# The Performance Research of LTE-A Cellular Network Based on Relay and Pass-Through D2D Technology

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**Abstract.** With the development of 5G technology, the demand for communication quality has shown exponential growth. Regarding the problem of low communication quality at the edge of the LTE-Advanced cellular network, we can expand the coverage and share base station traffic by introducing relay technology and D2D technology, thereby increasing the throughput of the communication system. The research background of this paper is to combine pass-through (direct) D2D mode with LTE-A relay cellular network in LTE-A cells, and then construct the MINLP optimization problem with maximizing system throughput as the objective function. The convex optimization method is used to realize the optimal allocation of power resources. The final simulation results show the growth of system throughput and user fairness.

**Keywords:** LTE-Advanced cellular network · Relay technology · D2D technology · Convex optimization · Throughput

## 1 Introduction

The addition of relay nodes in the LTE-A system can cover a wider area and achieve higher-quality communication, while it also greatly increases the complexity of the communication system, so it has become the research focus of scholars at home and abroad. To improve system throughput, we should first consider to control the channel interference, the solutions mainly adopt the partial frequency reuse. [1] proposes a method for selecting relay nodes in a mobile Ad hoc network. Based on the messages received from one or other nodes, the list of adjacent relay nodes is updated, and the adjacent relay nodes in the first hop is updated. In the relay nodes and the updated relay list, the relay with the highest value of the selection counter is selected as the MPR node. The method

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can prevent unnecessary channel competition and conflict between nodes, so it can improve network performance. A new round-robin scheduling algorithm using location information is proposed in [2], so that some frequencies can be reused by base stations or relays after selection. It has been proved that this algorithm can effectively control channel interference and improve the system throughput. [3] proposes an optimization algorithm based on energy efficiency in a large-scale LTE network in a dense urban area. The optimization variables include the relevant parameters of the base station and the relay antenna, such as the azimuth, transmission power, and antenna height. Through overall optimization, the energy consumption of users is reduced, and the communication performance of users is improved. [4] studies the power allocation scheme of subcarriers aiming at maximizing energy efficiency in a cooperative relay network, it mainly studies the situation that the destination nodes receive signals from the source nodes and relay nodes at the same time. [5] in the context of OFDMA downlink, resource allocation is performed by maximizing the minimum weighted rate under the constraint of the transmission power of each base station. This allocation method ensures that the information rate of each cell is roughly the same at the expense of the total data rate of the system, so as to make the system have better fairness. [6] is similar to [5], it also sacrifices the overall throughput of the system to ensure the QoS of each user. Under the condition of a certain total power, the allocation strategies of subcarrier, power and relay are combined at the same time to establish a complex optimization problem, which is solved combined with the optimization theory.

Due to the advantages of D2D technology in sharing data traffic in cellular networks, it has recently become a hot research topic. [7] studies the resource allocation algorithm for single-cell users and multiple D2D users using game theory methods in high-density D2D user scenarios. [8] discusses the link sharing mechanism and user resource allocation strategy of the multi-user D2D communication system. With the purpose of improving the energy efficiency of users, a distributed resource allocation scheme is proposed by using game and non-cooperative game methods. [9] studies the interference coordination problem of D2D communication under homogeneous network. Under the condition of satisfying QoS, a channel allocation and power control method based on partial location information is proposed.

The main research content of this paper is to establish the resource sharing model between direct D2D users and cellular users in LTE-A cellular network by combining relay technology and direct D2D technology, establish the objective function of maximizing system throughput, and use convex optimization method to solve this problem. Finally, the throughput and user fairness are analyzed.

Notation: $a^*$ represent the conjugate.

## 2 System Model for the Coexistence of Direct D2D Users and Cellular Users

The system model is shown in Fig. 1, which contains both direct D2D users and cellular users. The cellular network adopts Time Division Duplex (TDD) mode.

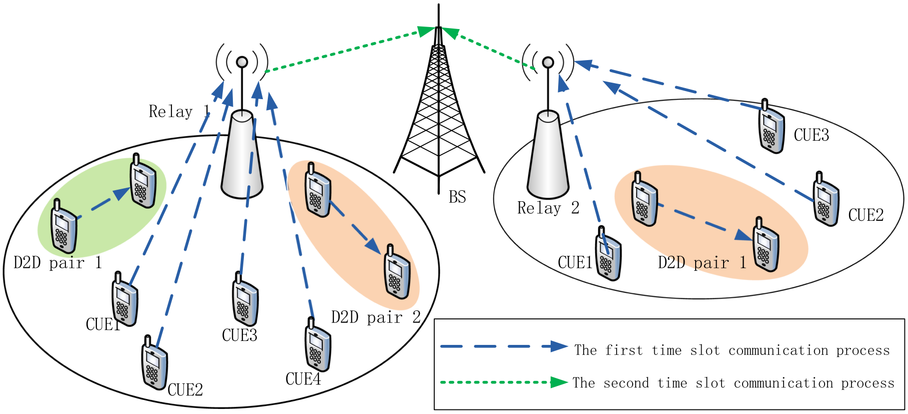


**Fig. 1.** System model of coexistence of direct D2D users and cellular users.

The entire communication process is divided into two time slots, and all relays communicate synchronously in these two time slots. All cellular users (not using D2D mode) are represented by set $M$, and all direct D2D users is represented by set $D^p$, the system bandwidth is divided into $N$ resource blocks (RBs), the set of resource blocks is $N = \{1, 2, \cdots, |N|\}$, the set of resource blocks that can be used in each relay is $N$, and the bandwidth of each resource block is represented by $B_{RB}$. The relay set is denoted by $L = \{1, 2, \cdots |L|\}$, $U_l, \forall l \in L$ represents the set of all users who communicate through relay $l$, the set of cellular users under the relay $l$ coverage is $M_l = M \cap U_l$, the set of direct D2D users under the relay $l$ coverage is $D_l^p = D^p \cap U_l$. Therefore, the following formulas are established: $U_l \subseteq \{D_l^p \cup M_l\}, \forall l \in L$, $\cup_l U_l = \{D^p \cup M\}$, $\cap_l U_l = \varphi, \forall l \in L$. The Signal to Interference plus Noise Ratio (SINR) of the direct D2D pair is:

$$\gamma_{u_l,u_l,1}^{(n)} = \frac{h_{u_l,u_l}^{(n)}}{\sum\limits_{\substack{u_j \in D_j^p \\ j \neq l, j \in L}} Q_{u_j,u_j}^{(n)} \cdot g_{u_j,u_l}^{(n)} + \sum\limits_{\substack{u_j \in M_j \\ j \neq l, j \in L}} Q_{u_j,j}^{(n)} \cdot g_{u_j,u_l}^{(n)} + \sigma^2} \tag{1}$$

Where, $Q_{a,b}^{(n)}$, $h_{a,b}^{(n)}$, $g_{a,b}^{(n)}$ respectively represent the transmit power, the channel coefficient of the communication link, and the gain of the interference link on the resource block $n$ in the communication process from the transmitter $a$ to the receiver $b$. $\sigma^2 = N_0 B_{RB}$, $N_0$ is the power spectral density of thermal noise. Therefore, the information rate of this communication process is:

$$R_{u_l,u_l}^{(n)} = B_{RB} \log_2 \left(1 + Q_{u_l,u_l}^{(n)} \gamma_{u_l,u_l,1}^{(n)}\right) \tag{2}$$

