdown, which stops the processing service of SUs. The interrupted SU can continue her service at the leaving of PU till her service is completed, and the quality-of-service experienced by PU will not be affected by SUs. However, in practice, the channels are not perfectly reliable due to the hardware or environmental issues. As a result, the imperfect sensing can exist, and the PU will inevitably be affected by the speculating SUs because of the unreliability of channels. That is to say, it is necessary to consider the sensing failures in the derivation of system performance.

In general, in CR systems there exist two kinds of spectrum sensing errors (misdetections) for arriving SUs, see [9,10]. The first spectrum sensing error, called Class-A misdetection, may occur to both PU and SU during transmission (i.e., during service) when an arriving SU senses the PU band incorrectly and miss-senses that the PU band is idle. When it happens, a collision is resulted in, and both the arriving SU and job (PU/SU) in service will be deleted. The second spectrum sensing error, called Class-B error or disruption, is a type of disruption events to a PU and may occur when an ongoing SU transmits through the PU band and it incorrectly detects the presence of an arriving PU and assesses there is no PU accessing to the PU band. Wang et al. [14] conducted the game-theoretic analysis of the behavior of SUs in a cognitive radio system with class-B misdetection. They formulated the problem as a retrial queueing system for characterizing SUs' retrying transmission due to the interruptions by PUs and the spectrum sensing failure. The equilibrium and socially optimal strategies for all SUs were derived. An appropriate admission fee on SUs who decide to join the system was imposed to eliminate the gap between equilibrium strategy and socially optimal strategy. Nevertheless, the class-A misdetection is often ignored in the literature because it has the following difficulties in tractability: (1) the throughput of SUs does not equal to its effective arrival rate in steady state due to the loss of jobs; and (2) the SUs who decide to join the system does not necessarily obtain the service reward eventually, that is, the received service reward of SUs is random. In this paper, we focus on the class-A misdetection, and investigate its impact on the CR system.

To our best knowledge, it is the first work to study the strategic behavior of SUs in CR systems with class-A misdetection. Comparing to the existing related works in CR networks, our major contributions are as follows. First, we consider the negative effect of SUs on both SUs and PU. In particular, when the collision probability is 0, our model degenerates to that in [2]. In comparison, Do et al. [2] studied a similar problem for queueing control by formulating it as an M/M/1 queueing-game with server breakdowns where each customer wants to optimize their benefit in a selfish distributed manner. To improve the service efficiency, an admission fee was proposed to correct the gap between individually optimal strategy and socially optimal strategy. However, it is evident that the issue of imperfect sensing has not been considered in Do et al. [2]. Second, we investigate the equilibrium joining strategy as well as the expected utility of SUs under different levels of sensing errors. Last but not least, we find that, interestingly, the system throughput and overall social welfare is not monotonically decreasing in the sensing error probability. That is to say, a higher sensing error does not necessarily worsen the system throughput and social welfare. And the social welfare can be maximized at a certain level of sensing error.

To summarize, the contribution of our study to the literature is three-fold and described as follows:

- A new random spectrum sensing error framework of secondary users (SUs) is proposed to study the SUs' behavior so as to make the best benefit for the CR system and by overcoming the perfect spectrum sensing in the literature. A novel queueing-game theoretical model is formulated and various system stationary performance measures are derived
- The equilibrium strategy of SUs under the unobservable situation by considering imperfect sensing is firstly identified and then obtained. Obviously, such

an imperfect sensing formulation is not only more realistic to real-world practices, but also increases the complexity of the research problem. For games without sensing error, it is well-known that avoid-the-crowd (ATC) typically leads to a unique symmetric equilibrium, while follow-the-crowd (FTC) (see [3] for the definitions of ATC and FTC) leads to multiple equilibria. However, in our setting, we firstly observed that both ATC and FTC behavior can exist, which depends on the level of sensing error and leads to multiple equilibria.
– The system throughput and social welfare are first verified to be non-monotone in the level of sensing error, respectively. These results further surprisingly disclose that some amount of sensing error (but not all) in the system can lead to more efficient outcomes in terms of throughput and social welfare.

The reminder of paper is organized as follows. Section 2 introduces the problem formulation considered in this paper. The system performance analysis is provided in Sect. 3. The equilibrium joining strategies of SUs are derived in Sect. 4. Section 5 is devoted to the SUs throughput analysis. In Sect. 6, the system's social welfare characteristics are investigated and the performance evaluations are illustrated accordingly. The conclusion and future works are provided in Sect. 7.

2 Problem Formulation

We focus on a cognitive radio system with a single PU band that opportunistically used by SUs, which means that the PU band can transmit either one PU packet or one SU packet at one time. This PU band can be regarded as a server. As PU packets have a preemptive priority over the SUs, an arriving PU packet can get the service immediately even if the service area is occupied by an SU job. The server resumes to be shared by SUs until the PU completes transmissions.

We construct this service system as a classic M/M/1 queue, in which potential arrival rate of SUs is Λ_s, and the corresponding service rate is μ_s. The service of an SU packet can be interrupted at the arrival of a primary user. A PU packet will access the system only when the PU band is not occupied by another PU packet. The arrival rate of primary user is λ_p, and its service time is exponentially distributed with rate μ_p. The probability of sensing error for SUs is denoted by $\epsilon \in [0, 1]$. Specifically, if the spectrum is occupied, but the arriving SUs sense (*mistakenly*) that the channel is idle, she will try to access to the spectrum, then a collision happens, and the two jobs will be cleared. On the other hand, as mentioned in the previous section, the SUs can also mistakenly sense an idle channel to be a busy one. This kind of sensing error is referred to as a false alarm and we do not consider it in this paper.

The SUs are strategic in the sense that every arriving SU who wants to get service at the CR base station has the right to decide whether to join the system or not. In this work we consider the unobservable case that SUs do not know the system information. That is, an arriving SU has no knowledge of whether

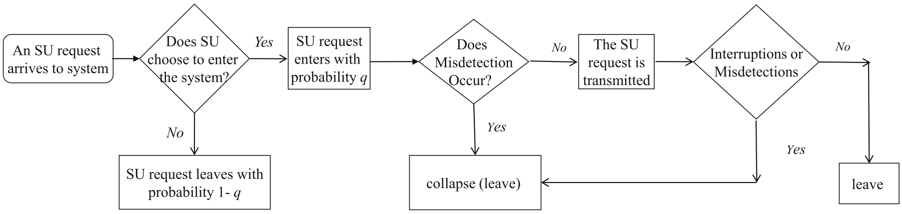


Fig. 1. Strategic behavior of SUs with imperfect sensing in a CR system

the PU band is available or not and the total number of SUs in the system. To characterize the economic structure of the decision process, we assume that at the completion of service, SUs and PU can receive a service reward R_s and R_p, respectively. During the sojourn time in system, including waiting time in queue and service time by the PU band, the SUs will suffer a waiting cost c per time unit. Note that the collision can only be caused by an arriving SU. And the primary user will definitely preempt the SUs on service. We do not consider the waiting cost of PU because its expected waiting time is constant, which equals to its service time, thus it can be subtracted from R_p. It is irrevocable for SUs' decisions on joining or balking according to their assessment on the reward against the costs. For convenience, Table 1 lists some important notations used in this paper.

Methodologically we adopt a queueing-game approach (see [3]) to study the SUs' strategic behavior, which is characterized by $q \in [0, 1]$ that is the probability an SU decides to enter the system. Equivalently, with probability $1 - q$ the SU decides to leave the system. Therefore, the effective entering probability for SUs is $\lambda_s \equiv \Lambda_s q$. The model is presented in Fig. 1. Since all SUs have the right to take their own decisions, we model this system as a non-cooperative queueing-game problem and aim to derive the symmetric Nash equilibria. In the sequel, we will give some results about SUs' equilibrium behavior and socially optimal strategies in the unobservable situation under the impact of sensing failures.

