




# Dynamics of a Predator and Prey-Hibernation Model with Genic Mutation and Impulsive Effects

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**Abstract.** A predator and prey-hibernation system with mutation and impulsive effects is presented in this work. The globally asymptotically stable solution  $(\tilde{x}, \tilde{y}, 0)$  of system (1) exists. System (1) is proved to be permanent. Finally, simulations are presented to explain the results.

**Keywords:** Mutation · Predator and prey-hibernation system · Permanence

## 1 Introduction

Many authors [1, 2] indicated that environmental pollutants caused many diseases. Population dynamics are studied by the theories of impulsive differential equations [3–6]. Jiao et al. [7] constructed a predator-prey system with periodic switches and impulses. Jiao and Chen [8] presented a predator-prey system with mutation and impulses. For convenience, we make notation  $N = n\tau$ , and  $N + l = (n + l)\tau$ .

## 2 The Model

In this work, we presented a predator and prey-hibernation system with mutation and impulses

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Supported by National Natural Science Foundation of China (11761019, 11361014), the Science Technology Foundation of Guizhou Education Department (20175736-001, 2008038), the Project of High Level Creative Talents in Guizhou Province (No. 20164035), Major Research Projects on Innovative Groups in Guizhou Provincial Education Department (No. [2018]019).

$$\left. \begin{aligned}
\left. \begin{aligned}
\frac{dx}{dt} &= -ax - cx^2 - \beta_1xz, \\
\frac{dy}{dt} &= -d_1y - \beta_2yz, \\
\frac{dz}{dt} &= -d_2z + k_1\beta_1xz \\
&\quad + k_2\beta_2yz,
\end{aligned} \right\} t \in (N, N+l], \\
\left. \begin{aligned}
\Delta x &= 0, \\
\Delta y &= -\mu_1y, \\
\Delta z &= -\mu_2z,
\end{aligned} \right\} t = (N+l), \\
\left. \begin{aligned}
\frac{dx}{dt} &= -d_3x - \beta_3xz, \\
\frac{dy}{dt} &= -d_4y - \beta_4yz, \\
\frac{dz}{dt} &= -d_5z + k_3\beta_3xz \\
&\quad + k_4\beta_4yz,
\end{aligned} \right\} t \in (N+l, N+1], \\
\left. \begin{aligned}
\Delta x &= (1-\theta_1)b_1x, \\
\Delta y &= \theta_1b_1x, \\
\Delta z &= 0,
\end{aligned} \right\} t = N+1,
\end{aligned} \right\} \tag{1}$$

The biological meanings of the varies and the parameters can reference to [7,8].

### 3 The Lemmas

If  $z = 0$ , we can easily have the subsystem of system (1)

$$\left. \begin{aligned}
\left. \begin{aligned}
\frac{dx}{dt} &= -ax - cx^2, \\
\frac{dy}{dt} &= -d_1y,
\end{aligned} \right\} t \in (N, N+l], \\
\left. \begin{aligned}
\Delta x &= 0, \\
\Delta y &= -\mu_1y,
\end{aligned} \right\} t = N+l, \\
\left. \begin{aligned}
\frac{dx}{dt} &= -d_3x, \\
\frac{dy}{dt} &= -d_4y,
\end{aligned} \right\} t \in (N+l, N+1], \\
\left. \begin{aligned}
\Delta x &= (1-\theta_1)b_1x, \\
\Delta y &= \theta_1b_1x,
\end{aligned} \right\} t = N+1.
\end{aligned} \right\} \tag{2}$$

Integrating (2), and the stroboscopic map of (2) is

$$\begin{cases} x((N+1)^+) = [1 + (1 - \theta_1)b_1] \times \frac{ax(N^+)e^{-[al+d_3(1-l)]\tau}}{a + cx(N^+)(1 - e^{-al\tau})}, \\ y((N+1)^+) = \theta_1 b_1 \times \frac{ax(N^+)e^{-[al+d_3(1-l)]\tau}}{a + cx(N^+)(1 - e^{-al\tau})} \\ \quad + (1 - \mu_1)e^{-[d_1l+d_4(1-l)]\tau}y(N^+). \end{cases} \quad (3)$$

Points  $G_1(0,0)$  and  $G_2(x^*, y^*)$  are gotten as

$$\begin{cases} x^* = Ay^*, \quad \theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1. \\ y^* = \frac{a}{cA(1 - e^{-al\tau})} \times \left[ \frac{\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau}}{1 - B} - 1 \right], \\ \quad \theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1, \end{cases} \quad (4)$$

here  $A = \frac{[1+(1-\theta_1)b_1](1-B)}{\theta_1 b_1} > 0$  and  $B = (1 - \mu_1)e^{-[d_1l+d_4(1-l)]\tau} > 0$ .

Similarly with reference [7], the following lemmas can easily be obtained.

**Lemma 1.** *i)* If  $\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B < 1$ , point  $G_1(0,0)$  is globally asymptotically stable;

*ii)* If  $\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1$ , point  $G_2(x^*, y^*)$  is globally asymptotically stable.