The SINR of the communication between the cellular user and the relay is:

$$\gamma_{u_l,l,1}^{(n)} = \frac{h_{u_l,l}^{(n)}}{\sum\limits_{\substack{u_j \in D_j^p \\ j \neq l, j \in L}} Q_{u_j,u_j}^{(n)} \cdot g_{u_j,l}^{(n)} + \sum\limits_{\substack{u_j \in M_j \\ j \neq l, j \in L}} Q_{u_j,j}^{(n)} \cdot g_{u_j,l}^{(n)} + \sigma^2} \tag{3}$$

Therefore, the information rate of this communication process is:

$$R_{u_l,l}^{(n)} = B_{RB} \log_2 \left(1 + Q_{u_l,l}^{(n)} \gamma_{u_l,l,1}^{(n)}\right) \tag{4}$$

The relay $l$ communicates with the base station, for $u_l \in M_l$, the SINR per unit power is:

$$\gamma_{l,eNB,2}^{(n)} = \frac{h_{l,eNB}^{(n)}}{\sigma^2} \tag{5}$$

Therefore, the information rate of this communication process is:

$$R_{l,eNB}^{(n)} = B_{\mathrm{RB}} \log_2 \left(1 + Q_{l,eNB}^{(n)} \gamma_{l,eNB,2}^{(n)}\right) \tag{6}$$

In summary, for user $u_l$ under relay $l$ coverage, the total information rate consists of two parts:

(1) direct D2D mode: the user $u_l$ communicates in direct D2D mode, and the information rate is:

$$R_{\mathrm{D}}^{(n)} = R_{u_l,u_l}^{(n)} \tag{7}$$

(2) Cellular mode: The user $u_l$ communicates in the cellular mode, and the information rate of this communication process is:

$$R_{\mathrm{M}}^{(n)} = \frac{1}{2} \min \left\{R_{u_l,l}^{(n)}, R_{l,eNB}^{(n)}\right\} \tag{8}$$

The above formula is multiplied by 1/2 because the cellular users within the relay range need two time slots to realize two-way communication through relay forwarding, while the direct D2D communication process only uses one time slot, so there is no need to multiply it.

Set the maximum value of user transmission power as $Q_{u_l}^{\max}$ and the maximum value of relay transmission power as $Q_l^{\max}$. Introduce resource block allocation factor $x_{u_l}^{(n)}$, $x_{u_l}^{(n)} \in \{0, 1\}$, they are binary integer variables: When $x_{u_l}^{(n)} = 1$, it means that the resource block $n$ is allocated to user $u_l$, otherwise $x_{u_l}^{(n)} = 0$, $\bar{x}_{u_l}^{(n)} = 1 - x_{u_l}^{(n)}$. The users' QoS requirement is expressed by $R_{QoS}$. This optimization problem can be described as:

$$\max_{x_{u_l}^{(n)}, Q_{u_l,u_l}^{(n)}, Q_{u_l,l}^{(n)}, Q_{l,eNB}^{(n)}} \sum_{l \in L} \sum_{u_l \in U_l} \sum_{n=1}^{N} x_{u_l}^{(n)} R_D^{(n)} + \bar{x}_{u_l}^{(n)} R_M^{(n)} \tag{9}$$

$$0 \leq \sum_{u_l \in U_l} x_{u_l}^{(n)} \leq 1, \quad \forall n \in N \tag{10}$$

$$\sum_{n=1}^{N} x_{u_l}^{(n)} Q_{u_l,u_l}^{(n)} \le Q_{u_l}^{\max} \ ,\forall u_l \in D_l^p, \sum_{n=1}^{N} \bar{x}_{u_l}^{(n)} Q_{u_l,l}^{(n)} \le Q_{u_l}^{\max} \ \ ,\forall u_l \in M_l \tag{11}$$

$$\sum_{u_l \in M_l} \sum_{n=1}^{N} \bar{x}_{u_l}^{(n)} Q_{l,eNB}^{(n)} \le Q_l^{\max} \tag{12}$$

$$\begin{aligned} &\sum_{u_l \in D_l^p} x_{u_l}^{(n)} Q_{u_l,u_l}^{(n)} g_{u_l,u_l^*,1}^{(n)} \le I_{th}^{(n)}, \ \sum_{u_l \in M_l} \bar{x}_{u_l}^{(n)} Q_{u_l,l}^{(n)} g_{u_l,l^*,1}^{(n)} \le I_{th}^{(n)}, \\ &\forall n \in N, \forall l \in L, l \ne l^*, \forall l^* \in L \end{aligned} \tag{13}$$

$$R_{u_l} \ge R_{\text{QoS}}, \qquad \forall u_l \in U_l \tag{14}$$

$$Q_{u_l,u_l}^{(n)} \ge 0, Q_{u_l,l}^{(n)} \ge 0, Q_{l,eNB}^{(n)} \ge 0, \forall n \in N, u_l \in U_l \tag{15}$$

Where, (10) is the condition that each indicator coefficient needs to meet, each resource block can only be allocated to one user under each relay, (11) and (12) limit the transmission power not to exceed the maximum power, (13) indicates that the interference from D2D users under other relays and the interference from cellular users under other relays must meet the interference threshold, (14) ensures that the system meets the minimum QoS requirements, (15) indicates that the transmission power is non-negative.

The unit power SINR of direct D2D users is:

$$\gamma_{u_l,u_l,1}^{(n)} = \frac{h_{u_l,u_l}^{(n)}}{I_{u_l,u_l,1}^{(n)} + \sigma^2} \tag{16}$$

Where, $I_{u_l,u_l,1}^{(n)}$ is the interference of direct D2D user $u_l$ on the resource block $n$.

$$I_{u_l,u_l,1}^{(n)} = \sum_{\substack{u_j \in D_j^p \\ j \ne l, j \in L}} x_{u_j}^{(n)} Q_{u_j,u_j}^{(n)} \cdot g_{u_j,u_l}^{(n)} + \sum_{\substack{u_j \in M_j \\ j \ne l, j \in L}} \bar{x}_{u_j}^{(n)} Q_{u_j,j}^{(n)} \cdot g_{u_j,u_l}^{(n)} \tag{17}$$

For cellular users, the unit power SINR during the first time slot communication process is:

$$\gamma_{u_l,l,1}^{(n)} = \frac{h_{u_l,l}^{(n)}}{I_{u_l,l,1}^{(n)} + \sigma^2} \tag{18}$$

Where, $I_{u_l,l,1}^{(n)}$ is the interference of the cellular user $u_l$ on the resource block $n$ in the first time slot.

$$I_{u_l,l,1}^{(n)} = \sum_{\substack{u_j \in D_j^p \\ j \ne l, j \in L}} x_{u_j}^{(n)} Q_{u_j,u_j}^{(n)} \cdot g_{u_j,l}^{(n)} + \sum_{\substack{u_j \in M_j \\ j \ne l, j \in L}} \bar{x}_{u_j}^{(n)} Q_{u_j,j}^{(n)} \cdot g_{u_j,l}^{(n)} \tag{19}$$