3 System Performance Analysis

In this section, we derive the system performance (i.e., the expected delay and the stationary probability) under a fixed effective arrival rate of SUs. The major results are given in the Theorem 1 as below:

Theorem 1. *When the sensing error probability is $\epsilon \in [0, 1]$,*

- *the system is stable if and only if $\lambda_s < \bar{\lambda}(\epsilon)$, where*

$$\bar{\lambda}(\epsilon) = \begin{cases} \frac{\sqrt{\vartheta^2 + 4\mu_p \mu_s \epsilon(1-2\epsilon)} - \vartheta}{2\epsilon(1-2\epsilon)}, & if\ \epsilon < 1/2; \\ \infty, & if\ \epsilon \geq 1/2, \end{cases}$$

and $\vartheta = \lambda_p(1 - \epsilon) + \mu_p - \epsilon(2\mu_p + \mu_s)$.

Table 1. Important notations in this paper.

Notation	Meaning
R_p	Reward for each service of PU
R_s	Reward for each service of SU
c	Waiting cost per time unit of SUs
Λ_s	Potential arrival rate for SUs
λ_s	Effective arrival rate for SUs
λ_p	Arrival rate for PU's packets
μ_s	Transmission rate for SUs
μ_p	Transmission rate for PUs
ϵ	The probability of misdetection
q	The probability an SU decides to join
$U(\lambda_s;\epsilon)$	Expected utility of a joining SUs with λ_s and ϵ

– *The stationary probability that the system is occupied by PU or not is given by*

$$\Pi_0 = \frac{\beta}{\lambda_p + \beta} \quad \text{and} \quad \Pi_1 = \frac{\lambda_p}{\lambda_p + \beta}. \tag{1}$$

– *The expected number of SUs in system is*

$$N_s = \frac{\lambda_s \xi((\lambda_p + \beta)^2 - \lambda_p[(\lambda_p + \beta)\epsilon - (1-\epsilon)(\xi + \lambda_s \epsilon)])}{(\lambda_p + \beta)(\xi + \lambda_s \epsilon)(\beta\xi - (\lambda_p + \beta)\lambda_s(1-\epsilon))}. \tag{2}$$

– *The expected waiting time for each joining SU is*

$$\bar{w}_s = \frac{\xi((\lambda_p + \beta)^2 - \lambda_p[(\lambda_p + \beta)\epsilon - (1-\epsilon)(\xi + \lambda_s \epsilon)])}{(\lambda_p + \beta)(\xi + \lambda_s \epsilon)[\beta\xi - (\lambda_p + \beta)\lambda_s(1-\epsilon)]}. \tag{3}$$

Proof. We define $I(t)$ and $N(t)$ as the spectrum state and the number of SUs in the system, respectively, where $I(t) = 0, 1$ corresponds to that the service area is occupied by $I(t)$ primary users. Thus $\{(I(t), N(t)), t \geq 0\}$ forms a two-dimensional Markov chain. The transition diagram is illustrated in Fig. 1, where $\xi = \mu_s + \epsilon\lambda_s$, $\beta = \mu_p + \epsilon\lambda_s$ (Fig. 2).

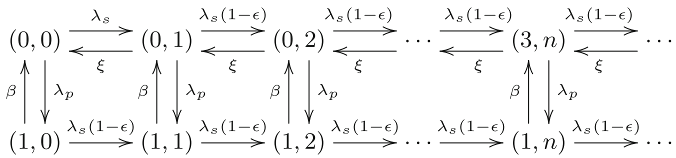


Fig. 2. Transition rate diagram.

Denote by $\pi_{i,j}$ the stationary probability at state (i, j), then we can have the following transition equations.

$$\pi_{0,0}(\lambda_p + \lambda_s) = \pi_{1,0}\beta + \pi_{0,1}\xi, \tag{4}$$
$$\pi_{0,1}(\lambda_p + \lambda_s + \mu_s) = \pi_{1,1}\beta + \pi_{0,0}\lambda_s + \pi_{0,2}\xi, \tag{5}$$
$$\pi_{0,i}(\lambda_p + \lambda_s + \mu_s) = \pi_{1,i}\beta + \pi_{0,i-1}\lambda_s(1-\epsilon) + \pi_{0,i+1}\xi, \tag{6}$$
$$\pi_{1,0}(\beta + \lambda_s(1-\epsilon)) = \pi_{0,0}\lambda_p, \tag{7}$$
$$\pi_{1,j}(\beta + \lambda_s(1-\epsilon)) = \pi_{1,j-1}\lambda_s(1-\epsilon) + \pi_{0,j}\lambda_p, \tag{8}$$

where $i \geq 2$ and $j \geq 1$. Define $\Pi_k(z) = \sum_{i=0}^{\infty} \pi_{k,i} z^i$ as the generating functions and $\Pi_k = \Pi_k(1)$ for $k = 0, 1$.

By combining (4)–(8), we can have the following equations:

$$\left[\lambda_p + (1-z)\left(\lambda_s - \frac{\mu_s}{z} - \frac{(1+z)\lambda_s\epsilon}{z}\right)\right]\Pi_0(z)$$
$$= \frac{z-1}{z}\pi_{0,0}[\xi + z\lambda_s\epsilon] + \beta\Pi_1(z), \tag{9}$$
$$[\beta + \lambda_s(1-\epsilon)(1-z)]\Pi_1(z) = \lambda_p\Pi_0(z). \tag{10}$$

By letting $z = 1$ in Eq. (10), we can get $\lambda_p\Pi_0 = \beta\Pi_1$. As $\Pi_0 + \Pi_1 = 1$, it gives the result (1). By taking the deviation of z on the both sides of (9) and (10), and letting $z = 1$, we have

$$\lambda_p\Pi_0' + [\xi - \lambda_s(1-\epsilon)]\Pi_0 = (\xi + \lambda_s\epsilon)\pi_{0,0} + \beta\Pi_1', \tag{11}$$
$$\beta\Pi_1' = \lambda_p\Pi_0' + \lambda_s(1-\epsilon)\Pi_1. \tag{12}$$

Combining (11)–(12), we can obtain that

$$\pi_{0,0} = \frac{\beta[\xi - \lambda_s(1-\epsilon)] - \lambda_s(1-\epsilon)\lambda_p}{(\lambda_p + \beta)(\xi + \lambda_s\epsilon)}.$$

Then the system can be stable if and only if $\pi_{0,0} > 0 \Leftrightarrow \lambda_s(1-\epsilon) < \frac{\beta\xi}{\lambda_p+\beta}$. By plugging the $\Pi_0(z)$ in (9) into (10), we have

$$\left(\left[\lambda_p + (1-z)\left(\lambda_s - \frac{\mu_s}{z} - \frac{(1+z)\lambda_s\epsilon}{z}\right)\right](\beta + \lambda_s(1-\epsilon)(1-z)) - \lambda_p\beta\right)\Pi_1(z)$$
$$= \lambda_p\frac{z-1}{z} \cdot \frac{(\beta[\xi - \lambda_s(1-\epsilon)] - \lambda_s(1-\epsilon)\lambda_p)(\xi + z\lambda_s\epsilon)}{(\lambda_p + \beta)(\xi + \lambda_s\epsilon)}.$$

By eliminating $(1 - z)$ on the both sides above, we can get

$$\left[\lambda_p\lambda_s(1-\epsilon) + \left(\lambda_s - \frac{\mu_s}{z} - \frac{(1+z)\lambda_s\epsilon}{z}\right) \times (\beta + \lambda_s(1-\epsilon)(1-z))\right]\Pi_1(z)$$
$$= -\frac{\lambda_p}{z} \cdot \frac{(\beta[\xi - \lambda_s(1-\epsilon)] - \lambda_s(1-\epsilon)\lambda_p)(\xi + z\lambda_s\epsilon)}{(\lambda_p + \beta)(\xi + \lambda_s\epsilon)}. \tag{13}$$

By taking the deviation of (13) with respect to z, and letting $z = 1$, we can derive that

$$\Pi_1' = \frac{\lambda_p}{\lambda_p+\beta}\left[\frac{\beta\xi-(\lambda_s(1-\epsilon)-\xi)\lambda_s(1-\epsilon)}{\beta\xi-(\lambda_p+\beta)\lambda_s(1-\epsilon)}-\frac{\xi}{\xi+\lambda_s\epsilon}\right].$$

