# Improved Pulse Shaping Algorithm for Reducing PAPR in OFDM System

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**Abstract.** Orthogonal frequency division multiplexing (OFDM) is the most widely used modulation technology because of its high spectrum efficiency and strong anti-interference ability, and it is a typical representative of multicarrier transmission technology. With the application of OFDM technology more and more common, OFDM also began to face the problem of high peak to average power ratio (PAPR). In this paper, pulse forming technology is mainly used to suppress the problem of too high peak average power. Pulse forming technology is to produce a new sequence by multiplying the formed pulse matrix and the original data sequence, so that each subcarrier symbol has a certain correlation, so as to improve the peak to average power ratio of the signal. Compared with the traditional pulse forming algorithm, the improved Nyquist pulse forming algorithm has better effect on the PAPR suppression, lower algorithm complexity and practicability.

**Keywords:** OFDM · PAPR · Pulse forming technology

## 1 Introduction

With the rapid development of science and technology, the application of intelligent products is more and more extensive, such as autopilot, UAV and so on. Therefore, the research of high-speed, efficient and high-quality information transmission system is becoming more and more important in communication. At the same time, OFDM technology came into being. OFDM is a kind of parallel signal transmission technology, which has the advantages of high spectrum efficiency, strong anti-multipath interference ability and strong anti inter symbol interference ability. OFDM signals are generated by superposition of multiple subcarrier signals. When the phases of multiple signals are the same, the instantaneous power of the superposition signal will be much higher than the average power of the signal, resulting in a large peak to average power ratio. In this way, the signal may be distorted, the spectrum of the signal will change, and the orthogonality of each sub channel will be destroyed, which will worsen the performance of the system [1].

At present, there are three main methods to reduce the PAPR of OFDM signals: signal distortion, coding and signal space expansion [6].

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Signal distortion technology includes compression expansion transformation and limiting technology. Compression transformation is a very simple technology, and it will not increase the number of modulation subcarriers and increase the additional computation. Compression expansion transform is a kind of nonlinear transform function based on μ rate non-uniform quantization, which amplifies the power of the smaller part of the original signal, and keeps the larger part unchanged. By increasing the power of the whole system, the peak to average power ratio is suppressed. The disadvantage of this method is the increase of the average transmit power and the distortion of the signal. Amplitude limiting is the simplest way to reduce the peak to average power ratio. It uses a nonlinear process to reduce the PAPR value of the signal directly in or near the peak amplitude of the OFDM signal. The method of amplitude reduction has strong adaptability and can be applied to any number of subcarriers system. Limiting is equivalent to adding a rectangular window to the original signal. If the signal amplitude is greater than the preset threshold value, a rectangular window with a value less than 1 will be added; if the OFDM signal amplitude is less than the preset threshold value, a rectangular window with a value of 1 will be added to the signal. Because the amplitude of the added rectangular window is not all 1, the OFDM signal will be distorted after passing through the rectangular window. Due to the distortion of the signal, one of the disadvantages of limiting amplitude is that it will produce a kind of self interference, which will cause the BER performance of the system to decline. In addition, because the signal goes through a nonlinear process, it will cause spectrum leakage. At present, there are many ways to solve spectrum leakage by adding no rectangular window function, but the effect is very weak.

The coding methods mainly include block code, gray complementary code and multiple complementary sequences. The coding technology mainly uses different code groups generated by different codes to select the code group with smaller PAPR as OFDM symbols for data transmission, so as to avoid signal peak value. This technology is a linear process. Using coding technology to reduce the PAPR will not cause distortion of the original signal, but the process of coding and decoding is relatively complex, and the calculation complexity is very high, so this kind of technology can be used when the number of subcarriers is relatively small. There are many advantages to reduce PAPR by coding, for example, the system is stable and simple, and the effect of reducing PAPR is very good. But the technology also has many disadvantages.

In this paper, the peak to average power ratio is suppressed by improved pulse forming technology. The pulse shaping technology multiplies the original data and the shaping pulse matrix to produce a new sequence, which makes the subcarriers have a certain correlation, so as to reduce the peak to average power ratio of the signal. This method only needs to select the time-domain waveform of each subcarrier properly, so it can avoid the inverse process of the additional FFT. It can also leave room for channel coding while effectively maintaining the system bandwidth efficiency. Therefore, pulse forming technology is a very effective method to suppress the peak to average power ratio.