The total information rate of all cellular users on the resource block $n$ is $R_M^{(n)}$:

$$\begin{aligned} R_{\text{M}}^{(n)} &= \frac{1}{2} \min \left\{ R_{u_l,l}^{(n)}, R_{l,eNB}^{(n)} \right\} \\ &= \frac{1}{2} \min \left\{ B_{\text{RB}} \log_2 \left( 1 + Q_{u_l,l}^{(n)} \gamma_{u_l,l,1}^{(n)} \right), B_{\text{RB}} \log_2 \left( 1 + Q_{l,eNB}^{(n)} \gamma_{l,eNB,2}^{(n)} \right) \right\} \end{aligned} \tag{20}$$

When $Q_{u_l,l}^{(n)}\gamma_{u_l,l,1}^{(n)} = Q_{l,eNB}^{(n)}\gamma_{l,eNB,2}^{(n)}$ is established, $R_M^{(n)}$ can reach the maximum value. At this time, $Q_{l,eNB}^{(n)}$ in the second time slot can be expressed by the power in the first time slot, that is $Q_{l,eNB}^{(n)} = \frac{\gamma_{u_l,l,1}^{(n)}}{\gamma_{l,eNB,2}^{(n)}}Q_{u_l,l}^{(n)}$. Therefore, the total information rate of cellular users on the resource block $n$ can be rewritten as

$$R_{\mathrm{M}}^{(n)} = \frac{1}{2}B_{\mathrm{RB}}\log_2\left(1 + Q_{u_l,l}^{(n)}\gamma_{u_l,l,1}^{(n)}\right), \qquad u_l \in M_l \tag{21}$$

## 3 Power Distribution Method Based on Lagrangian Multiplier Method

Optimization problem (9) is difficult to solve because it contains both continuous variables and binary integer variables, and the objective function is nonlinear. Such problems can be called mixed-integer nonlinear programming (MINLP). In order to simplify the problem, we first relax the resource block allocation factor $x_{u_l}^{(n)}$ into a continuous variable, that is $x_{u_l}^{(n)} \in [0,1]$, which represents the proportion of time that the resource block $n$ is allocated to user $u_l$, and it still meets the aforementioned restriction conditions $0 \leq \sum_{u_l \in U_l} x_{u_l}^{(n)} \leq 1, \forall n \in N$, $0 < \sum_{u_l \in U_l} x_{u_l}^{(n)} \leq 1, \forall n \in N$. In addition, two new variables $S_{u_l,u_l}^{(n)} = x_{u_l}^{(n)}Q_{u_l,u_l}^{(n)}$, $T_{u_l,l}^{(n)} = \bar{x}_{u_l}^{(n)}Q_{u_l,l}^{(n)}$ are introduced, which are respectively used as power allocation variables for direct D2D users and cellular users to represent the actual transmit power of the user $u_l$ on the resource block $n$. The optimization problem after relaxation and adjustment can be expressed as

$$\max_{x_{u_l}^{(n)},S_{u_l,u_l}^{(n)},T_{u_l,l}^{(n)}} \sum_{l \in L}\sum_{u_l \in U_l}\sum_{n=1}^{N}\left[x_{u_l}^{(n)}B_{RB}\log_2\left(1 + \frac{S_{u_l,u_l}^{(n)}h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}\right)\right. \tag{22}$$
$$\left. + \bar{x}_{u_l}^{(n)}\frac{1}{2}B_{RB}\log_2\left(1 + \frac{T_{u_l,l}^{(n)}h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)}\mu_{u_l}^{(n)}}\right)\right]$$

$$\text{s.t. } 0 < \sum_{u_l \in U_l} x_{u_l}^{(n)} \leq 1, \qquad \forall n \in N \tag{23}$$

$$\sum_{n=1}^{N} S_{u_l,u_l}^{(n)} \leq Q_{u_l}^{\max}, \forall u_l \in D_l^p, \sum_{n=1}^{N} T_{u_l,l}^{(n)} \leq Q_{u_l}^{\max}, \forall u_l \in M_l \tag{24}$$

$$\sum_{u_l \in M_l}\sum_{n=1}^{N}\frac{\gamma_{u_l,l,1}^{(n)}}{\gamma_{l,eNB,2}^{(n)}}T_{u_l,l}^{(n)} \leq Q_l^{\max} \tag{25}$$

$$\sum_{u_l \in D_l^p} S_{u_l,u_l}^{(n)}g_{u_l,u_l^*,1}^{(n)} \leq I_{th}^{(n)}, \sum_{u_l \in M_l} T_{u_l,l}^{(n)}g_{u_l,l^*,1}^{(n)} \leq I_{th}^{(n)}, \forall n \in N \tag{26}$$

$$\sum_{n=1}^{N}\left[x_{u_l}^{(n)}B_{RB}\log_2\left(1+\frac{S_{u_l,u_l}^{(n)}h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}\right)\right. \\ \left. +\ \bar{x}_{u_l}^{(n)}\frac{1}{2}B_{RB}\log_2\left(1+\frac{T_{u_l,u_l}^{(n)}h_{u_l,u_l}^{(n)}}{\bar{x}_{u_l}^{(n)}\mu_{u_l}^{(n)}}\right)\right] \geq R_{QoS}, \forall u_l \in U_l \tag{27}$$

$$S_{u_l,u_l}^{(n)} \geq 0, u_l \in D_l^p, T_{u_l,l}^{(n)} \geq 0\,, u_l \in M_l\,, \forall n \in N \tag{28}$$

$$I_{u_l,u_l,1}^{(n)} + \sigma^2 \leq \omega_{u_l}^{(n)}, u_l \in D_l^p\,, I_{u_l,l,1}^{(n)} + \sigma^2 \leq \mu_{u_l}^{(n)}, u_l \in M_l, \forall n \in N \tag{29}$$

Where, $\omega_{u_l}^{(n)}, \mu_{u_l}^{(n)}$ are auxiliary variables, when the number of RBs is extremely large, the dual spacing of the optimization problem that satisfies the time allocation condition can be ignored. The optimization problem studied in this paper satisfies the time allocation condition, so the relaxed optimization problem has an asymptotic optimal solution. From the above formulas, the constraint condition (27) is convex, and other constraints are linear. If the objective function is a concave function, then the optimization problem (22) is a convex problem, and there is an optimal solution. Next, we must first prove that the objective function is a concave function.