**Lemma 2.** *i)* If  $\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B < 1$ , periodic solution  $(0,0)$  of system (2) is globally asymptotically stable;

*ii)* If  $\theta_1 b_1 A e^{-[al+d_3(1-l)]\tau} + B > 1$ , periodic solution  $(\tilde{x}, \tilde{y})$  of system (2) is globally asymptotically stable, where

$$\begin{cases} \tilde{x} = \begin{cases} \frac{ax^*e^{-a(t-N)}}{a + cx^*(1 - e^{-a(t-N)})}, t \in (N, (N+1)], \\ x^{**}e^{-d_3(t-(N+l))}, t \in ((N+l), (N+1)], \end{cases} \\ \tilde{y} = \begin{cases} y^*e^{-d_1(t-N)}, t \in (N, (N+l)], \\ y^{**}e^{-d_4(t-(N+l))}, t \in ((N+l), (N+1)], \end{cases} \end{cases} \quad (5)$$

here  $x^*, y^*$  are determined as (5) and  $x^{**}, y^{**}$  are determined as

$$\begin{cases} x^{**} = \frac{ax^*e^{-al\tau}}{a + cx^*(1 - e^{-al\tau})}, \\ y^{**} = (1 - \mu_1)y^*e^{-d_1l\tau}. \end{cases} \quad (6)$$

For (1), we know

$$\left. \begin{cases} \frac{dx}{dt} \leq -ax - cx^2, \\ \frac{dy}{dt} \leq -d_1y, \end{cases} \right\} t \in (N, (N+l)],$$

$$\left. \begin{cases} \Delta x = 0, \\ \Delta y = -\mu_1y, \end{cases} \right\} t = (N+l),$$

$$\left. \begin{cases} \frac{dx}{dt} \leq -d_3x, \\ \frac{dy}{dt} \leq -d_4y, \end{cases} \right\} t \in ((N+l), (N+1)],$$

$$\left. \begin{cases} \Delta x = (1 - \theta_1)b_1x, \\ \Delta y = \theta_1b_1x, \end{cases} \right\} t = (N+1).$$
(7)

Then, we can obtain the following remark.

**Remark 3.** If  $\theta_1b_1Ae^{-[al+d_3(1-l)]\tau} + B > 1$ , then

$$x(t) \leq x^* + x^{**},$$

and

$$y(t) \leq y^* + y^{**},$$

with  $(x, y)$  of system (1).

Similar to Ref. [13], we get

**Lemma 4.** There exists a constant  $M > 0$ , which makes  $x \leq \frac{M}{k}, y \leq M$  and  $z \leq M$  for all  $t$  large enough.

## 4 The Dynamics

**Theorem 1.** Suppose

(H<sub>1</sub>):

$$\theta_1b_1Ae^{-[al+d_3(1-l)]\tau} + B > 1,$$

(H<sub>2</sub>):

$$\ln \frac{1}{1 + (1 - \theta_1)b_1} + al\tau + 2 \ln \frac{a + cx^*(1 - e^{-al\tau})}{a} + d_3(1 - l)\tau > 0,$$

(H<sub>3</sub>):

$$\begin{aligned} & \ln \frac{1}{1 - \mu_2} + d_2l\tau + d_5(1 - l)\tau \\ & > \frac{k_1\beta_1}{c} \ln \frac{a + cx^*(1 - e^{-al\tau})}{a} + \frac{k_2\beta_2y^*(1 - e^{-d_1l\tau})}{d_1} \end{aligned}$$

$$+ \frac{k_3\beta_3x^{**}e^{-d_3(1-l)\tau}(1 - e^{-d_1(1-l)\tau})}{d_3} + \frac{k_4\beta_4y^{**}e^{-d_5(1-l)\tau}(1 - e^{-d_5(1-l)\tau})}{d_5},$$

hold, the solution  $(\tilde{x}, \tilde{y}, 0)$  of (1) is globally asymptotically stable, and  $x^*, x^{**}$  are by (5),  $y^*, y^{**}$  are by (7).

**Proof.** Defining  $x_1 = x - \tilde{x}, y_1 = y - \tilde{y}, z_1 = z$ , for  $t \in (N, (N + l)]$ , the linear system of (1) are as

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dy_1}{dt} \\ \frac{dz_1}{dt} \end{pmatrix} = \begin{pmatrix} -a - 2c\tilde{x} & 0 & -\beta_1\tilde{x} \\ 0 & -d_1 & -\beta_2\tilde{y} \\ 0 & 0 & -d_2 + k_1\beta_1\tilde{x} + k_2\beta_2\tilde{y} \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}.$$

Then, we get the fundamental solution matrix

$$\Phi_1(t) = \begin{pmatrix} \exp(\int_N^t (-a - 2c\tilde{x})ds) & *_1 & *_2 \\ 0 & \exp(-d_1(t - N)) & *_3 \\ 0 & 0 & M_1 \end{pmatrix}$$

where  $M_1 = \exp[\int_N^t (-d_2 + k_1\beta_1\tilde{x} + k_2\beta_2\tilde{y})ds]$ .