## 1.1 OFDM System Composition

The composition block diagram of OFDM system is shown in the figure below. It mainly consists of transmitter link and receiver. The system mainly includes channel coding/decoding, modulation/demodulation, FFT/IFFT, digital up and down conversion [5]. The flow chart is shown in Fig. 1.

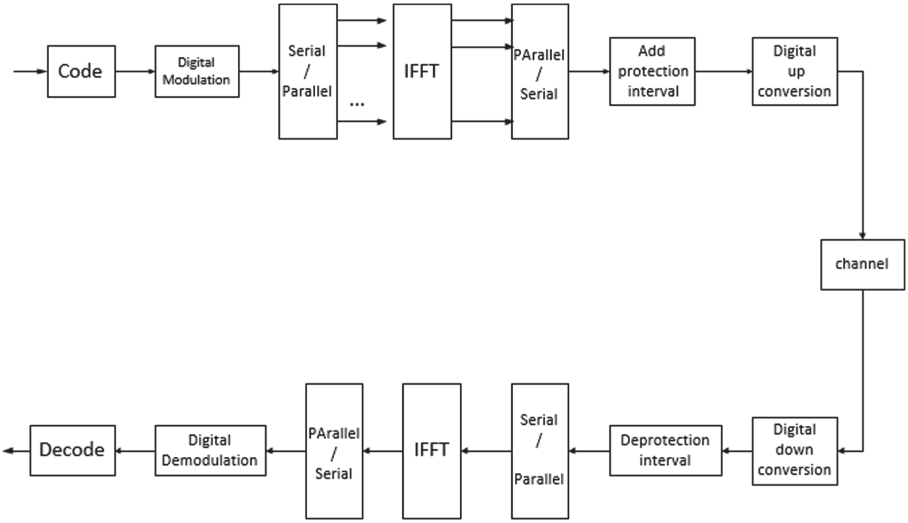

**Fig. 1.** Flow chart of OFDM system

After the channel coding of the input signal sequence is completed, the modulation is completed according to the modulation mode of the system, and the sequence $\{X(n)\}$ is obtained. The inverse Fourier transform is performed on the $\{X(n)\}$, and the sampling sequence in time domain of the modulated signal of orthogonal frequency division multiplexing is obtained. The cyclic prefix is added, and then the digital up conversion is carried out. The receiver first down converts the received signal, removes the protection interval, and obtains the sampling sequence of the modulated signal of OFDM, then carries on the FFT transformation, and then obtains the $\{X(n)\}$.

## 1.2 The Basic Principle of OFDM System

OFDM is a multicarrier modulation technology. Its principle is to use N subcarriers to divide the whole channel into N subchannels, N subcarriers with equal interval in frequency are modulated and added to transmit at the same time, so that n subchannels can transmit information in parallel. In this way, the spectrum of each symbol only occupies 1/N of the channel bandwidth, and each subcarrier keeps the spectrum orthogonality within the symbol period T of OFDM.

Figure two shows an example of an orthogonal frequency division multiplexing symbol containing four subcarriers. All subcarriers have the same amplitude and phase. It can also be seen from the figure that each subcarrier contains an integer multiple cycles in an OFDM symbol cycle, and the difference between adjacent subcarriers is one cycle. This characteristic can be used to explain the orthogonality between subcarriers. From the perspective of time domain, orthogonality meets the following formula (Fig. 2):

$$\frac{1}{T}\int_0^t \mathrm{e}^{\mathrm{jau}} \cdot \mathrm{e}^{-\mathrm{jev}'}\mathrm{d}t = \begin{cases} 1, & n = m \\ 0, & n \neq m \end{cases} \tag{1}$$