Combining (12) and (1), we can get

$$\Pi_0' = \frac{\beta}{\lambda_p+\beta}\left[\frac{\beta\xi-(\lambda_s(1-\epsilon)-\xi)\lambda_s(1-\epsilon)}{\beta\xi-(\lambda_p+\beta)\lambda_s(1-\epsilon)}-\frac{\xi}{\xi+\lambda_s\epsilon}\right]-\frac{\lambda_s(1-\epsilon)}{\lambda_p+\beta}.$$

Therefore, the expected number of SUs in system is

$$N_s = \Pi_0' + \Pi_1' = \frac{\lambda_s\xi((\lambda_p+\beta)^2-\lambda_p[(\lambda_p+\beta)\epsilon-(1-\epsilon)(\xi+\lambda_s\epsilon)])}{(\lambda_p+\beta)(\xi+\lambda_s\epsilon)(\beta\xi-(\lambda_p+\beta)\lambda_s(1-\epsilon))}.$$

Notice that the actual joining rate of SUs is $\lambda_s(1-\sum_{i=1}^{\infty}\epsilon\pi_{0,i}-\sum_{i=0}^{\infty}\epsilon\pi_{1,i})$, based on the Little's law, the expected sojourn time of a joining SUs who are not rejected (i.e., no collision happens upon arrival) is given by

$$w_s = \frac{N_s}{\lambda_s(1-\sum_{i=1}^{\infty}\epsilon\pi_{0,i}-\sum_{i=0}^{\infty}\epsilon\pi_{1,i})}.$$

And the overall expected delay is

$$\begin{aligned}\bar{w}_s =&(1-\sum_{i=1}^{\infty}\epsilon\pi_{0,i}-\sum_{i=0}^{\infty}\epsilon\pi_{1,i})w_s+(\sum_{i=1}^{\infty}\epsilon\pi_{0,i}+\sum_{i=0}^{\infty}\epsilon\pi_{1,i})\cdot 0\\ =&\frac{N_s}{\lambda_s}=\frac{\xi((\lambda_p+\beta)^2-\lambda_p[(\lambda_p+\beta)\epsilon-(1-\epsilon)(\xi+\lambda_s\epsilon)])}{(\lambda_p+\beta)(\xi+\lambda_s\epsilon)[\beta\xi-(\lambda_p+\beta)\lambda_s(1-\epsilon)]}.\end{aligned}$$

Finally, notice that the stability condition

$$\lambda_s(1-\epsilon) < \frac{\beta\xi}{\lambda_p+\beta} \Leftrightarrow \lambda_s(1-\epsilon) < \frac{(\mu_p+\lambda_s\epsilon)(\mu_s+\lambda_s\epsilon)}{\lambda_p+\mu_p+\lambda_s\epsilon}.$$

If $\epsilon < 1/2$, then it holds if and only if

$$\lambda_s < \frac{\sqrt{(\lambda_p(1-\epsilon)+\mu_p-\epsilon(2\mu_p+\mu_s))^2+4\mu_p\mu_s\epsilon(1-2\epsilon)}+\lambda_p(\epsilon-1)-\mu_p+(\mu_s+2\mu_p)\epsilon}{2\epsilon\,(1-2\epsilon)}.$$

When $\epsilon \geq 1/2$, it is readily seen that the stability condition holds for any $\lambda_s > 0$. This completes the proof. ■

Remark 1. It should be noted that by considering the error probability of imperfect spectrum sensing, the stability condition of the underlying system is quite different from the previous works such as Do et al. [2], in which the stability condition is $\lambda_s < \frac{\mu_p\mu_s}{\mu_p+\lambda_p}$ by our notations. We find that the system can be stable if $\lambda_s < \bar{\lambda}(\epsilon)$, when ϵ is relatively large, the system can be stable for any effective arrival rate of SUs because each arriving SU who finds an occupied PU band will be deleted due to the definite sensing error. In particular, when $\epsilon = 0$, our model can degenerate to the situation in [2] which considered the perfect spectrum sensing mechanism.

Theorem 2. *For any given λ_s, the overall expected delay of an SU request is decreasing in the service rate of SU request, i.e., $\bar{w}_s$ is decreasing in μ_s for any $\epsilon \in [0,1]$.*

Proof. To show that $\bar{w}_s$ is decreasing in μ_s, it is sufficient to prove that $\bar{w}_s$ is decreasing in $\xi = \mu_s + \epsilon\lambda_s$. Notice that $\bar{w}_s$ can be rewritten as

$$\bar{w}_s = \frac{((\lambda_p + \beta)(\lambda_p + \beta - \lambda_p\epsilon)/(\xi + \lambda_s\epsilon) - (1-\epsilon)])}{(\lambda_p + \beta)[\beta - (\lambda_p + \beta)\lambda_s(1-\epsilon)/\xi]},$$

which is obviously decreasing in ξ. Next, we will prove that $\bar{w}_s$ is decreasing in ϵ. Notice that $\bar{w}_s$ can be rewritten as $\bar{w}_s(\beta, \xi, \epsilon)$, where $\beta(\epsilon)$ and $\xi(\epsilon)$ are both increasing in ϵ. Then we use three steps to complete this proof.

(1) We need to show that $\partial\bar{w}_s(\beta, \xi, \epsilon)/\partial\beta < 0$. It is sufficient to show that

$$\frac{(\lambda_p + \beta)[\beta + \lambda_p(1-\epsilon)]}{(\lambda_p + \beta)[\beta\xi - (\lambda_p + \beta)\lambda_s(1-\epsilon)]} + \frac{\lambda_p(1-\epsilon)(\xi + \lambda_s\epsilon)}{(\lambda_p + \beta)[\beta\xi - (\lambda_p + \beta)\lambda_s(1-\epsilon)]}$$

is decreasing in β. The second term is obviously decreasing in β because $\xi > \lambda_s(1-\epsilon)$. Then it suffices to prove that $\phi(\beta) = \frac{\beta + \lambda_p(1-\epsilon)}{\beta[\xi - \lambda_s(1-\epsilon)] - \lambda_s(1-\epsilon)\lambda_p}$ is decreasing in β. And it can be verified directly because

$$\frac{d\phi(\beta)}{d\beta} = \frac{\lambda_p(\epsilon - 1)(\epsilon\lambda_s + \xi)}{[\beta(\xi - (1-\epsilon)\lambda_s) - \lambda_p(1-\epsilon)\lambda_s]^2} < 0.$$

(2) We need to show that $\partial\bar{w}_s(\beta, \xi, \epsilon)/\partial\xi < 0$. As $\bar{w}_s$ can be rewritten as

$$\bar{w}_s = \frac{\frac{(\lambda_p+\beta)(\lambda_p+\beta-\lambda_p\epsilon)}{\xi+\lambda_s\epsilon} + \lambda_p(1-\epsilon)}{(\lambda_p + \beta)[\beta - \frac{(\lambda_p+\beta)\lambda_s(1-\epsilon)}{\xi}]}. \tag{14}$$

Then it is not difficult to verify that $\frac{(\lambda_p+\beta)(\lambda_p+\beta-\lambda_p\epsilon)}{\xi+\lambda_s\epsilon}$ and $\beta - \frac{(\lambda_p+\beta)\lambda_s(1-\epsilon)}{\xi}$ are decreasing and increasing in ξ, respectively. Thus we can get that $\bar{w}_s$ is decreasing in ξ.