For  $t \in ((N + l), (N + 1)]$ , the linear system of (1) are also as

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dy_1}{dt} \\ \frac{dz_1}{dt} \end{pmatrix} = \begin{pmatrix} -d_3 & 0 & -\beta_3\tilde{x} \\ 0 & -d_4 & -\beta_4\tilde{y} \\ 0 & 0 & -d_5 + k_3\beta_3\tilde{x} + k_4\beta_4\tilde{y} \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}.$$

Then, we also get the fundamental solution matrix

$$\Phi_2(t) = \begin{pmatrix} \exp(-d_3(t - (N + l))) & *_4 & *_5 \\ 0 & \exp(-d_4(t - (N + l))) & *_6 \\ 0 & 0 & M_2 \end{pmatrix},$$

where  $M_2 = \exp[\int_{(N+l)}^t (-d_5 + k_3\beta_3\tilde{x} + k_4\beta_4\tilde{y})ds]$ . There is no need to calculate the form of  $*_i (i = 1, 2, 3, 4, 5, 6)$ .

The linearization of the 4th, to 6th equations of (1) is

$$\begin{pmatrix} x_1((N + l)^+) \\ y_1((N + l)^+) \\ z_1((N + l)^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \mu_1 & 0 \\ 0 & 0 & 1 - \mu_2 \end{pmatrix} \begin{pmatrix} x_1((N + l)) \\ y_1((N + l)) \\ z_1((N + l)) \end{pmatrix}.$$

The linearization of the 10th, to 12th equations of (1) is

$$\begin{pmatrix} x_1((N+1)^+) \\ y_1((N+1)^+) \\ z_1((N+1)^+) \end{pmatrix} = \begin{pmatrix} 1 + (1 - \theta_1)b_1 & 0 & 0 \\ \theta_1 b_1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1((N+1)) \\ y_1((N+1)) \\ z_1((N+1)) \end{pmatrix}.$$

The stability of  $(\tilde{x}, \tilde{y}, 0)$  is by eigenvalues of

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \mu_1 & 0 \\ 0 & 0 & 1 - \mu_2 \end{pmatrix} \begin{pmatrix} 1 + (1 - \theta_1)b_1 & 0 & 0 \\ \theta_1 b_1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(\tau),$$

which are

$$\lambda_1 = [1 + (1 - \theta_1)b_1] \exp\left(\int_0^{l\tau} (-a - 2c\tilde{x})ds + d_3(1 - l)\tau\right),$$

$$\lambda_2 = (1 - \mu_1) \exp(-[d_1 l + d_4(1 - l)]\tau) < 1,$$

and

$$\lambda_3 = (1 - \mu_2) \exp\left[\int_0^{l\tau} (-d_2 + k_1\beta_1\tilde{x} + k_2\beta_2\tilde{y})ds + \int_{l\tau}^{\tau} (-d_5 + k_3\beta_3\tilde{x} + k_4\beta_4\tilde{y})ds\right].$$

According the conditions of Theorem 1, then  $\lambda_1 < 1$ , and  $\lambda_3 < 1$ . Therefore, the  $(\tilde{x}, \tilde{y}, 0)$  is locally stable.

Choose  $\varepsilon > 0$ , we get

$$\begin{aligned} \rho_1 &= (1 - \mu_2) \exp\left\{\int_0^{l\tau} [-d_2 + k_1\beta_1(\tilde{x} + \varepsilon) + k_2\beta_2(\tilde{y} + \varepsilon)]ds\right. \\ &\quad \left.+ \int_{l\tau}^{\tau} [-d_5 + k_3\beta_3(\tilde{x} + \varepsilon) + k_4\beta_4(\tilde{y} + \varepsilon)]ds\right\} < 1. \end{aligned}$$

From (1), we get  $\frac{dx}{dt} \leq -ax - cx^2$ ,  $\frac{dy}{dt} \leq -d_1x$ , and  $\frac{dx}{dt} \leq -d_3x$ ,  $\frac{dy}{dt} \leq -d_4x$ , so

$$\left. \begin{aligned} &\left. \begin{aligned} \frac{dx_1}{dt} &= -ax_1(t) - cy_1^2, \\ \frac{dy_1}{dt} &= -d_1y_1, \end{aligned} \right\} t \neq (N, (N+l)), \\ &\left. \begin{aligned} \Delta x_1 &= 0, \\ \Delta y_1 &= -\mu_1 y_1, \end{aligned} \right\} t = (N+l), \\ &\left. \begin{aligned} \frac{dx_1}{dt} &= -d_3x_1, \\ \frac{dy_1}{dt} &= -d_4y_1, \end{aligned} \right\} t \in ((N+l), (N+1)), \\ &\left. \begin{aligned} \Delta x_1 &= (1 - \theta_1)b_1x_1, \\ \Delta y_1 &= \theta_1b_1y_1, \end{aligned} \right\} t = (N+1), \end{aligned} \right\} \tag{8}$$