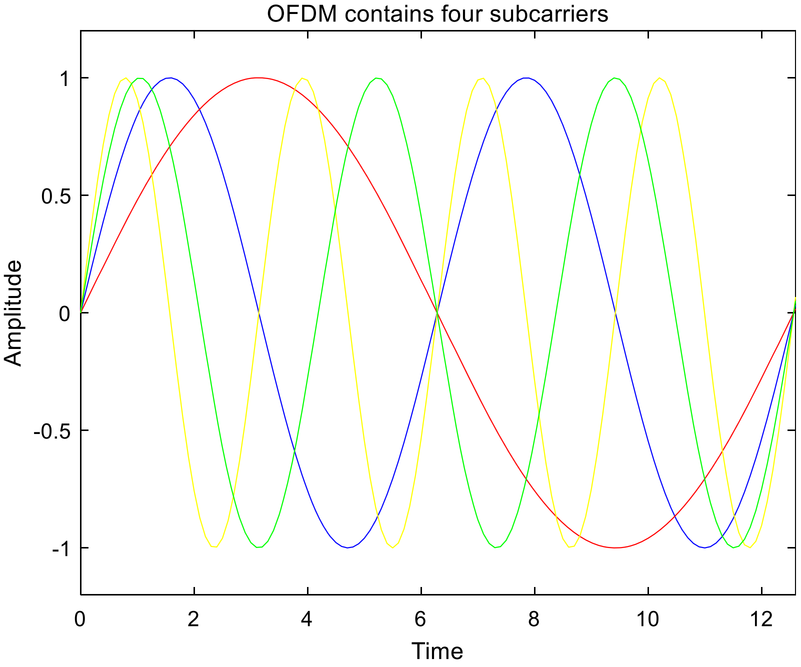

**Fig. 2.** Four subcarriers of OFDM

Figure three shows the spectrum diagram of four orthogonal subcarriers in an orthogonal frequency division multiplexing symbol. From the diagram, we can see that the four subcarriers reach the peak value in the same frequency, and the frequency value attenuates to 0 at the same time, that is, their spectrum changes are always consistent (Fig. 3).

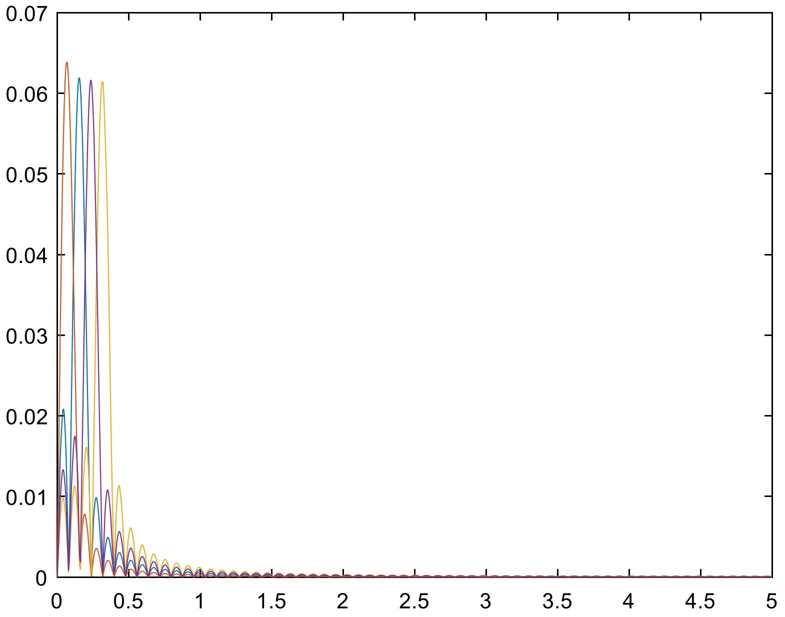

**Fig. 3.** Spectrum of four orthogonal subcarriers

All subcarriers of a symbol in OFDM are shaped by rectangular wave to obtain sinc function spectrum. At the maximum frequency of each subcarrier, the spectrum value of all the remaining subchannels is exactly 0. In the process of demodulation of OFDM symbols, it is necessary to calculate the maximum value of each subcarrier frequency corresponding to these points, so each subcarrier symbol can be extracted from multiple overlapping subcarrier symbols without interference from other subchannels. Since the OFDM symbol spectrum satisfies the Nyquist criterion, that is, there is no mutual interference, so the interference between carriers can be avoided.

There are three main advantages of the OFDM system, which are high spectrum efficiency, strong ability to resist ISI, frequency selective fading and narrow-band interference.

In the traditional frequency division multiplexing multicarrier modulation technology, the frequency spectrum of each subcarrier does not overlap with each other, but it is necessary to keep enough frequency interval to avoid the interference between each subcarrier. In the OFDM multicarrier modulation technology, the spectrum of each subcarrier overlaps with each other, and meets the orthogonality in the whole symbol period, which not only reduces the mutual interference between the subcarriers, but also reduces the frequency protection interval and improves the spectrum utilization.

The OFDM system inserts a protection interval greater than the channel impulse response time between each transmitted data block, so it effectively reduces the inter code interference. In a single carrier system, a single fading or interference will lead to the failure of the whole link, but in OFDM system, there are multiple subcarriers, and the fading at a certain time can only affect a part of the molecular channel. OFDM can enhance the resistance to impulse noise and channel fast fading by joint coding of subcarriers.