Define the function $\Re\left(S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}\right)$ as

$$\Re\left(S_{u_1,u_l}^{(n)}, T_{u_l,l}^{(n)}\right) = -\left[x_{u_l}^{(n)}B_{\mathrm{RB}}\log_2\left(1+\frac{S_{u_l,u_l}^{(n)}h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}\right)\right. \\ \left. +\bar{x}_{u_l}^{(n)}\frac{1}{2}B_{\mathrm{RB}}\log_2\left(1+\frac{T_{u_l,l}^{(n)}h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)}\mu_{u_l}^{(n)}}\right)\right] \tag{30}$$

Find the Hessian matrix $H$ about $\left(S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}\right)$ for function $\Re\left(S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}\right)$, and find the first derivative about $S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}$ for function $\Re\left(S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}\right)$ respectively, and get formula (31) and formula (32)

$$\frac{\partial\Re}{\partial S_{u_l,u_l}^{(n)}} = -\frac{1}{\ln 2}x_{u_l}^{(n)}B_{RB}\frac{1}{1+\frac{S_{u_l,u_l}^{(n)}h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}}\frac{h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}} = -\frac{x_{u_l}^{(n)}B_{RB}h_{u_l,u_l}^{(n)}}{\ln 2\left(x_{u_l}^{(n)}\omega_{u_l}^{(n)} + S_{u_l,u_l}^{(n)}h_{u_l,u_l}^{(n)}\right)} \tag{31}$$

$$\frac{\partial\Re}{\partial T_{u_l,l}^{(n)}} = -\frac{1}{2\ln 2}\bar{x}_{u_l}^{(n)}B_{RB}\frac{1}{1+\frac{T_{u_l,l}^{(n)}h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)}\mu_{u_l}^{(n)}}}\frac{h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)}\mu_{u_l}^{(n)}} = -\frac{\bar{x}_{u_l}^{(n)}B_{RB}h_{u_l,l}^{(n)}}{2\ln 2\left(\bar{x}_{u_l}^{(n)}\mu_{u_l}^{(n)} + T_{u_l,l}^{(n)}h_{u_l,l}^{(n)}\right)} \tag{32}$$

Continue to find the derivative about $S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}$ respectively to obtain the Hessian matrix $H$ in the following formula

$$H = \begin{vmatrix} \frac{\partial^2 \Re\left(S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}\right)}{\partial S_{u_l,u_l}^{(n)}{}^2} & \frac{\partial^2 \Re\left(S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}\right)}{\partial S_{u_l,u_l}^{(n)} \partial T_{u_l,l}^{(n)}} \\ \frac{\partial^2 \Re\left(S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}\right)}{\partial T_{u_l,l}^{(n)} \partial S_{u_l,u_l}^{(n)}} & \frac{\partial^2 \Re\left(S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}\right)}{\partial T_{u_l,l}^{(n)}{}^2} \end{vmatrix} = \begin{vmatrix} \frac{x_{u_l}^{(n)} B_{RB} h_{u_l,u_l}^{(n)}{}^2}{\ln 2\left(x_{u_l}^{(n)} \omega_{u_l}^{(n)} + S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}\right)^2} & 0 \\ 0 & \frac{\bar{x}_{u_l}^{(n)} B_{RB} h_{u_l,l}^{(n)}{}^2}{2\ln 2\left(\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)} + T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}\right)^2} \end{vmatrix} \tag{33}$$

The matrix $H$ is a second-order matrix with two eigenvalues, the two eigenvalues are

$$\tilde{\lambda}_1 = \frac{x_{u_l}^{(n)} B_{\mathrm{RB}} h_{u_l,u_l}^{(n)2}}{\ln 2\left(x_{u_l}^{(n)} \omega_{u_l}^{(n)} + S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}\right)^2}, \quad \tilde{\lambda}_2 = \frac{\bar{x}_{u_l}^{(n)} B_{\mathrm{RB}} h_{u_l,l}^{(n)2}}{2\ln 2\left(\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)} + T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}\right)^2} \tag{34}$$

Because $\tilde{\lambda}_1 > 0$, $\tilde{\lambda}_2 > 0$, $\Re\left(S_{u_l,u_l}^{(n)}, T_{u_l,l}^{(n)}\right)$ is convex, the objective function (22) is concave. Therefore, the optimization problem is a convex problem, and there is an optimal solution. Therefore, the KKT conditions in convex optimization theory can be used to solve this problem.

Next, use the Lagrangian multiplier method to solve the problem. Let the Lagrangian multipliers of the constraints (23) to (29) in Eq. (22) be $\delta_n$, $\xi_{u_l}$, $\varsigma_{u_l}$, $\upsilon_l$, $\psi_n$, $\varepsilon_n$, $\lambda_{u_l}$, $\rho_{u_l}^{(n)}$, $\kappa_{u_l}^{(n)}$, then the Lagrangian function is:

$$\begin{aligned} L = &- \sum_{l\in L} \sum_{u_l \in U_l} \sum_{n=1}^{N} \left[ x_{u_l}^{(n)} B_{RB} \log_2\left(1 + \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)} \omega_{u_l}^{(n)}}\right) + \bar{x}_{u_l}^{(n)} \tfrac{1}{2} B_{RB} \log_2\left(1 + \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\left(1 - x_{u_l}^{(n)}\right)\mu_{u_l}^{(n)}}\right)\right] \\ &+ \sum_{n=1}^{N} \delta_n \left(\sum_{u_l \in U_l} x_{u_l}^{(n)} - 1\right) + \sum_{u_l \in D_l^p} \xi_{u_l} \left(\sum_{n=1}^{N} S_{u_l,u_l}^{(n)} - Q_{u_l}^{\max}\right) \\ &+ \sum_{u_l \in M_l} \varsigma_{u_l} \left(\sum_{n=1}^{N} T_{u_l,l}^{(n)} - Q_{u_l}^{\max}\right) + \upsilon_l \left(\sum_{u_l \in M_l} \sum_{n=1}^{N} \frac{\gamma_{u_l,l,1}^{(n)}}{\gamma_{l,eNB,2}^{(n)}} T_{u_l,l}^{(n)} - Q_l^{\max}\right) \\ &+ \sum_{n=1}^{N} \psi_n \left(\sum_{u_l \in D_l^p} S_{u_l,u_l}^{(n)} g_{u_l,u_l^*,1}^{(n)} - I_{th}^{(n)}\right) + \sum_{n=1}^{N} \varepsilon_n \left(\sum_{u_l \in M_l} T_{u_l,l}^{(n)} g_{u_l,l^*,1}^{(n)} - I_{th}^{(n)}\right) \\ &+ \sum_{u_l \in U_l} \lambda_{u_l} \left[ R_{QoS} - \sum_{n=1}^{N} \left( x_{u_l}^{(n)} B_{RB} \log_2\left(1 + \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)} \omega_{u_l}^{(n)}}\right) \right.\right. \\ &\qquad \left.\left. + \bar{x}_{u_l}^{(n)} \tfrac{1}{2} B_{RB} \log_2\left(1 + \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)}}\right)\right)\right] \\ &+ \sum_{u_l \in D_l^p} \sum_{n=1}^{N} \rho_{u_l}^{(n)} \left(I_{u_l,u_l,1}^{(n)} + \sigma^2 - \omega_{u_l}^{(n)}\right) + \sum_{u_l \in M_l} \sum_{n=1}^{N} \kappa_{u_l}^{(n)} \left(I_{u_l,l}^{(n)} + \sigma^2 - \mu_{u_l}^{(n)}\right) \end{aligned} \tag{35}$$