From Lemma 3, we have  $x \leq x_1, y \leq y_1$ , and

$$\begin{cases} x \leq x_1 \leq \tilde{x} + \varepsilon, \\ y \leq y_1 \leq \tilde{y} + \varepsilon, \end{cases} \quad (9)$$

From (1) and (14), we also get

$$\begin{cases} \frac{dz}{dt} \leq z[-d_2 + k_1\beta_1(\tilde{x} + \varepsilon) + k_2\beta_2(\tilde{y} + \varepsilon)], \\ \quad \quad \quad t \in (N, (N + l)] \\ \Delta z = -\mu_2 z, t = (N + l), \\ \frac{dz}{dt} \leq z[-d_5 + k_3\beta_3(\tilde{x} + \varepsilon) + k_4\beta_4(\tilde{y} + \varepsilon)], \\ \quad \quad \quad t \in ((N + l), (N + 1)], \end{cases} \quad (10)$$

Then,

$$\begin{aligned} z((N + 1)) &\leq (1 - \mu_2)z(N^+) \exp\left[\int_N^{(N+l)} (-d_2 + k_1\beta_1(\tilde{x} + \varepsilon) + k_2\beta_2(\tilde{y} + \varepsilon))ds\right. \\ &\quad \left. + \int_{(N+l)}^{(N+1)} (-d_5 + k_3\beta_3(\tilde{x} + \varepsilon) + k_4\beta_4(\tilde{y} + \varepsilon))ds\right]. \end{aligned}$$

and  $z((N + l)) \leq z(l\tau^+)\rho_1^n$  with  $z((N + l)) \rightarrow 0$  as  $n \rightarrow \infty$ . For  $0 < z \leq z((N + l))e^{(-d_5(1-l)\tau + \frac{k_3\beta_3}{d_3}x^{**} + \frac{k_4\beta_4}{d_4}y^{**})}$  for  $N < t \leq (N + l)$ . Therefore  $z \rightarrow 0$  as  $t \rightarrow \infty$ .

For (1) and  $t \in (N, (N + l)]$ , we have

$$\begin{cases} -(a + \beta_1\varepsilon)x - cx^2 \leq \frac{dx}{dt} \leq -ax - cx^2, \\ -(d_1 + \beta_2\varepsilon)y \leq \frac{dy}{dt} \leq -d_1y, \end{cases} \quad (11)$$

and for  $t \in ((N + l), (N + 1)]$ ,

$$\begin{cases} -(d_3 + \beta_3\varepsilon)x \leq \frac{dx}{dt} \leq -d_1x, \\ -(d_4 + \beta_4\varepsilon)y \leq \frac{dy}{dt} \leq -d_4y, \end{cases} \quad (12)$$

then,  $w_1 \leq x \leq m_1, w_2 \leq y \leq m_2$ , with  $w_1 \rightarrow \widetilde{w}_1, w_2 \rightarrow \widetilde{w}_2, m_1 \rightarrow \widetilde{x}, m_2 \rightarrow \widetilde{y}$ , as  $t \rightarrow \infty$ . While  $(w_1, w_2)$  and  $(m_1, m_2)$  are the solutions of

$$\left. \begin{aligned} \left. \begin{aligned} \frac{dw_1}{dt} &= -(a + \beta_1 \varepsilon)w_1 - cw_1^2, \\ \frac{dw_2}{dt} &= -(d_1 + \beta_2 \varepsilon)w_2, \end{aligned} \right\} t \in (N, (N + l)), \\ \left. \begin{aligned} \Delta w_1 &= 0, \\ \Delta w_2 &= -\mu_1 w_2, \end{aligned} \right\} t = (N + l), \\ \left. \begin{aligned} \frac{dw_1}{dt} &= -(d_3 + \beta_3 \varepsilon)w_1, \\ \frac{dw_2}{dt} &= -(d_4 + \beta_4 \varepsilon)w_2, \end{aligned} \right\} t \in ((N + l), (N + 1)], \\ \left. \begin{aligned} \Delta w_1 &= (1 - \theta_1)b_1 w_1, \\ \Delta w_2 &= \theta_1 b_1 w_1, \end{aligned} \right\} t = (N + 1), \end{aligned} \right\} \quad (13)$$

and

$$\left. \begin{aligned} \left. \begin{aligned} \frac{dm_1}{dt} &= -am_1 - cm_1^2, \\ \frac{dm_2}{dt} &= -d_1 m_2, \end{aligned} \right\} t \in (N, (N + l)), \\ \left. \begin{aligned} \Delta m_1 &= 0, \\ \Delta m_2 &= -\mu_1 m_2, \end{aligned} \right\} t = (N + l), \\ \left. \begin{aligned} \frac{dm_1}{dt} &= -d_3 m_1, \\ \frac{dm_2}{dt} &= -d_4 m_2, \end{aligned} \right\} t \in ((N + l), (N + 1)], \\ \left. \begin{aligned} \Delta m_1 &= (1 - \theta_1)b_1 m_1, \\ \Delta m_2 &= \theta_1 b_1 m_1, \end{aligned} \right\} t = (N + 1). \end{aligned} \right\} \quad (14)$$