OFDM has many advantages, but it inevitably has some disadvantages. It is easy to be affected by frequency deviation and has high peak to average power ratio.

# 2 PAPR Suppression Algorithm Based on Improved Pulse Forming Technology

## 2.1 Definition of PAPR

Compared with the single carrier system, since the OFDM symbol is composed of multiple independent and modulated subcarrier signals, such a composite signal may generate relatively large peak power, which will result in a large peak to average power ratio. The definition of peak to average power ratio in the OFDM system is as follows:

$$\mathrm{PAPR(dB)} = 10\,\mathrm{lg}\frac{\max\left\{|x_n|^2\right\}}{E\left\{|x_n|^2\right\}} \tag{2}$$

Where a is the output signal after IFFT operation,

$$x_n = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} X_k W_N^{nk} \tag{3}$$

For OFDM systems with N subchannels, when n subchannels are summed with the same phase, the peak power of the signal obtained is n times of the average power, so the peak to average ratio of baseband signal can be: PAPR = 10log10n.

## 2.2 Principle of PAPR Suppression in Pulse Forming

The pulse shaping technology is to multiply the original data sequence and the shaping pulse matrix to produce a new sequence, so that the subcarrier symbols of the multicarrier have certain correlation, so as to improve the PAPR characteristics of the signal [3].

The flow chart of OFDM system transmitter based on pulse forming technology is shown in Fig. 4.

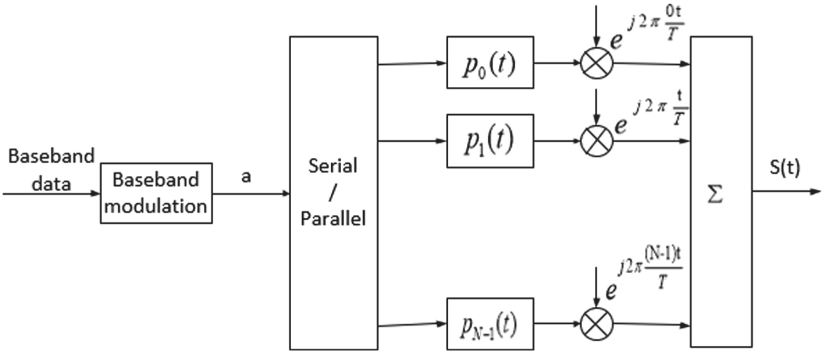

**Fig. 4.** Flow chart of OFDM system transmitter

After serial/parallel conversion, the baseband data sequence is multiplied by N shaping pulses, and N orthogonal subcarriers are modulated. The symbol period of OFDM is T, $a_n(n = 0, 1, \ldots., N - 1)$ represents the modulation data of each subcarrier, $f_n$ represents the frequency of the nth subcarrier, and $\mathrm{p_n}(\mathrm{t})$ represents the shaping pulse of the subcarrier $f_n$ whose period is t. The OFDM complex signal in $0 \leq t \leq T$ is expressed as:

$$s(t) = \sum_{n=0}^{N-1} a_n p_n(t) \exp(j2\pi f_n t) \quad 0 \leqslant t \leqslant T \tag{4}$$

Sub carriers $f_n = n/T$. The real part and the virtual part of $s(t)$ correspond to the orthogonal component and the in-phase of the OFDM signal respectively. In the actual system, they can be multiplied by the in-phase component and the orthogonal component of the corresponding subcarrier respectively to synthesize the final OFDM signal [2].

Forming pulse $p_n(t)$ with period T must meet the following two conditions:

1. Orthogonal:

$$\int_0^T p_m(t) p_n^*(t) \exp[j2\pi(f_m - f_n)t] \mathrm{d}t = \begin{cases} T, & m = n \\ 0, & m \neq n \end{cases} \tag{5}$$

2. Equal energy:

$$\int_0^\tau |p_n(t)|^2 dt = T \tag{6}$$

Another expression of PAPR of OFDM signal is:

$$\text{PAPR} = \max_{0<1<T} |s(t)|^2 \Big/ \mathop{E}_{0<1<T}\left[|s(t)|^2\right] \tag{7}$$

When the subcarrier modulation phase is the same, the peak value of OFDM signal will be superposed to produce a large peak power, resulting in a high peak to average power ratio. If there is a certain correlation between the subcarrier symbols, the PAPR will be suppressed by reducing the probability of phase congruence.