First, take the derivative of the allocation variable of the transmit power of the direct D2D users $S_{u_l,u_l}^{(n)}$

$$\begin{aligned}\frac{\partial L}{\partial S_{u_l,u_l}^{(n)}} &= -\frac{1}{\ln 2} x_{u_l}^{(n)} B_{\mathrm{RB}} \frac{1}{1+\frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)} \omega_{u_l}^{(n)}}} \frac{h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)} \omega_{u_l}^{(n)}} + \xi_{u_l} + \psi_n g_{u_l,u_l^*,1}^{(n)} \\ &- \lambda_{u_l} \frac{1}{\ln 2} x_{u_l}^{(n)} B_{\mathrm{RB}} \frac{1}{1+\frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)} \omega_{u_l}^{(n)}}} \frac{h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)} \omega_{u_l}^{(n)}}\end{aligned} \tag{36}$$

According to the KKT conditions, let $\frac{\partial L}{\partial S_{u_l,u_l}^{(n)}} = 0$, we can obtain $S_{u_l,u_l}^{(n)} = \frac{(\lambda_{u_l}+1) x_{u_l}^{(n)} B_{RB}}{\ln 2\left(\xi_{u_l}+\psi_n g_{u_l,u_l^*,1}^{(n)}\right)} - \frac{x_{u_l}^{(n)} \omega_{u_l}^{(n)}}{h_{u_l,u_l}^{(n)}}$. Assume that $\Delta_{u_l,u_l}^{(n)} = \frac{(\lambda_{u_l}+1) B_{RB}}{\ln 2\left(\xi_{u_l}+\psi_n g_{u_l,u_l^*,1}^{(n)}\right)}$, the optimal value of the transmit power of the direct D2D user is expressed as

$$Q_{u_l,u_l}^{(n)^*} = \frac{S_{u_l,u_l}^{(n)^*}}{x_{u_l}^{(n)^*}} = \left[\Delta_{u_l,u_l}^{(n)} - \frac{\omega_{u_l}^{(n)}}{h_{u_l,u_l}^{(n)}}\right]^* \tag{37}$$

$[\xi]^+$ means that the result takes a value not less than zero as the effective value, that is $[\xi]^+ = \max\{\xi, 0\}$. Next, take the derivative of the allocation variable of the cellular user transmit power $T_{u_l,l}^{(n)}$

$$\begin{aligned}\frac{\partial L}{\partial T_{u_l,l}^{(n)}} &= -\frac{1}{2\ln 2} y_{u_l}^{(n)} B_{\mathrm{RB}} \frac{1}{1+\frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)}}} \frac{h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)}} + \varsigma_{u_l} + v_l \frac{\gamma_{u_l,l,1}^{(n)}}{\gamma_{l,eNB,2}^{(n)}} + \varepsilon_n g_{u_l,l^*,1}^{(n)} \\ &- \frac{1}{2\ln 2} \lambda_{u_l} \bar{x}_{u_l}^{(n)} B_{\mathrm{RB}} \frac{1}{1+\frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)}}} \frac{h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)}}\end{aligned} \tag{38}$$

In the same way, according to the KKT conditions in the convex optimization theory, let $\frac{\partial L}{\partial T_{u_l,l}^{(n)}} = 0$, we can obtain

$$T_{u_l,l}^{(n)} = \frac{(\lambda_{u_l}+1) \bar{x}_{u_l}^{(n)} B_{\mathrm{RB}}}{2\ln 2\left(\zeta_{u_l} + v_l \frac{\gamma_{u_l,l,1}^{(n)}}{\gamma_{l,eNB,2}^{(n)}} + \varepsilon_n g_{u_l,l^*,1}^{(n)}\right)} - \frac{\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)}}{h_{u_l,l}^{(n)}} \tag{39}$$

Then the optimal transmit power of the cellular user is

$$Q_{u_l,l}^{(n)^*} = \frac{T_{u_l,l}^{(n)^*}}{\bar{x}_{u_l}^{(n)^*}} = \left[\Delta_{u_l,l}^{(n)} - \frac{\mu_{u_l}^{(n)}}{h_{u_l,l}^{(n)}}\right]^* \tag{40}$$

$$\Delta_{u_l,l}^{(n)} = \frac{(\lambda_{u_l}+1) B_{\mathrm{RB}}}{2\ln 2\left(\varsigma_{u_l} + v_l \frac{\gamma_{u_l,l,1}^{(n)}}{\gamma_{l,eNB,2}^{(n)}} + \varepsilon_n g_{u_l,l^*,1}^{(n)}\right)} \tag{41}$$

Substituting $\bar{x}_{u_l}^{(n)} = 1 - x_{u_l}^{(n)}$ into the Lagrangian function and deriving a derivative about $x_{u_l}^{(n)}$, we can obtain the formula (42). According to the KKT conditions, let $\frac{\partial L}{\partial x_{u_l}^{(n)}} = 0$, the expression related to the Lagrange multiplier $\delta_n$ is obtained as (43). In formula (43), there are $\theta_{u_l,u_l}^{(n)} = \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{\ln 2\left(x_{u_l}^{(n)} \omega_{u_l}^{(n)} + S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}\right)}$, $\theta_{u_l,l}^{(n)} = \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\ln 2\left(\left(1-x_{u_l}^{(n)}\right)\mu_{u_l}^{(n)} + T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}\right)}$.

$$
\begin{aligned}
\frac{\partial L}{\partial x_{u_l}^{(n)}} = & -B_{\mathrm{RB}}\log_2\left(1 + \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}\right) + \frac{1}{2}B_{\mathrm{RB}}\log_2\left(1 + \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\left(1-x_{u_l}^{(n)}\right)\mu_{u_l}^{(n)}}\right) \\
& - \frac{1}{\ln 2} x_{u_l}^{(n)} B_{\mathrm{RB}} \frac{1}{1+\frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}} \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{\omega_{u_l}^{(n)}}\left(-\frac{1}{\left(x_{u_l}^{(n)}\right)^2}\right) \\
& - \frac{1}{2\ln 2}\left(1 - x_{u_l}^{(n)}\right) B_{\mathrm{RB}} \frac{1}{1+\frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\left(1-x_{u_l}^{(n)}\right)\mu_{u_l}^{(n)}}} \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\mu_{u_l}^{(n)}}\left(-\frac{-1}{\left(1-x_{u_l}^{(n)}\right)^2}\right) \\
& + \delta_n + \lambda_{u_l}\left(-B_{\mathrm{RB}}\log_2\left(1 + \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}\right) + \frac{1}{2}B_{\mathrm{RB}}\log_2\left(1 + \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\left(1 - x_{u_l}^{(n)}\right)\mu_{u_l}^{(n)}}\right)\right) \\
& - \frac{1}{\ln 2}\lambda_{u_l} x_{u_l}^{(n)} B_{\mathrm{RB}} \frac{1}{1 + \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}} \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{\omega_{u_l}^{(n)}}\left(-\frac{1}{\left(x_{u_l}^{(n)}\right)^2}\right) \\
& - \frac{1}{2\ln 2}\lambda_{u_l}\left(1 - x_{u_l}^{(n)}\right) B_{\mathrm{RB}} \frac{1}{1 + \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\left(1-x_{u_1}^{(n)}\right)\mu_{u_l}^{(n)}}} \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\mu_{u_l}^{(n)}}\left(-\frac{-1}{\left(1 - x_{u_l}^{(n)}\right)^2}\right)
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
\delta_n = \left(\lambda_{u_l} + 1\right) B_{\mathrm{RB}} & \left(\frac{1}{2}\theta_{u_l,l}^{(n)} - \theta_{u_l,u_l}^{(n)} + \log_2\left(1 + \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}\right)\right. \\
& \left. - \frac{1}{2}\log_2\left(1 + \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\left(1 - x_{u_1}^{(n)}\right)\mu_{u_l}^{(n)}}\right)\right)
\end{aligned}
\tag{43}
$$