(3) We need to show that $\partial\bar{w}_s(\beta, \xi, \epsilon)/\partial\epsilon < 0$. By 14, we can verify that $\frac{(\lambda_p+\beta)(\lambda_p+\beta-\lambda_p\epsilon)}{\xi+\lambda_s\epsilon} + \lambda_p(1-\epsilon)$ and $\beta - \frac{(\lambda_p+\beta)\lambda_s(1-\epsilon)}{\xi}$ are decreasing and increasing in ϵ, respectively. Thus we can get that $\partial\bar{w}_s(\beta, \xi, \epsilon)/\partial\epsilon < 0$. So far, we can establish the monotonicity of $\bar{w}_s$ because

$$\frac{d\bar{w}_s(\beta, \xi, \epsilon)}{d\epsilon} = \frac{\partial\bar{w}_s(\beta, \xi, \epsilon)}{\partial\beta} \cdot \frac{\partial\beta}{\partial\epsilon} + \frac{\partial\bar{w}_s(\beta, \xi, \epsilon)}{\partial\xi} \cdot \frac{\partial\xi}{\partial\epsilon} + \frac{\partial\bar{w}_s(\beta, \xi, \epsilon)}{\partial\epsilon} < 0$$

by combining the three results derived above, which completes this proof. ■

Remark 2. Theorem 2 shows that the expected waiting time is decreasing in the error probability. It is intuitive because when ϵ increases, more SUs are blocked or squeezed by the collisions. Thus the system congestion is reduced, which results in a lower expected waiting time. Similarly, the expected waiting time is decreasing in the service rate of SUs because a shorter service time is needed.

4 Equilibrium Strategy

In this section, we will investigate the equilibrium strategy of SUs. First of all, we tend to determine the monotonicity of $\bar{w}_s$ in the effective arrival rate λ_s. Different from the results derived before, counter to our intuition, the expected delay of SUs is not always increasing in λ_s. Let denote by

$$\begin{aligned} A = & (\lambda_p + \mu_p + \epsilon\lambda_s)(2\epsilon\lambda_s + \mu_s)(\lambda_p(\epsilon - 1)\lambda_s + (\mu_p + \epsilon\lambda_s)((2\epsilon - 1)\lambda_s + \mu_s))\times \\ & [(\epsilon\lambda_s + \mu_s)(2\lambda_s(\lambda_p + \mu_p + \epsilon\lambda_s) - \lambda_p(\lambda_p + \mu_p - 2\lambda_s + 6\epsilon\lambda_s + \mu_s)) \\ & + \lambda_s \left((\lambda_p + \mu_p + \epsilon\lambda_s)^2 - \lambda_p(\epsilon(\lambda_p + \mu_p + \epsilon\lambda_s) + (\epsilon - 1)(2\epsilon\lambda_s + \mu_s))\right)], \end{aligned} \quad (15)$$

$$\begin{aligned} B = & \lambda_s(\epsilon\lambda_s + \mu_s)\left[(\lambda_p + \mu_p + \epsilon\lambda_s)^2 - \lambda_p(\epsilon(\lambda_p + \mu_p + \epsilon\lambda_s) + (\epsilon - 1)(2\epsilon\lambda_s + \mu_s))\right]\times \\ & [(\lambda_p + \mu_p + \epsilon\lambda_s)(2\epsilon\lambda_s + \mu_s)(\lambda_p + 2\mu_p - \lambda_s + 4\epsilon\lambda_s + \mu_s) \\ & + 2(\lambda_p + \mu_p + \epsilon\lambda_s)(\lambda_p(\epsilon - 1)\lambda_s + (\mu_p + \epsilon\lambda_s)((2\epsilon - 1)\lambda_s + \mu_s)) \\ & + (2\epsilon\lambda_s + \mu_s)(\lambda_p(\epsilon - 1)\lambda_s + (\mu_p + \epsilon\lambda_s)((2\epsilon - 1)\lambda_s + \mu_s))], \end{aligned} \quad (16)$$

we will have the following result:

Theorem 3. *The monotonicity of overall expected delay of an SU request with regards to the service rate of the SUs are characterized through following three situations:*

- *When $\epsilon \in (0,1)$, the overall expected delay of an SU request is decreasing in the service rate of SU request, i.e., $\bar{w}_s$ is increasing in λ_s if and only if $A > B$, where A and B are given in the following Eq. (15)–(16).*
- *In case if $\epsilon = 0$, $\bar{w}_s$ is increasing in λ_s;*
- *In case if $\epsilon = 1$, $\bar{w}_s$ is decreasing in λ_s.*

Proof. For any $\epsilon \in [0,1]$, by taking the deviation of $\bar{w}_s$ with respect to λ_s, one can check that $d\bar{w}_s/d\lambda_s > 0 \Leftrightarrow A > B$, where A and B are given in (15)–(16), respectively. If $\epsilon = 0$, we have that $\bar{w}_s = \frac{(\lambda_p+\mu_p)^2+\lambda_p\mu_s}{(\lambda_p+\mu_p)[\mu_p\mu_s-(\lambda_p+\mu_p)\lambda_s]}$, then

$$\frac{\partial \bar{w}_s}{\partial \lambda_s} = \frac{(\lambda_p + \mu_p)[(\lambda_p + \mu_p)^2 + \lambda_p\mu_s]}{(\lambda_p + \mu_p)[\mu_p\mu_s - (\lambda_p + \mu_p)\lambda_s]^2} > 0,$$

which implies that $\bar{w}_s$ is increasing in λ_s. If $\epsilon = 1$, we can get that $\bar{w}_s = \frac{1}{\mu_s+2\lambda_s}$, which is obviously decreasing in λ_s. This completes our proof. ∎

When $\epsilon \in (0,1)$, it is hard to obtain the exact monotonicity of $\bar{w}_s$. However, we can still investigate its property numerically. Figure 3 illustrates the $\bar{w}_s$ for different λ_s under three error probabilities (i.e., $\epsilon = 0.1$, $\epsilon = 0.45$ and $\epsilon = 0.8$). From Fig. 3, we can observe that if ϵ is small (large), $\bar{w}_s$ is increasing (decreasing) in λ_s, which is consistent with our Theorem 3. Otherwise, if ϵ drops into the intermediate interval, $\bar{w}_s$ decreases in λ_s firstly and then increases in λ_s. That is to say, the monotonicity of $\bar{w}_s$ in λ_s is quite sensitive to ϵ.

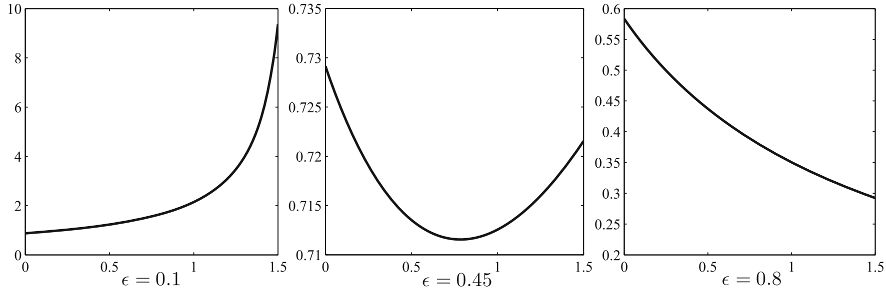


Fig. 3. The $\bar{w}_s$ vs. λ_s when $\mu_s = \mu_p = 2$ and $\lambda_p = 1$.

So far, we have derived the expected waiting times of the SUs, next, we aim to characterize the expected utilities of SUs. Define $U(\lambda_s;\epsilon)$ as the expected utility of a joining SUs when the effective arrival rate is λ_s and the error probability is ϵ. We also define the probability that the SUs who are already in system can be served successfully as P_s. Because such collision can happen only when the tagged user is in the service area, we have that

$$P_s = \frac{\mu_s}{\mu_s+\lambda_s+\lambda_p}\cdot 1 + \frac{\lambda_s(1-\epsilon)}{\mu_s+\lambda_s+\lambda_p}\cdot P_s + \frac{\lambda_p}{\mu_s+\lambda_s+\lambda_p}\cdot P_s + \frac{\lambda_s\epsilon}{\mu_s+\lambda_s+\lambda_p}\cdot 0,$$

which gives $P_s = \frac{\mu_s}{\xi}$. Then $U(\lambda_s;\epsilon)$ can be divided into three parts in (17).