From the perspective of the sampling values of OFDM symbols, the $R_S(t_1, t_2)$ cross-correlation function is as follows:

$$R_s(t_1, t_2) = \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} E\left[a_n a_m^*\right] p_n(t_1) p_m^*(t_2) \exp[j2\pi(nt_1 - mt_2)/T] \tag{8}$$

From Eq. 8, we can see that the cross-correlation function $R_S(t_1, t_2)$ is a function of shaping pulse waveform and baseband data. We can introduce the correlation between baseband data by encoding the input information. But this will introduce redundant information and reduce the efficiency of system bandwidth.

## 2.3 Improvement of Nyquist Pulse Forming Technology to Suppress PAPR

The Nyquist pulse set is:

$$p_m(t) e^{j2\pi\frac{m}{T}t} = p_n(t - \tau_{m-n}) e^{j2\pi\frac{n}{T}(t-\tau_{m-n})}, \quad n, m = 0, 1, \cdots, N-1 \tag{9}$$

Where $\tau_{m-n} = [(m-n) \bmod N]T_s, p_n(t)$ (n = 0, 1,…, N − 1 ) is Nyquist pulse, which has no ISI property:

$$p_n(kT_s) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}, k \in \mathbf{Z} \tag{10}$$

The maximum PAPR value of OFDM signal corresponding to Nyquist pulse set defined by the formula is:

$$\text{PAPR}_{\max} = \frac{1}{N} \max_{0<\tau<T} \left( \sum_{n=0}^{N-1} |p_n(t)| \right)^2 \leqslant \frac{1}{N} \left( \sum_{n=0}^{N-1} \max_{0<\tau<T} |p_n(t)| \right)^2 = N \tag{11}$$

The maximum value of PAPR is n when and only when rectangular pulse. The derivation of the above formula is based on the no ISI property of Nyquist pulse. It is shown that all Nyquist pulse sets constructed according to the above formula can be used for PAPR suppression of OFDM signals [4].

Since the pulse forming $p_n(t)$ (n = 0, 1,…, N – 1) is a time limited signal within the symbol period T, it can be approximated by Fourier series, that is:

$$p_n(t) = \sum_{i=-l}^{N+L-1} c_{n,l} \mathrm{e}^{j2\pi \frac{l}{T} t}, \quad 0 \leqslant t \leqslant T \tag{12}$$

$L = N\beta/2$, $c_{n,l}$ is the coefficient of the Fourier series of $p_n(t)$:

$$c_{n,l} = \frac{1}{T} \int_0^T p_n(t) \mathrm{e}^{-j2\pi \frac{l}{T} t} \mathrm{d}t = \frac{1}{T} P_n\left(\frac{l}{T}\right) \tag{13}$$

Substituting Eq. 12 into 9, we can get:

$$p_n(t) = \sum_{i=-L}^{N+L-1} c_{n,l} e^{-j2\pi \frac{nl}{N}} e^{j2\pi \frac{l-n}{T}} \tag{14}$$

By substituting the subcarrier waveforms of expression 14 into expression 4, we can get:

$$\begin{aligned} s(t) &= \sum_{n=0}^{N-1} a_n p_n(t) \mathrm{e}^{j2\pi^n t} \\ &= \sum_{n=0}^{N-1} a_n \sum_{l=-L}^{N+L-1} c_{n,l} \mathrm{e}^{j2\pi \frac{nl}{N}} \mathrm{e}^{j2\pi \frac{l-n}{T} t} \mathrm{e}^{j2\pi n \frac{t}{T}} \\ &= \sum_{n=0}^{N-1} a_n \sum_{l=-L}^{N+L-1} c_{n,l} \mathrm{e}^{-j2\pi \frac{nl}{N}} \mathrm{e}^{j2\pi \frac{l}{T} t} \\ &= \sum_{l=-L}^{N+L-1} \left[ \sum_{n=0}^{N-1} a_n c_{n,l} \mathrm{e}^{j2\pi \frac{nl}{N}} \right] \mathrm{e}^{j2\pi \frac{l}{T} t} \\ &= \sum_{l=L}^{N+L-1} b_l \mathrm{e}^{j2\pi \frac{l}{T} t} \\ &= \mathrm{IFFT(b)} \end{aligned} \tag{15}$$

Where $b_l = \sum_{n=0}^{N-1} a_n c_{n,l} \mathrm{e}^{-j2\pi \frac{nl}{N}}$ and $b = \{b_l\}$ is the vector containing N + 2L elements. $p_{n,j} = c_{nj} e^{-j2\pi \frac{nl}{N}}$ (n = 0, 1, …, N − 1, l = − L,…,…N + L − 1), Then the orthogonal matrix representing $N \times (N + 2L)$ is called the shaping matrix, and B = AP is the new sequence after transformation.