The actual $\delta_n$ is not calculated by Eq. (43), but updated by sub-gradient iteration method, which will be described in detail in the following content. In order to obtain the integer value of the resource block allocation factor $x_{u_l}^{(n)}$, a threshold variable needs to be set as

$$
\begin{aligned}
\chi_{u_l}^{(n)} = \left(\lambda_{u_l} + 1\right) B_{RB} & \left(\frac{1}{2}\theta_{u_l,l}^{(n)} - \theta_{u_l,u_l}^{(n)} + \log_2\left(1 + \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)}\omega_{u_l}^{(n)}}\right)\right. \\
& \left. - \frac{1}{2}\log_2\left(1 + \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\left(1 - x_{u_l}^{(n)}\right)\mu_{u_l}^{(n)}}\right)\right)
\end{aligned}
\tag{44}
$$

The discriminant formula of resource block allocation factor $x_{u_l}^{(n)}$ is shown in the following formula

$$x_{u_l}^{(n)^*} = \begin{cases} 1, \, \delta_n \leq \chi_{u_l}^{(n)} \\ 0, \, \delta_n > \chi_{u_l}^{(n)} \end{cases} \tag{45}$$

After obtaining the optimal solution of the user-to-relay communication transmission power $Q_{u_l,l}^{(n)^*}$, $Q_{u_l,u_l}^{(n)^*}$ and resource block allocation factor $x_{u_l}^{(n)^*}$, the sub-gradient iteration method is used to update each Lagrangian multiplier, and the Lagrangian multiplier of the $(t+1)$th iteration is updated according to the following formula, $\Lambda_\alpha^{(t)}$ is the step length during the first iteration, $\Lambda_\alpha^{(t)} = a/\sqrt{t}$ and $a$ is a constant.

$$\delta_n(t+1) = \left[\delta_n(t) + \Lambda_{\delta_n}^{(t)} \left( \sum_{u_l \in U_l} x_{u_l}^{(n)} - 1 \right)\right]^+ \tag{46}$$

$$\xi_{u_l}(t+1) = \left[\xi_{u_l}(t) + \Lambda_{\xi_{ul}}^{(t)} \left( \sum_{n=1}^{N} S_{u_l,u_l}^{(n)} - Q_{u_l}^{\max} \right)\right]^+ \tag{47}$$

$$\varsigma_{u_l}(t+1) = \left[\varsigma_{u_l}(t) + \Lambda_{\varsigma_{u_l}}^{(t)} \left( \sum_{n=1}^{N} T_{u_l,l}^{(n)} - Q_{u_l}^{\max} \right)\right]^+ \tag{48}$$

$$v_l(t+1) = \left[v_l(t) + \Lambda_{v_l}^{(t)} \left( \sum_{u_l \in M_l} \sum_{n=1}^{N} \frac{\gamma_{u_l l,1}^{(n)}}{\gamma_{l,eNB,2}^{(n)}} T_{u_l,l}^{(n)} - Q_l^{\max} \right)\right]^+ \tag{49}$$

$$\psi_n(t+1) = \left[\psi_n(t) + \Lambda_{\psi_n}^{(t)} \left( \sum_{u_l \in D_l^p} S_{u_l,u_l}^{(n)} g_{u_l,u_l^*,1}^{(n)} - I_{th}^{(n)} \right)\right]^+ \tag{50}$$

$$\varepsilon_n(t+1) = \left[\varepsilon_n(t) + \Lambda_{\varepsilon_n}^{(t)} \left( \sum_{u_l \in M_l} T_{u_l,l}^{(n)} g_{u_l,l^*,1}^{(n)} - I_{th}^{(n)} \right)\right]^+ \tag{51}$$

$$\begin{aligned} \lambda_{u_l}(t+1) = \Bigg[\lambda_{u_l}(t) + \Lambda_{\lambda_{u_l}}^{(t)} \Big[R_{QoS} - \sum_{n=1}^{N} \Bigg( x_{u_l}^{(n)} B_{RB} \log_2 \left( 1 + \frac{S_{u_l,u_l}^{(n)} h_{u_l,u_l}^{(n)}}{x_{u_l}^{(n)} \omega_{u_l}^{(n)}} \right) \\ + \; \bar{x}_{u_l}^{(n)} \tfrac{1}{2} B_{RB} \log_2 \left( 1 + \frac{T_{u_l,l}^{(n)} h_{u_l,l}^{(n)}}{\bar{x}_{u_l}^{(n)} \mu_{u_l}^{(n)}} \right) \Bigg)\Bigg]^+ \end{aligned} \tag{52}$$

$$\rho_{u_l}^{(n)}(t+1) = \left[\rho_{u_l}^{(n)}(t) + \Lambda_{\rho_{ul}^{(n)}}^{(t)} \left( I_{u_l,u_l,1}^{(n)} + \sigma^2 - \omega_{u_l}^{(n)} \right)\right]^+ \tag{53}$$

$$\boldsymbol{\kappa}_{u_l}^{(n)}(t+1) = \left[\boldsymbol{\kappa}_{u_l}^{(n)}(t) + \Lambda_{\kappa_{ul}^{(n)}}^{(t)} \left( I_{u_l,l}^{(n)} + \sigma^2 - \mu_{u_l}^{(n)} \right)\right]^+ \tag{54}$$

## 4 Performance Analysis

Since there are random variables in each channel coefficient and interference link, the information rate of each resource block is also random. In order to reduce the influence of randomness on the simulation results, we have carried out a large number of simulations to calculate the average rate. This will eliminate the randomness of channel coefficients and obtain the fairness index of information rate in each resource block. This paper adopts RajJain fairness index to judge the information rate in each resource block. The definition of RajJain index is: $F = \left(\sum_{n=1}^{N} R_n\right)^2 \Big/ N \sum_{n=1}^{N} R_n^2$, where $N$ denotes the number of total resource blocks in system, $R_n$ stands for the information rate in resource block $n$. The parameters of simulation are shown in Table 1.