Lemma 1. *The expected utility of a joining SU, i.e., $U(\lambda_s;\epsilon)$, is increasing in μ_s.*

Proof. Because $U(\lambda_s;\epsilon) = \frac{R_s\mu_s}{\xi}[1-(1-\pi_{0,0})\epsilon] - c\bar{w}_s$. It is not difficult to verify that $\mu_s/\xi = \mu_s/(\mu_s+\lambda_s\epsilon)$ and $\pi_{0,0}$ are both increasing in μ_s, then $\frac{R_s\mu_s}{\xi}[1-(1-\pi_{0,0})\epsilon]$ is increasing in μ_s. On the other hand, we have shown that $\bar{w}_s$ is decreasing in μ_s (see Theorem 2), thus $U(\lambda_s;\epsilon)$ is increasing in μ_s. ■

To characterize the equilibrium joining strategy for SUs, we need to investigate the property of $U(\lambda_s;\epsilon)$. At the first sight, we can observe that the expected service reward is decreasing in λ_s because of the collisions become to be more frequently. But as we have shown that, the expected delay can be non-monotone in λ_s. Then the monotonicity of $U(\lambda_s;\epsilon)$ is equivocal. Denote by q_e the equilibrium joining probability of SUs. Next, we study the equilibrium strategy for SUs under two special cases to give some insights. Since it is very difficult to determine the monotonicity of $\bar{w}_s$ for the case $\epsilon \in (0,1)$ and thus it is still an open problem to get the equilibrium joining probability in general. However, we have figured out the following two equilibrium joining probabilities in a popular situations for the case when $\epsilon = 0$ or $\epsilon = 1$.

Theorem 4. *The two equilibrium joining probabilities for the case when $\epsilon = 0$ or $\epsilon = 1$ are:*

$$\begin{aligned} U(\lambda_s;\epsilon) = &\underbrace{0\cdot[\sum_{i=1}^{\infty}\epsilon\pi_{0,i}+\sum_{i=0}^{\infty}\epsilon\pi_{1,i}]}_{collision\ happens\ upon\ arrival} \underbrace{-cw_s\cdot[(1-\sum_{i=1}^{\infty}\epsilon\pi_{0,i}-\sum_{i=0}^{\infty}\epsilon\pi_{1,i})](1-P_s)}_{collision\ happens\ during\ service} \\ &\underbrace{+(R_s-cw_s)\cdot P_s[(1-\sum_{i=1}^{\infty}\epsilon\pi_{0,i}-\sum_{i=0}^{\infty}\epsilon\pi_{1,i})]}_{collision\ never\ happens} \qquad (17) \\ &= R_s\cdot P_s[(1-\sum_{i=1}^{\infty}\epsilon\pi_{0,i}-\sum_{i=0}^{\infty}\epsilon\pi_{1,i})]-c\bar{w}_s \\ &= \frac{R_s\mu_s}{\xi}[1-(1-\pi_{0,0})\epsilon]-c\bar{w}_s \\ &= \frac{[(\lambda_p+\beta)[\xi+2\lambda_s\epsilon(1-\epsilon)]-\lambda_p\xi\epsilon]\mu_s R_s}{(\lambda_p+\beta)(\xi+\lambda_s\epsilon)\xi} \\ &- \frac{c\xi((\lambda_p+\beta)^2-\lambda_p[(\lambda_p+\beta)\epsilon-(1-\epsilon)(\xi+\lambda_s\epsilon)])}{(\lambda_p+\beta)(\xi+\lambda_s\epsilon)[\beta\xi-(\lambda_p+\beta)\lambda_s(1-\epsilon)]} \\ &= \frac{[(\lambda_p+\beta)[\xi+2\lambda_s\epsilon(1-\epsilon)]-\lambda_p\xi\epsilon]\mu_s}{(\lambda_p+\beta)(\xi+\lambda_s\epsilon)\xi} \\ &\cdot\left(R_s-\frac{c\xi^2((\lambda_p+\beta)^2-\lambda_p[(\lambda_p+\beta)\epsilon-(1-\epsilon)(\xi+\lambda_s\epsilon)])}{\mu_s((\lambda_p+\beta)[\xi+2\lambda_s\epsilon(1-\epsilon)]-\lambda_p\xi\epsilon)[\beta\xi-(\lambda_p+\beta)\lambda_s(1-\epsilon)]}\right). \end{aligned}$$

– *When $\epsilon = 0$, the equilibrium strategy is given by*

$$q_e = \begin{cases} 1, & if\ \Lambda_s < \frac{\beta\mu_s}{\lambda_p+\beta}\ and U(\Lambda_s;0) > 0; \\ 0, & if\ U(0;0) \le 0; \\ \frac{\left(\mu_p\mu_s - \frac{c[(\lambda_p+\mu_p)^2+\lambda_p\mu_s]}{R_s(\lambda_p+\mu_p)}\right)}{\Lambda_s(\lambda_p+\mu_p)}, & otherwise. \end{cases}$$

– *When $\epsilon = 1$, the equilibrium strategy is given by*

$$q_e = \begin{cases} 1, & if\ U(0;1) \ge 0; \\ \frac{(\lambda_p+\mu_p)c-\mu_p R_s\mu_s}{\mu_p R_s\mu_s - c}, & if\ U(\Lambda_s;1) \ge 0 > U(0;1); \\ 0, & if\ U(\Lambda_s;1) < 0, \end{cases}$$

Proof. (1) When $\epsilon = 0$, we have that $U(\lambda_s;0) = R_s - c\frac{(\lambda_p+\mu_p)^2+\lambda_p\mu_s}{(\lambda_p+\mu_p)[\mu_p\mu_s-(\lambda_p+\mu_p)\lambda_s]}$, which is decreasing in λ_s because $\frac{(\lambda_p+\mu_p)^2+\lambda_p\mu_s}{(\lambda_p+\mu_p)[\mu_p\mu_s-(\lambda_p+\mu_p)\lambda_s]}$ is increasing in λ_s (see Theorem 3). Therefore, when all other SUs adopt joining strategy $q = \lambda_s/\Lambda_s$, the best response for the tagged one is

$$q_{\lambda_s} = \arg\max_{\hat{q}\in[0,1]} \hat{q}\cdot U(\lambda_s;0).$$

• When $\Lambda_s < \frac{\beta\mu_s}{\lambda_p+\beta}$, we consider the following three cases:

If $U(0;0) < 0$, then $q_{\lambda_s} = 0$ for all $\lambda_s \in [0, \Lambda_s]$. Similarly, if $U(\lambda_s;0) \geq 0$, then $q_{\lambda_s} = 1$. Otherwise, if $U(0;0) \geq 0 > U(\Lambda_s;0)$, there exists unique λ_s' such that $U(\lambda_s;0) \geq 0$ and $U(\lambda_s;0) < 0$ for $\lambda_s \in [0, \lambda_s']$ and $\lambda_s \in (\lambda_s', \Lambda_s]$, respectively. Then we have $q_{\lambda_s} = 1$ and $q_{\lambda_s} = 0$ for $\lambda_s \in [0, \lambda_s']$ and $\lambda_s \in (\lambda_s', \Lambda_s]$, accordingly. In summary, we have that q_{λ_s} is weakly decreasing in $\lambda_s \in [0, \Lambda_s]$, which characterizes an ATC type behavior. And the equilibrium strategy is

$$q_e = \begin{cases} 1, & \text{if } U(\Lambda_s;0) > 0; \\ \frac{\mu_p\mu_s - \frac{c[(\lambda_p+\mu_p)^2+\lambda_p\mu_s]}{R_s(\lambda_p+\mu_p)}}{\Lambda_s(\lambda_p+\mu_p)}, & \text{if } U(\Lambda_s;0) \leq 0 < U(0;0); \\ 0, & \text{if } U(0;0) \leq 0. \end{cases}$$

- When $\Lambda_s \geq \frac{\beta\mu_s}{\lambda_p+\beta}$, from the same argument, we have

$$q_e = \begin{cases} \frac{1}{\Lambda_s(\lambda_p+\mu_p)} \cdot \left(\mu_p\mu_s - \frac{c[(\lambda_p+\mu_p)^2+\lambda_p\mu_s]}{R_s(\lambda_p+\mu_p)}\right), & \text{if } U(0;0) > 0; \\ 0, & \text{if } U(0;0) \leq 0. \end{cases}$$

And the same ATC type behavior can be justified.