The frequency response and time domain signal of the improved Nyquist pulse are as follows:

$$P_2(f) = \begin{cases} 1, & |f| \leqslant B(1-\alpha) \\ e^{\lambda[B(1-\alpha)-|f|]}, & B(1-\alpha) < |f| \leqslant B \\ 1 - e^{\lambda[|f|-B(1+\alpha)]}, & B < |f| < B(1+\alpha) \\ 0, & |f| \geqslant B(1+\alpha) \end{cases} \tag{16}$$

$$p_2(t) = \frac{1}{T_s} \sin c\left(\frac{t}{T_s}\right) \frac{4\lambda\pi t \sin(\pi\alpha t/T_s) + 2\lambda^2 \cos(\pi\alpha t/T_s) - \lambda^2}{(2\pi t)^2 + \lambda^2} \tag{17}$$

Where parameter $\lambda = \frac{\ln 2}{\alpha\beta}$.

Nyquist pulse is a real symmetric signal, and it is zero at Nyquist sampling frequency. From formula 17, it can be seen that the time-domain waveform tailing of the improved pulse is gradually attenuated, and the interference to other values caused by superposition of different sampling times is less.

## 3 Simulation Results and Analysis

In this paper, the simulation is carried out in the MATLAB environment. QPSK modulation is adopted, the number of subcarriers is n = 128, and the roll off coefficient of Nyquist pulse is α = 0.3. The following figure shows the comparison of PAPR suppression between the traditional pulse forming technology and the improved Nyquist pulse forming technology. From the Fig. 5, we can see that the improved Nyquist pulse forming algorithm has better inhibition effect on PAPR than the traditional pulse forming algorithm

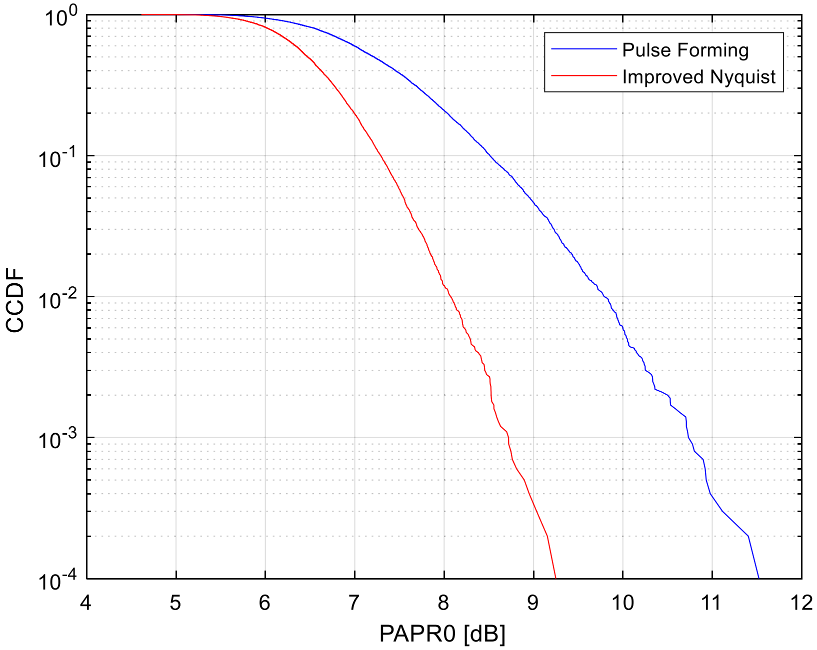

**Fig. 5.** Comparison of PAPR inhibition effect

## 4 Conclusion

In this paper, an improved Nyquist pulse shaping algorithm is proposed to solve the problem of high peak to average power ratio in OFDM system. In this algorithm, a pulse is circularly shifted to form a pulse set, which effectively avoids the phenomenon that the peak value of each subcarrier appears at the same time. The results show that compared with the traditional pulse forming algorithm, the improved Nyquist pulse forming algorithm can reduce the inter symbol interference, and it has a very good inhibition effect on the peak to average power ratio. This algorithm has greatly reduced the computational complexity, so this algorithm has certain research value.

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