**Table 1.** System parameters of simulation.

| Parameter of system | Value |
|---|---|
| Bandwidth of system | 2.5 MHz |
| Number of total resource block | 13 |
| Path loss among D2D users | $102.9 + 18.7log[d(km)]$ |
| Path loss between users and relay | $103.8 + 20.9log[d(km)]$ |
| Path loss between relay and base station | $100.7 + 23.5log[d(km)]$ |
| Standard deviation of shadow fading between D2D users | 3 dB |
| Standard deviation of shadow fading between user and relay | 10 dB |
| Standard deviation of shadow fading between base station and relay | 6 dB |
| Range of transmitting power of relay | 20–30 dBm |
| Range of transmitting power of user | 13–23 dBm |
| Maximum distance among D2D direct users | 20 m |
| Radius of coverage of relay | 200 m |
| Distance between base station and relay | 125 m |
| Power spectrum density of noise | −174 dBm/Hz |
| Interfering threshold | −70 dBm |

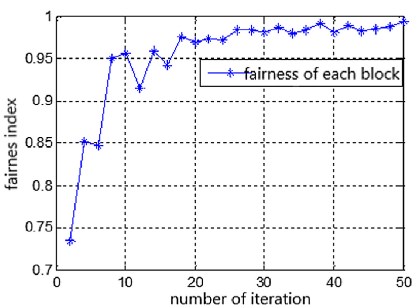


**Fig. 2.** Fairness index of each resource block.

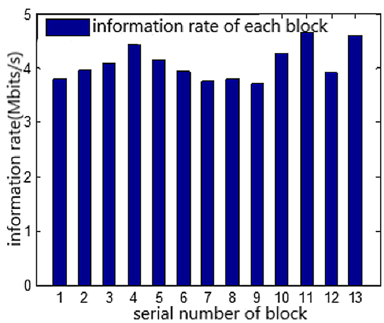


**Fig. 3.** Distribution of information rates in each resource block.

To check the correctness of theoretical analysis, we implement the following simulations. Assuming the number of resource blocks and total bandwidth of system are fixed. In our communication scenario, there are two relays, and the number of D2D pairs in each relay coverage area is the same as that of cellular users. For example, assuming there are four D2D pairs and four cellular users in relay 1 and relay 2 respectively. The simulation result in Fig. 2 shows that with the increase of number of iterations, the fairness index of information rate in each resource block is close to 1, which represents the user fairness becomes better gradually. These results show that averaging results of a lot of simulations indeed reduces the effect of random of channel. After 50 iterations, the information rate of each resource block is close to 4 Mbits/s as shown in Fig. 3.

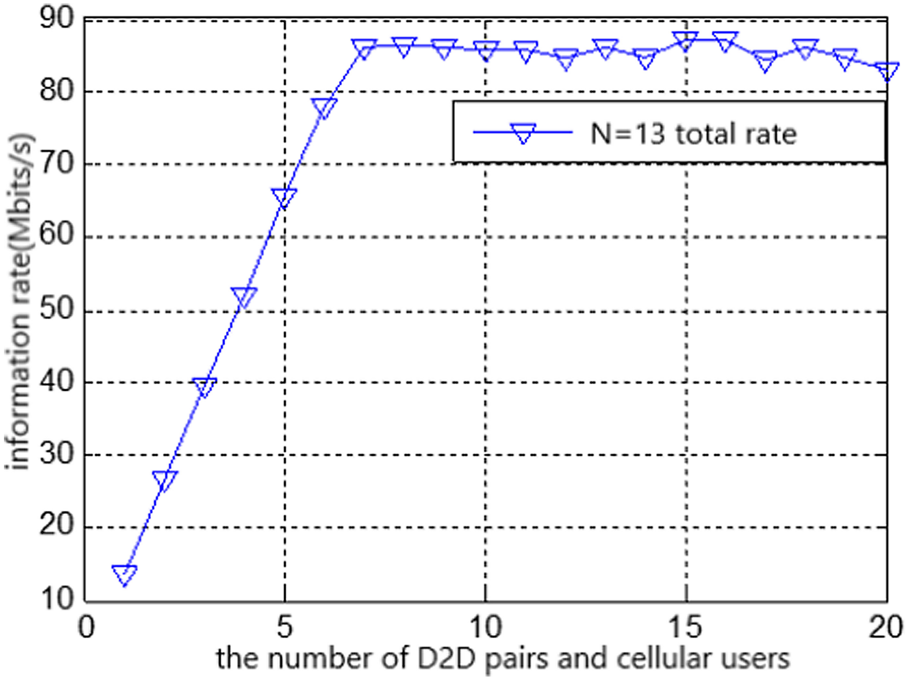


**Fig. 4.** Total information rate of system.

The simulation parameters of Fig. 4 is nearly 50 iterations and 13 resource blocks. Observing this figure, we can find that with the increase of number of D2D pairs and cellular users, the information rate of whole system first increases linearly, and then tends to a fixed value. Particularly, when the number of D2D pairs and cellular users are equal to 7 respectively, the peak information rate can be increased up to 85 Mbits/s.

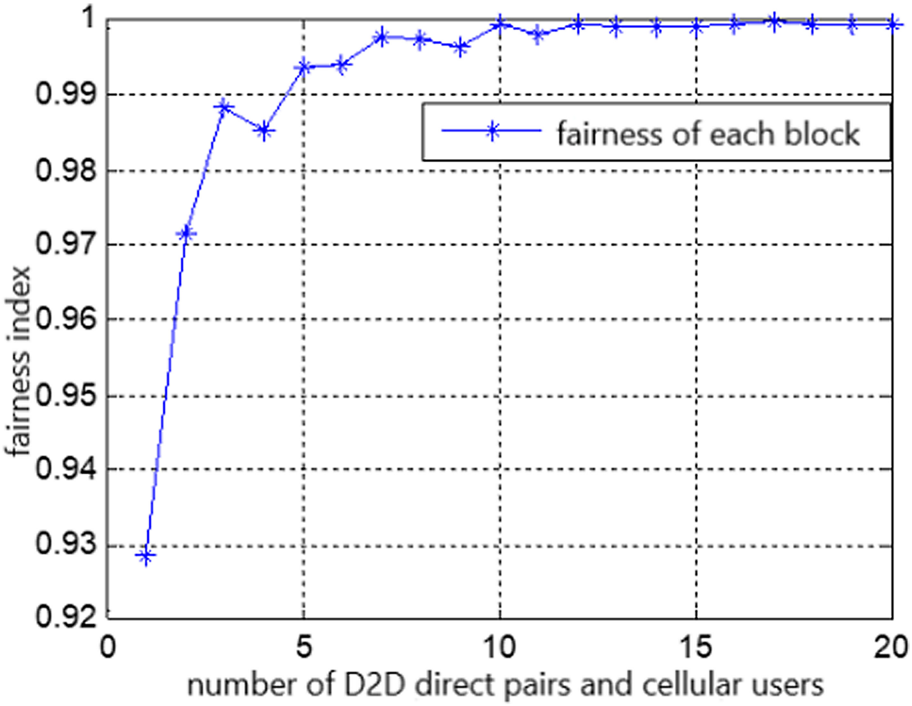


**Fig. 5.** Fairness of information rate of each resource block.

Next, use the same simulation parameters as Fig. 1, we analyze the fairness of information rate of each resource block. The corresponding simulation result is shown in Fig. 5. Observing this figure, we can find that, with the increase of number of D2D pairs and cellular users in coverage area of each relay, the fairness index also grows close to 1. The reason is that, when there are few users, each resource block may not be fully utilized. However, with the increase of users or links, especially the number of users or links exceeds the number of resource block, the resources allocated to users in system are almost the same, and total resource blocks are fully utilized, this improves the fairness of each resource block.