(2) When $\epsilon = 1$, we can get that $U(\lambda_s;1) = \frac{1}{\mu_s+2\lambda_s}\left[\frac{(\mu_p+\lambda_s)\mu_s R_s}{(\lambda_p+\mu_p+\lambda_s)} - c\right]$. By taking the deviation of $(\mu_s + 2\lambda_s)U(\lambda_s;1)$ with respect to λ_s, we have

$$\frac{\partial(\mu_s + 2\lambda_s)U(\lambda_s;1)}{\partial\lambda_s} = \frac{\lambda_p\mu_s R_s}{(\lambda_s + \lambda_p + \mu_p)^2} > 0.$$

Therefore, if $U(0;1) \geq 0$, then $q_{\lambda_s} = 1$ for all $\lambda_s \in [0, \Lambda_s]$. If $U(\lambda_S;0) < 0$, then $q_{\lambda_s} = 0$ for all $\lambda_s \in [0, \Lambda_s]$. Otherwise, if $U(0;0) < 0 \leq U(\Lambda_s;0)$, there exists unique λ_s' such that $U(\lambda_s;0) < 0$ and $U(\lambda_s;0) \geq 0$ for $\lambda_s \in [0, \lambda_s']$ and $\lambda_s \in (\lambda_s', \Lambda_s]$, respectively.

Thus we have $q_{\lambda_s} = 0$ and $q_{\lambda_s} = 1$ for $\lambda_s \in [0, \lambda_s']$ and $\lambda_s \in (\lambda_s', \Lambda_s]$, accordingly. That is, q_{λ_s} is weakly increasing in $\lambda_s \in [0, \Lambda_s]$, which characterizes an FTC type behavior. And the equilibrium joining strategy is given by

$$q_e = \begin{cases} 1, & \text{if } U(0;1) \geq 0; \\ \frac{(\lambda_p+\mu_p)c-\mu_p R_s\mu_s}{\mu_p R_s\mu_s - c}, & \text{if } U(\Lambda_s;1) \geq 0 > U(0;1); \\ 0, & \text{if } U(\Lambda_s;1) < 0, \end{cases}$$

which completes this proof. ∎

Remark 3. Theorem 4 discloses that customers present different behaviors at two extreme cases $\epsilon = 0$ and $\epsilon = 1$. In particular, when ϵ is small, the expected utility of SUs is decreasing in ϵ, then SUs are more hesitate to join the system when others do. Thus an ATC type behavior can be observed. However, when ϵ is large, the decreased amount of delay is higher than that of reduced service value. Thus, the SUs are more inclined to join when others do, which presents an FTC type behavior, and multiple equilibria can exist. That is to say, both FTC and ATC joining behavior can exist in the system, which depends on the level of misdetection. More discussions on the ATC and FTC can be found in [3].

5 Throughput of the Secondary Users

In this part, we tend to investigate how the throughput of SUs (denote by T_s) changes with the sensing error level. Notice that when the sensing error is considered, the throughput of SUs is given by $T_s = \frac{\lambda_s[(\lambda_p+\beta)[\xi+2\lambda_s\epsilon(1-\epsilon)]-\lambda_p\xi\epsilon]\mu_s}{(\lambda_p+\beta)(\xi+\lambda_s\epsilon)\xi}$. Then we can have the following result.

Theorem 5. *The monotonic properties of the throughput of SUs regard to the reward for completing the service of a SU are characterized by the following results:*

- *If $\epsilon \in (0,1)$, the throughput of SUs is not monotonically increasing in the reward for completing the service of a SU, i.e., T_s is not monotonically increasing function of R_s;*
- *If $\epsilon = 0$, the throughput of SUs is increasing in the reward for completing the service of a SU, i.e., T_s is an increasing function of R_s;*
- *If $\epsilon = 1$, the throughput of SUs is decreasing in the reward for completing the service of a SU, i.e., T_s is an decreasing function of R_s;*

Proof. When $\epsilon = 0$, we have $T_s = \lambda_s$. By Theorem 4, the equilibrium λ_s is determined by $\max\{\lambda_s | U(\lambda_s; 0) \geq 0\}$, which increases in R_s. Then T_s is increasing in R_s. On the other hand, when $\epsilon = 1$, $T_s = \frac{\lambda_s}{\mu_s+2\lambda_s}$. Because $U(\lambda_s; 1) = \frac{1}{\mu_s+2\lambda_s}\left[\frac{(\mu_p+\lambda_s)\mu_s R_s}{(\lambda_p+\mu_p+\lambda_s)} - c\right]$, which implies that the equilibrium λ_s is decreasing in R_s. Then T_s is decreasing in R_s by noticing that T_s increases in λ_s, which completes this proof. ■

Remark 4. Theorem 5 discloses that, under the equilibrium strategy of SUs, the throughput of SUs is not necessarily increasing in their service reward. Specifically, when the error probability is large, the throughput T_s can be even decreasing in R_s. That is to say, when the service reward of SUs increases, a lower throughput can be resulted in, which is led by the decrease of equilibrium joining rate.

Denote by λ_s^e and T_s^e the effective arrival rate and throughput of SUs in equilibrium. Next, we compare the effective arrival rate λ_s and throughput T_s in equilibrium for different sensing error level in Fig. 4.

By Fig. 4, we can observe that the throughput is always lower than the effective arrival rate when $\epsilon > 0$ because of the possible collisions. In addition, the effective arrival rate is weakly increasing in the sensing error level. It is intuitive by recalling that the expected waiting time of SUs is decreasing in ϵ by Theorem 2. Interestingly, the throughput is non-monotone in the sensing error level. To explain this phenomenon, we need to understand the essential characteristics when the sensing error is considered. On the positive side, increasing the sensing error level can help SUs crowd out the PU in service, which cut down their expected waiting time greatly, thus more SUs could be served. On the negative side, it should be noted that sensing error can also happen to the SUs themselves, which lowers the probability of SUs to complete this service. In particular, when

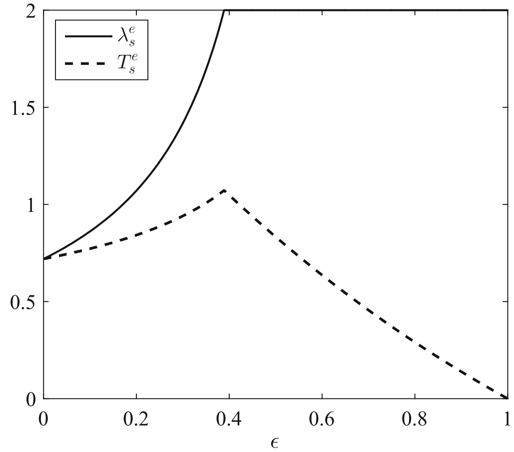


Fig. 4. The comparison of λ_s^e and throughput T_s^e for different ϵ when $R_s = \mu_s = \mu_p = \mu_p = 2$ and $\lambda_p = 1$

ϵ is small, the PU is less affected by the arriving SUs, thus increasing sensing error level can efficiently access the PU band by deleting the PU in service. By contrast, when ϵ is large, the expected service reward received by SUs decreases greatly, which outperforms the amount of reduced waiting time of them, thus the negative effect plays a dominant role, and a lower throughput is resulted in.

6 System Social Welfare

In this section, we investigate how the sensing error affects the social welfare. Recall that the benefit of SUs and PU generated per time unit is $\lambda_s U(\lambda_s;\epsilon)$ and $\Pi_1 R_p \mu_p = \frac{R_p \lambda_p \mu_p}{\lambda_p + \beta} = \frac{R_p \lambda_p \mu_p}{\lambda_p + \mu_s + \epsilon\lambda_s}$, respectively. Then the social welfare is $SW(\lambda_s;\epsilon) = \lambda_s U(\lambda_s;\epsilon) + \frac{R_p \lambda_p \mu_p}{\lambda_p + \mu_p + \epsilon\lambda_s}$. The following results provides the property of social welfare.