Here  $(\widetilde{w}_1, \widetilde{w}_2)$  can be expressed as

$$\left\{ \begin{aligned} \widetilde{w}_1 &= \begin{cases} \frac{(a + \beta_1 \varepsilon)w_1^* e^{-(a + \beta_1 \varepsilon)(t - N)}}{(a + \beta_1 \varepsilon) + cw_1^*(1 - e^{-(a + \beta_1 \varepsilon)(t - N)})}, & t \in (N, (N + 1)], \\ w_1^{**} e^{-(d_3 + \beta_3 \varepsilon)(t - (N + l))}, & t \in ((N + l), (N + 1)], \end{cases} \\ \widetilde{w}_2 &= \begin{cases} w_2^* e^{-(d_1 + \beta_2 \varepsilon)(t - N)}, & t \in (N, (N + l)], \\ w_2^{**} e^{-(d_4 + \beta_4 \varepsilon)(t - (N + l))}, & t \in ((N + l), (N + 1)], \end{cases} \end{aligned} \right\} \quad (15)$$

while  $w_1^*$ ,  $w_2^*$  are determined as

$$\begin{cases} w_1^* = A_1 w_2^*, & \theta_1 b_1 A_1 e^{-[(a+\beta_1\varepsilon)l+(d_3+\beta_3\varepsilon)(1-l)]\tau} + B_1 > 1, \\ w_2^* = \frac{(a + \beta_1\varepsilon)}{c A_1 (1 - e^{-(a+\beta_1\varepsilon)l\tau})} \times \left[ \frac{\theta_1 b_1 A_1 e^{-[(a+\beta_1\varepsilon)l+(d_3+\beta_3\varepsilon)(1-l)]\tau}}{1 - B_1} - 1 \right], \\ \theta_1 b_1 A_1 e^{-[(a+\beta_1\varepsilon)l+(d_3+\beta_3\varepsilon)(1-l)]\tau} + B_1 > 1, \end{cases} \quad (16)$$

where  $A_1 = \frac{[1+(1-\theta_1)b_1](1-B_1)}{\theta_1 b_1} > 0$  and  $B_1 = (1-\mu)e^{-[(d_1+\beta_2\varepsilon)l+(d_4+\beta_4\varepsilon)(1-l)]\tau} > 0$ , and  $w_1^{**}$ ,  $w_2^{**}$  are determined as

$$\begin{cases} w_1^{**} = \frac{(a + \beta_1\varepsilon)w_1^* e^{-(a+\beta_1\varepsilon)l\tau}}{(a + \beta_1\varepsilon) + c w_1^* (1 - e^{-(a+\beta_1\varepsilon)l\tau})}, \\ w_2^{**} = (1 - \mu_1)w_2^* e^{-(d_1+\beta_2\varepsilon)l\tau}. \end{cases} \quad (17)$$

For any  $\varepsilon_1 > 0$ , there exists a  $t_1, t > t_1$  such that

$$\widetilde{w}_1 - \varepsilon_1 < x < \widetilde{x} + \varepsilon,$$

and

$$\widetilde{w}_2 - \varepsilon_1 < y < \widetilde{y} + \varepsilon.$$

Let  $\varepsilon \rightarrow 0$ , we have

$$\widetilde{x} - \varepsilon_1 < x < \widetilde{x} + \varepsilon_1,$$

and

$$\widetilde{y} - \varepsilon_1 < y < \widetilde{y} + \varepsilon_1,$$

for  $t$  large enough, which indicates  $x \rightarrow \widetilde{x}, y \rightarrow \widetilde{y}$  as  $t \rightarrow \infty$ .

**Theorem 2.** If  $(H_1)$ ,  $(H_2)$  and  $(H_4)$ :

$$\begin{aligned} & \ln \frac{1}{1 - \mu_2} + d_2 l \tau + d_5 (1 - l) \tau \\ & < \frac{k_1 \beta_1}{c} \ln \frac{a + c x^* (1 - e^{-a l \tau})}{a} + \frac{k_2 \beta_2 y^* (1 - e^{-d_1 l \tau})}{d_1} \\ & + \frac{k_3 \beta_3 x^{**} e^{-d_3 (1-l)\tau} (1 - e^{-d_1 (1-l)\tau})}{d_3} + \frac{k_4 \beta_4 y^{**} e^{-d_5 (1-l)\tau} (1 - e^{-d_5 (1-l)\tau})}{d_5}, \end{aligned}$$

hold, (1) is permanent, where  $x^*, x^{**}$  are by (5) and  $y^*, y^{**}$  are by (7).