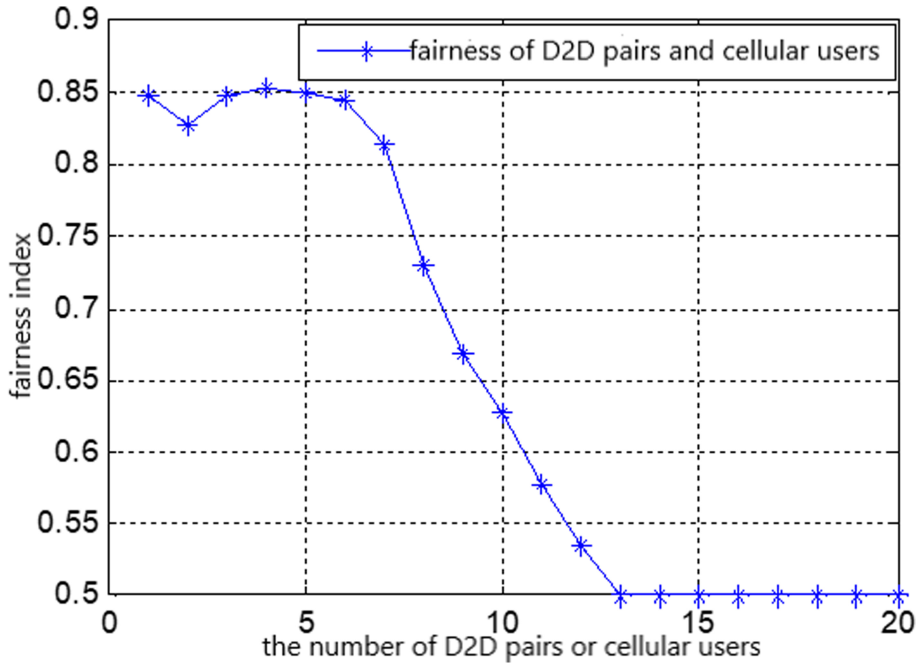


**Fig. 6.** The fairness of information rate of different users.

Use the same simulation parameters as Fig. 5, the fairness of information rate of different types of users are shown in Fig. 6. With the increase of number of D2D pairs and cellular users in covering area of each relay, the fairness of information rate of D2D pairs and cellular users decreases gradually. The reason is that, for each resource block, each slot can only be occupied by one user served by relays. When the number of users is larger than that of resource blocks, there is resource block competition between communication links. The above resource allocation scheme ensures that some of users have priority to use the current resource blocks and the other users have to wait for resource blocks. This leads to the decrease of fairness index of two kind of users in terms of information rate.

## 5 Conclusion

In this paper, we firstly set up a complex scenario, which includes base station of LTE-A, relays, and D2D pairs. Next, under the mode of direct D2D, we analyze the communication links of D2D pairs and cellular users, especially the interference of these links. Based on the above elements in scenario, we build the system model of information rate. Then we formulate this model into a

MINLP problem, utilize Lagrange multiplier and KKT conditions to obtain the optimal power allocation. The simulation results show that with the increase of number of users, the fairness index of information rate of each resource block also increases, and gradually approaches 1.

## References


1. Cha, W.S., Song, J.K., Kim, S.T., Lee, T.J., Park, J.H., Oh, C.Y.: Method of selecting relay mode in mobile ad-hoc network. US 9060386 B2 (2015)
2. Yang, D., Bastos, J., Verikoukis, C., Rodriguez, J.: Location-aided round robin scheduling for fractional frequency reused LTE-A relay network. In: IEEE International Workshop on Computer Aided Modeling and Design of Communication Links and Networks, pp. 11–15 (2012)
3. Deng, J., Dowhuszko, A.A., Freij, R., Tirkkonen, O.: Relay selection and resource allocation for D2D-relaying under uplink cellular power control. In: IEEE Globecom Workshops. IEEE (2015)
4. Joung, J., Sun, S.: Power efficient resource allocation for downlink OFDMA relay cellular networks. IEEE Trans. Sig. Process. **60**(5), 2447–2459 (2012). https://doi.org/10.1109/TSP.2012.2187643
5. Wang, T., Vandendorpe, L.: Resource allocation for maximizing weighted sum min-rate in downlink cellular OFDMA systems. In: IEEE International Conference on Communications, pp. 1–6 (2010). https://doi.org/10.1109/ICC.2010.5501904
6. Alam, M.S., Mark, J.W., Shen, X.S.: Relay selection and resource allocation for multi-user cooperative OFDMA networks. IEEE Trans. Wirel. Commun. **12**(5), 2193–2205 (2013). https://doi.org/10.1109/TWC.2013.032113.120652
7. Wen, S., Zhu, X., Lin, Z., Zhang, X., Yang, D.: Optimization of interference coordination schemes in device-to-device (D2D) communication. In: 7th International Conference on Communications and Networking in China, pp. 542–547 (2012). https://doi.org/10.1109/ChinaCom.2012.6417542
8. Yin, R., Zhong, C., Yu, G., Zhang, Z., Wong, K.K., Chen, X.: Joint spectrum and power allocation for D2D communications underlaying cellular networks. IEEE Trans. Veh. Technol. **65**(4), 2182–2195 (2016). https://doi.org/10.1109/TVT.2015.2424395
9. Ryu, H., Park, S.H.: Performance comparison of resource allocation schemes for D2D communications. In: 2014 IEEE Wireless Communications and Networking Conference Workshops (WCNCW), pp. 266–270 (2014). https://doi.org/10.1109/WCNCW.2014.6934897
10. Khosrowpour, M.: An overview of 3GPP long term evolution (LTE), 13–19 (2015)
11. Phunchongharn, P., Hossain, E., Kim, D.I.: Resource allocation for device-to-device communications underlaying LTE-advanced networks. IEEE Wirel. Commun. **20**(4), 91–100 (2013). https://doi.org/10.1109/MWC.2013.6590055
12. Periyalwar, S., Zhang, H., Senarath, N., Yu, D., Au, K.K., Mann, K.: System and method for peer-to-peer communication in cellular systems. US, US7548758 B2 (2018)
13. Park, K.J., Cho, H.G., Kwon, Y.H., Chung, J.H.: Method for transmitting an uplink signal and feedback information, and relay apparatus using the method. US, US20140286265 A1[P] (2016)

14. Yu, W., Lui, R.: Dual methods for nonconvex spectrum optimization of multicarrier systems. IEEE Trans. Commun. **54**(7), 1310–1322 (2006). https://doi.org/10.1109/TCOMM.2006.877962
15. Xu, Y., Yin, R., Chen, Q., Yu, G.: Joint licensed and unlicensed spectrum allocation for unlicensed LTE. In: 2015 IEEE 26th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC), pp. 1912–1917 (2015). https://doi.org/10.1109/PIMRC.2015.7343611