Theorem 6. *The system's social welfare is not monotonically decreasing in either the effective arrival rate for SUs or the spectrum error probability, i.e., the function $SW(\lambda_s;\epsilon)$ is not monotonically decreasing in either λ_s or ϵ.*

Proof. Notice that the expected utility of SUs is given by

$$U(\lambda_s;\epsilon) = \frac{[(\lambda_p+\beta)[\xi + 2\lambda_s\epsilon(1-\epsilon)] - \lambda_p\xi\epsilon]\mu_s}{(\lambda_p+\beta)(\xi+\lambda_s\epsilon)\xi} \cdot \left(R_s - \frac{c\xi^2((\lambda_p+\beta)^2 - \lambda_p[(\lambda_p+\beta)\epsilon - (1-\epsilon)(\xi+\lambda_s\epsilon)])}{\mu_s((\lambda_p+\beta)[\xi+2\lambda_s\epsilon(1-\epsilon)] - \lambda_p\xi\epsilon)[\beta\xi - (\lambda_p+\beta)\lambda_s(1-\epsilon)]}\right).$$

Denote by $\bar{\lambda}_s$ the unique solution of

$$R_s - \frac{c\xi^2((\lambda_p+\beta)^2 - \lambda_p[(\lambda_p+\beta)\epsilon - (1-\epsilon)(\xi+\lambda_s\epsilon)])}{\mu_s((\lambda_p+\beta)[\xi+2\lambda_s\epsilon(1-\epsilon)] - \lambda_p\xi\epsilon)[\beta\xi - (\lambda_p+\beta)\lambda_s(1-\epsilon)]},$$

we have $U(\lambda_s;\epsilon) > 0$ for any $\lambda_s \in [0, \bar{\lambda}_s)$. Then it gives $SW(\bar{\lambda}_s;\epsilon) = \frac{R_p\lambda_p\mu_p}{\lambda_p+\mu_s+\bar{\lambda}_s+\epsilon}$. Also, we have $SW(0;\epsilon) = \frac{R_p\lambda_p\mu_p}{\lambda_p+\mu_s}$. It immediately follows that $SW(0;\epsilon) > SW(\bar{\lambda}_s;\epsilon)$. Thus $SW(\lambda_s;\epsilon)$ is not monotonically decreasing in λ_s.

On the other hand, when $\epsilon = 0$, we have that

$$SW(\lambda_s;0) = \lambda_s\left[R_s - c\frac{(\lambda_p+\mu_p)^2+\lambda_p\mu_s}{(\lambda_p+\mu_p)[\mu_p\mu_s-(\lambda_p+\mu_p)\lambda_s]}\right] + \frac{R_p\lambda_p\mu_p}{\lambda_p+\mu_p}.$$

When $\epsilon = 1$, we have that $SW(\lambda_s;1) = \frac{\lambda_s}{\mu_s+2\lambda_s}\left[\frac{(\mu_p+\lambda_s)\mu_s R_s}{(\lambda_p+\mu_p+\lambda_s)} - c\right] + \frac{R_p\lambda_p\mu_p}{\lambda_p+\mu_p+\lambda_s}$. Notice that

$$\begin{aligned} &SW(\lambda_s;1) - SW(\lambda_s;0) > 0 \\ \Leftrightarrow &R_p < \left(\frac{1}{\lambda_p}+\frac{1}{\mu_p}\right)(\lambda_p+\mu_p+\lambda_s)\cdot \\ &\left(\frac{c\left((\lambda_p+\mu_p)^2+\lambda_p\mu_s\right)}{(\lambda_p+\mu_p)(\mu_p\mu_s-\lambda_s(\lambda_p+\mu_p))} - \frac{(c+2R_s\lambda_s)(\lambda_p+\mu_p+\lambda_s)+\lambda_p\mu R_s}{(\mu+2\lambda_s)(\lambda_p+\mu_p+\lambda_s)}\right). \end{aligned}$$

Thus, when R_p is small, a higher social welfare can be obtained when $\epsilon = 1$. That is, $SW(\lambda_s;\epsilon)$ is not monotonically decreasing in ϵ. ■

Theorem 6 indicates that a higher congestion level of SUs and a higher level of sensing error do not necessarily worsen the social welfare. The first result in intuitive since the SUs is also a part of society, which can bring positive benefit to the system. Even though it puts a negative effect on the PU when sensing error happens, it could improve the social welfare by carefully controlling its arrival rate. In addition, the second result reveals that some amount of occurrence of sensing error can indeed induce a higher social welfare than that without sensing error. It is because that increasing the level of sensing error allows the SUs to crowd out the users in service, which reduce the system congestion and in turn, improves the total benefits of SUs. When R_p is small, the social welfare generated by PU is less sensitive to the sensing error, where the marginal loss of PU's benefit is lower than the marginal benefit of SUs, then a higher social welfare can be reached. The following corollary is a follow-up result of Theorem 6.

Corollary 1. *When $\epsilon = 0$, $SW(\lambda_s;0)$ is unimodal in λ_s. When $\epsilon = 1$, if $\mu_s \geq 2\lambda_p$, $SW(\lambda_s;1)$ is unimodal in λ_s.*

Proof. (i) When $\epsilon = 0$, then we have that

$$SW(\lambda_s;0) = \lambda_s\left[R_s - c\frac{(\lambda_p+\mu_p)^2+\lambda_p\mu_s}{(\lambda_p+\mu_p)[\mu_p\mu_s-(\lambda_p+\mu_p)\lambda_s]}\right] + \frac{R_p\lambda_p\mu_p}{\lambda_p+\mu_p}.$$

By taking the deviation of λ_s with respect to $SW(\lambda_s;0)$, we can get

$$\frac{dSW(\lambda_s;0)}{d\lambda_s} = R_s - c\frac{[(\lambda_p+\mu_p)^2+\lambda_p\mu_s]\mu_p\mu_s}{(\lambda_p+\mu_p)[\mu_p\mu_s-(\lambda_p+\mu_p)\lambda_s]^2},$$

which is decreasing in λ_s. If $\frac{dSW(\lambda_s;0)}{d\lambda_s}|_{\lambda_s=0} = R_s - c\frac{(\lambda_p+\mu_p)^2+\lambda_p\mu_s}{(\lambda_p+\mu_p)\mu_p\mu_s} < 0$, $SW(\lambda_s;0)$ is decreasing in λ_s. Otherwise, $SW(\lambda_s;0)$ is increasing first and then decreasing in λ_s. In summary, $SW(\lambda_s;0)$ is unimodal in λ_s.

(ii) When $\epsilon = 1$, then we have that $SW(\lambda_s;1) = \frac{\lambda_s}{\mu_s+2\lambda_s}\left[\frac{(\mu_p+\lambda_s)\mu_s R_s}{(\lambda_p+\mu_p+\lambda_s)} - c\right] + \frac{R_p\lambda_p\mu_p}{\lambda_p+\mu_p+\lambda_s}$. Let $x = \frac{\lambda_p}{\lambda_p+\mu_p+\lambda_s}$ and $f(x) = \frac{\lambda_s}{\mu_s+2\lambda_s} = \frac{x(\lambda_p+\mu_p)-\lambda_p}{x(2\lambda_p+2\mu_p-\mu_s)-2\lambda_p}$, where $x \in \left(0, \frac{\lambda_p}{\mu_p+\lambda_p}\right]$. Then $SW(\lambda_s;1)$ can be rewritten as $F(x) = f(x)[\mu_s R_s(1-x) - c] + \lambda_p R_p x$. It is not difficult to verify that

$$
\begin{aligned}
f'(x) &= -\frac{\lambda_p\mu_s}{[x(2\lambda_p+2\mu_p-\mu_s)-2\lambda_p]^2} < 0,\\
f''(x) &= \frac{2\lambda_p(2\lambda_p+2\mu_p-\mu_s)\mu_s}{[x(2\lambda_p+2\mu_p-\mu_s)-2\lambda_p]^3}.
\end{aligned}
$$