**Proof.** By Lemma 3, we get that  $x(t) \leq \frac{M}{k}$ ,  $y(t) \leq \frac{M}{k}$ ,  $z(t) \leq M$  for  $t$  large enough. From Theorem 1, we know  $x(t) \geq \frac{ax^*e^{-at\tau}}{a+cx^*(1-e^{-at\tau})} + x^{**}e^{-d_3(1-l)\tau} - \varepsilon_2 = m_{21}$  and  $y(t) \geq y^*e^{-d_1l\tau} + y^{**}e^{-d_4(1-l)\tau} - \varepsilon_2 = m_{22}$  for  $t$ . Thus, we will seek out  $m_1 > 0$  making  $y(t) \geq m_1$ .

By the conditions  $H_4$ , we can choose  $m_3 > 0, \varepsilon_1 > 0$  small enough to have

$$\begin{aligned} \sigma = & \frac{k_1\beta_1}{c} \ln \frac{(a + \beta_1 m_3) + cz_1^*(1 - e^{-(a+\beta_1 m_3)l\tau})}{(a + \beta_1 m_3)} + k_1\beta_1\varepsilon_1 l\tau \\ & + \frac{k_2 + \beta_2 z_2^*(1 - e^{-(d_1+\beta_1 m_3)l\tau})}{(d_1 + \beta_1 m_3)l\tau + k_2\beta_2 m_3 l\tau} + k_2\beta_2\varepsilon_1 l\tau \\ & + \frac{k_3\beta_3 z_1^{**} e^{-(d_3+\beta_1 m_3)(1-l)\tau} (1 - e^{-(d_1+\beta_1 m_3)(1-l)\tau})}{(d_3 + \beta_1 m_3)} + k_3\beta_3\varepsilon_1(1-l)\tau \\ & + \frac{k_4\beta_4 z_2^{**} e^{-(d_5+\beta_5 m_3)(1-l)\tau} (1 - e^{-(d_5+\beta_5 m_3)(1-l)\tau})}{(d_5 + \beta_5 m_3)} + k_4\beta_4\varepsilon_1(1-l)\tau, \\ & - \ln \frac{1}{1 - \mu_2} - d_2 l\tau - d_5(1-l)\tau] > 0, \end{aligned}$$

here  $z_1^*$ ,  $z_2^*$  are by

$$\left\{ \begin{array}{l} z_1^* = A_2 z_2^*, \quad \theta_1 b_1 A_2 e^{-[(a+\beta_1 m_3)l+(d_3+\beta_3 m_3)(1-l)]\tau} + B_2 > 1, \\ z_2^* = \frac{(a + \beta_1 m_3)}{c A_2 (1 - e^{-(a+\beta_1 m_3)l\tau})} \\ \quad \times \left[ \frac{\theta_1 b_1 A_2 e^{-[(a+\beta_1 m_3)l+(d_3+\beta_3 m_3)(1-l)]\tau}}{1 - B_2} - 1 \right], \\ \theta_1 b_1 A_2 e^{-[(a+\beta_1 m_3)l+(d_3+\beta_3 m_3)(1-l)]\tau} + B_2 > 1, \end{array} \right. \quad (18)$$

and  $z_1^{**}$ ,  $z_2^{**}$  are by

$$\left\{ \begin{array}{l} z_1^{**} = \frac{(a + \beta_1 m_3) z_1^* e^{-(a+\beta_1 m_3)l\tau}}{(a + \beta_1 m_3) + cz_1^*(1 - e^{-(a+\beta_1 m_3)l\tau})}, \\ z_2^{**} = (1 - \mu_1) z_2^* e^{-(d_1+\beta_2 m_3)l\tau}. \end{array} \right. \quad (19)$$

with  $A_2 = \frac{[1+(1-\theta_1)b_1](1-B_2)}{\theta_1 b_1} > 0$  and  $B_2 = (1 - \mu) e^{-[(d_1+\beta_2 m_3)l+(d_4+\beta_4 m_3)(1-l)]\tau} > 0$ .  $y(t) < m_3$  will be proved that it can not establish. Otherwise,

$$\left. \begin{array}{l} \left. \begin{array}{l} \frac{dx}{dt} \geq -(a + \beta_1 m_3)x - cx^2, \\ \frac{dy}{dt} \geq -(d_1 + \beta_2 m_3)y, \end{array} \right\} t \in (N, (N + l)], \\ \left. \begin{array}{l} \Delta x = 0, \\ \Delta y = -\mu_1 y, \end{array} \right\} t = (N + l), \end{array} \right\} (20)$$

$$\left. \begin{array}{l} \left. \begin{array}{l} \frac{dx}{dt} \geq -(d_3 + \beta_3 m_3)x, \\ \frac{dy}{dt} \geq -(d_4 + \beta_4 m_3)y, \end{array} \right\} t \in ((N + l), (N + 1)], \\ \left. \begin{array}{l} \Delta x = (1 - \theta_1)b_1 x, \\ \Delta x = \theta_1 b_1 x, \end{array} \right\} t = (N + 1). \end{array} \right\}$$