If $\mu_s \geq 2(\lambda_p+\mu_p)$, we have $f''(x) > 0$, which implies that $F''(x) = f''(x)[\mu_s R_s(1-x)-c] - 2\mu_s R_s f'(x) > 0$ because $\frac{\mu_p\mu_s R_s}{\mu_p+\lambda_p} > c$. Otherwise, if $\mu_s < 2(\lambda_p+\mu_p)$, notice that

$$
\begin{aligned}
F''(x) > 0 \Leftrightarrow & f''(x)[\mu_s R_s(1-x)-c] - 2\mu_s R_s f'(x) > 0\\
\Leftrightarrow & (2\lambda_p+2\mu_p-\mu_s)[\mu_s R_s(1-x)-c]\\
& < -\mu_s R_s[x(2\lambda_p+2\mu_p-\mu_s)-2\lambda_p]\\
\Leftrightarrow & (2\mu_p-\mu_s)\mu_s R_s < c(2\lambda_p+2\mu_p-\mu_s).
\end{aligned}
$$

If $2\mu_p \leq \mu_s < 2(\lambda_p+\mu_p)$, we still have $F''(x) > 0$. Therefore, for $\mu_s \geq 2\lambda_p$, $F'(x)$ is increasing in $x \in \left(0, \frac{\lambda_p}{\mu_p+\lambda_p}\right]$. In this case, if $F'(0) \geq 0$, then $F(x)$ is increasing in $x \in \left(0, \frac{\lambda_p}{\mu_p+\lambda_p}\right]$, i.e., $SW(\lambda_s;1)$ is decreasing in λ_s. If $F'(\frac{\lambda_p}{\mu_p+\lambda_p}) \leq 0$, i.e., $SW(\lambda_s;1)$ is increasing in λ_s. Otherwise, we can obtain that $SW(\lambda_s;1)$ is increasing first and then decreasing in λ_s.

In summary, $SW(\lambda_s;1)$ is unimodal in λ_s if $\mu_s \geq 2\lambda_p$. ■

Corollary 1 presents the property of social welfare in λ_s under two special cases (i.e., $\epsilon = 0$ and $\epsilon = 1$), under which the unique social welfare maximizer $\hat{\lambda}_s$ can be derived.

Figure 5 illustrates how the social welfare changes with the effective arrival rate for different ϵ. We can observe that the social welfare shows a different trend when ϵ varies. In particular, when ϵ is small, the social welfare is unimodal in the arrival rate of SUs (i.e., λ_s). When ϵ is intermediate, it is increasing in λ_s. However, when ϵ is large, the social welfare becomes to be decreasing in λ_s. Denote by $\hat{\lambda}_s$ the effective arrival rate of SUs that maximizes the social welfare. Obviously, $\hat{\lambda}_s$ is non-monotone in ϵ by Fig. 5.

To have a better understanding for the impacts of ϵ on the social welfare, we examine the socially optimal effective arrival rate of SUs $\hat{\lambda}_s$ and the

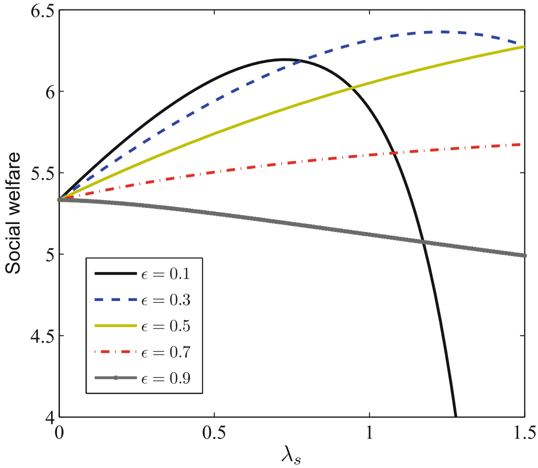
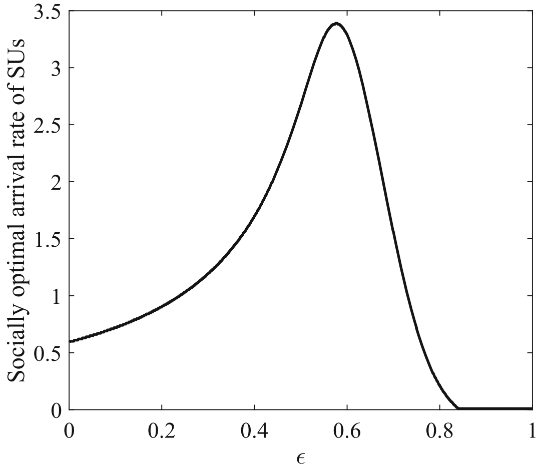
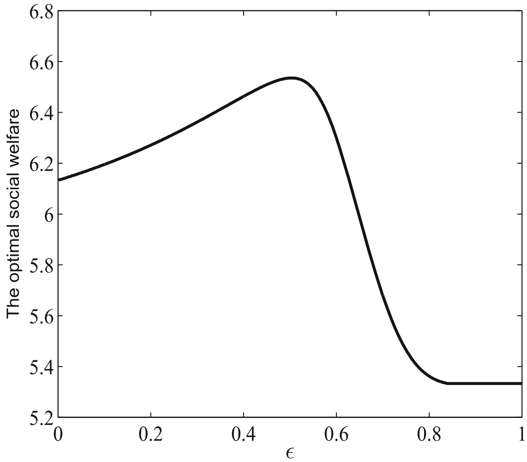


Fig. 5. The social welfare vs. λ_s for different ϵ when $R_s = 3$, $R_p = 8$, $\mu_s = \mu_p = 2$ and $\lambda_p = 1$

corresponding social welfare for different levels of sensing error ϵ in 6. Figure 6/(a) indicates that $\hat{\lambda}_s$ is unimodal in the level of sensing error. When the sensing error level is low, the negative impact of SUs on PU is small, thus increasing the arrival rate of SUs can efficiently improve the social welfare. By contrast, when the sensing error level is high, the frequent visiting of SUs can lead to large amount of sensing errors, which greatly reduce the benefit of PU. As a result, the SUs should be forbidden. Interestingly, we can observe that by Fig. 6/(b) that the optimal social welfare is non-monotone in the error probability ϵ. When ϵ is small, optimal social welfare can be increasing in ϵ because it can reduce the delay of SUs, which increases the expected utility of SUs, which results in a higher social welfare. On the other hand, when ϵ is large, the increased welfare of SUs cannot compensate for the decreased utility of PU, which worsens the social welfare. In sum, combining Fig. 4 and Fig. 6 discloses that some amount of sensing error can lead to a higher throughput and social welfare. The similar result can be found in Wang and Wang [18], which reveals that some amount of lack of information (but not all) in the population can lead to more efficient outcomes in retrial queues.



(a) Socially optimal $\hat{\lambda}_s$



(b) Optimal social welfare

Fig. 6. The socially optimal throughput and welfare for different ϵ when $R_s = 3$, $R_p = 8$, $\mu_s = \mu_p = 2$ and $\lambda_p = 1$

7 Conclusion

This paper conducted a theoretical investigation of equilibrium joining behavior of SUs in CR system when the spectrum sensing error is taken into account. The sensitivity of both SUs' behavior and system performance to the error rate were studied, and some major performance insights were provided accordingly. By adopting the queueing-game approach, the equilibrium strategy of SUs was firstly derived by considering the spectrum sensing error. It is more realistic and also more difficult than the scenario without sensing error in the literature.

In our setting, both ATC and FTC behavior exist (depending on the level of sensing error) which lead to multiple equilibria of the queueing-game model. It is observed that when the sensing error is considered, the system throughput and social welfare are non-monotone in the level of sensing error. It is pretty interesting to disclose the fact that some amount of sensing error (but not all) in the system can lead to more efficient outcomes in terms of throughput and social welfare. For the future work, it would be quite meaningful to incorporate both class-A and class-B sensing errors, as well as a non-linear reward-cost structure.

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