By Lemma 4, we have  $x \geq z_1$ ,  $y \geq z_2$  with  $z_1 \rightarrow \tilde{z}_1$ ,  $z_2 \rightarrow \tilde{z}_2$ ,  $t \rightarrow \infty$ , where  $(z_1, z_2)$  is the solution of

$$\left. \begin{array}{l} \left. \begin{array}{l} \frac{dz_1}{dt} = -(a + \beta_1 m_3)z_1 - cz_1^2, \\ \frac{dz_2}{dt} = -(d_1 + \beta_2 m_3)z_2 \end{array} \right\} t \in (N, (N + l)], \\ \left. \begin{array}{l} \Delta z_1 = 0, \\ \Delta z_2 = -\mu_1 z_2, \end{array} \right\} t = (N + l), \end{array} \right\} (21)$$

$$\left. \begin{array}{l} \left. \begin{array}{l} \frac{dz_1}{dt} = -(d_3 + \beta_3 m_3)z_1, \\ \frac{dz_2}{dt} = -(d_4 + \beta_4 m_3)z_2, \end{array} \right\} t \in ((N + l), (N + 1)], \\ \left. \begin{array}{l} \Delta z_1 = (1 - \theta_1)b_1 z_1, \\ \Delta z_2 = \theta_1 b_1 z_2, \end{array} \right\} t = (N + 1), \end{array} \right\}$$

and

$$\left. \begin{array}{l} \tilde{z}_1 = \begin{cases} \frac{(a + \beta_1 m_3)z_1^* e^{-(a + \beta_1 m_3)(t - n\tau)}}{(a + \beta_1 m_3) + cz_1^*(1 - e^{-(a + \beta_1 m_3)(t - N)})}, & t \in (N, (N + 1)], \\ z_1^{**} e^{-(d_3 + \beta_3 m_3)(t - (N + l))}, & t \in ((N + l), (N + 1)], \end{cases} \\ \tilde{z}_2 = \begin{cases} z_2^* e^{-(d_1 + \beta_2 m_3)(t - N)}, & t \in (N, (N + l)], \\ z_2^{**} e^{-(d_4 + \beta_4 m_3)(t - (N + l))}, & t \in ((N + l), (N + 1)], \end{cases} \end{array} \right\} (22)$$

here  $z_1^*$ ,  $z_2^*$  are by (18) and  $z_1^{**}$ ,  $z_2^{**}$  are by (19) with  $A_2 = \frac{[1 + (1 - \theta_1)b_1](1 - B_2)}{\theta_1 b_1} > 0$  and  $B_2 = (1 - \mu)e^{-[(d_1 + \beta_2 m_3)l + (d_4 + \beta_4 m_3)(1 - l)]\tau} > 0$ . Therefore, there exists a  $T_1 > 0$  such that

$$x \geq z_1 \geq \tilde{z}_1 - \varepsilon_1,$$

and

$$y \geq z_2 \geq \tilde{z}_2 - \varepsilon_1.$$

Then

$$\begin{cases} \frac{dz}{dt} \geq [-d_1 + k_1\beta_1(\tilde{z}_1 - \varepsilon_1) + k_2\beta_2(\tilde{z}_2 - \varepsilon_1)]z, \\ \qquad \qquad \qquad t \in (N, (N + l)), \\ \Delta z = -\mu_2 z, t = (N + l), \\ \frac{dz}{dt} \geq [-d_5 + k_3\beta_3(\tilde{z}_1 - \varepsilon_1) + k_4\beta_4(\tilde{z}_2 - \varepsilon_1)]z, \\ \qquad \qquad \qquad t \in ((N + l), (N + 1)]. \end{cases} \quad (23)$$

For  $t \geq T_1$ , Let  $N_1 \in N$  and  $N_1\tau > T_1$ , integrating (28) on  $(N, (N + 1)), n \geq N_1$ , we have

$$\begin{aligned} y((N + 1)) &\geq z(N^+)(1 - \mu_1) \\ &\times e^{\int_N^{(N+1)} [-d_1 + k_1\beta_1(\tilde{z}_1 - \varepsilon_1) + k_2\beta_2(\tilde{z}_2 - \varepsilon_1)] ds + \int_{(N+1)}^{(N+1)} [-d_5 + k_3\beta_3(\tilde{z}_1 - \varepsilon_1) + k_4\beta_4(\tilde{z}_2 - \varepsilon_1)] ds} \\ &= (1 - \mu_1)z(N^+)e^\sigma, \end{aligned}$$

then  $z((N_1 + k)\tau) \geq (1 - \mu_1)^k z(N_1\tau^+)e^{k\sigma} \rightarrow \infty$ , as  $k \rightarrow \infty$ , which is a contradiction to the boundedness of  $z$ . Hence there exists a  $t_1 > 0$  such that  $z(t) \geq m_1$ . The proof is complete.

### 5 Discussion

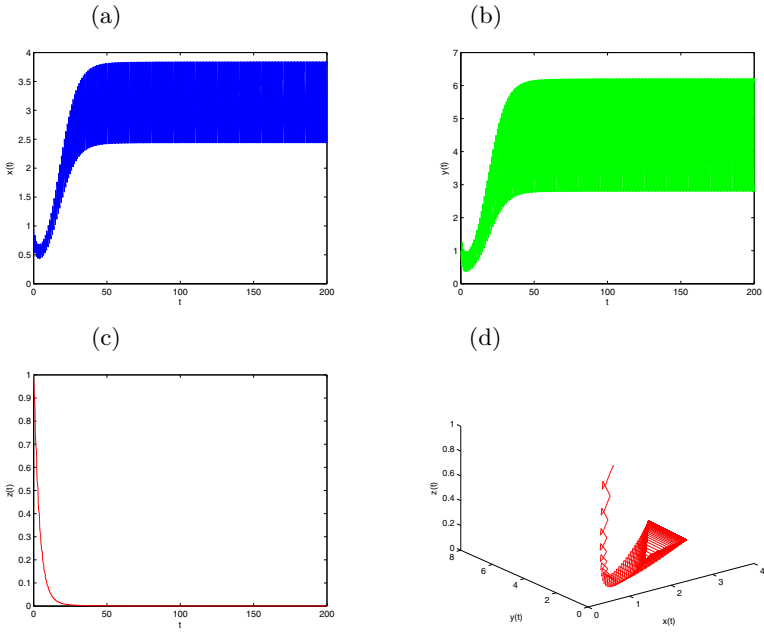
In this work, we consider a predator and prey-hibernation model with genic mutation and impulsive effects. If it is supposed that the variables are shown in the table below:

$x(0)$	$y(0)$	$z(0)$	$a$	$c$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$k_1$	$k_2$	$k_3$	$k_4$	$\theta_1$	$b_1$	$\mu_1$	$\mu_2$	$l$	$\tau$
1	1	1	0.3	0.1	0.3	0.3	0.3	0.1	0.1	0.4	0.6	0.3	0.4	0.1	0.1	0.1	0.1	0.6	1.5	0.4	0.1	0.5	1

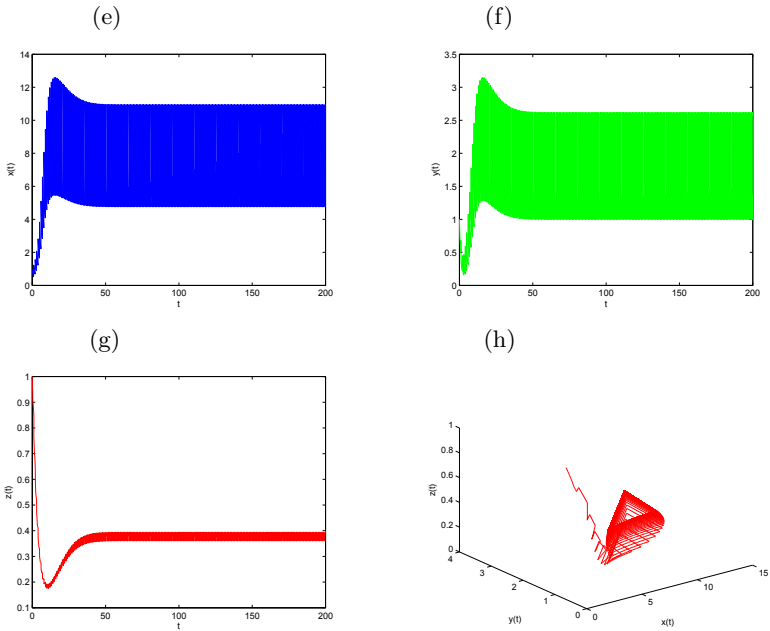
system (1) is permanent (one can see Fig.1). If it is supposed that another variables are shown in the table below:

$x(0)$	$y(0)$	$z(0)$	$a$	$c$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$k_1$	$k_2$	$k_3$	$k_4$	$\theta_1$	$b_1$	$\mu_1$	$\mu_2$	$l$	$\tau$
1	1	1	0.3	0.1	0.3	0.3	0.3	0.1	0.1	0.4	0.6	0.3	0.4	0.1	0.1	0.1	0.1	0.6	1.5	0.4	0.1	0.5	1

there exists a globally asymptotically stable solution  $(\tilde{x}, \tilde{y}, 0)$  of system (1) (one can see Fig. 2). Our results show that the environmental pollution will reduce biological diversity of the nature world. So we must be in harmony with the environment.



**Fig. 1.** The permanence of system (2.1) with parameters in the first table.



**Fig. 2.** The dynamics of the globally asymptotically stable microorganism-extinction with parameters in the second table.

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