





# Robust RSS-Based Localization in Mixed LOS/NLOS Environments

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**Abstract.** In this paper, we propose a robust received signal strength (RSS) based localization method in mixed line-of-sight/non-line-of-sight (LOS/NLOS) environments, where additional path losses caused by NLOS signal propagations are included. Considering that the additional path losses vary in a dramatic range, we express the additional path losses as the sum of a balancing parameter and some error terms. By doing so, we formulate a robust weighted least squares (RWLS) problem with the source location and the balancing parameter as unknown variables, which is, simultaneously, robust to the error terms. By employing the S-Lemma, the RWLS problem is transformed into a non-convex optimization problem, which is then approximately solved by applying the semidefinite relaxation (SDR) technique. The proposed method releases the requirement of knowing specific information about the additional path losses in the previous study. Simulation results show that the proposed method works well in both dense and sparse NLOS environments.

**Keywords:** Source localization · Received signal strength (RSS) · Line-of-sight/non-line-of-sight (LOS/NLOS) · Robust weighted least squares (RWLS) · Semidefinite relaxation (SDR)

## 1 Introduction

In recent years, the development of wireless sensor network (WSN) is rapid because of its wide applications. One of the important functions of the WSN is to provide location estimate for some objects. However, the WSN is typically composed of cheap and small sensor nodes with limited communication range and computational ability, which limits its application in localization using time of arrival (TOA) [1] or time difference of arrival (TDOA) [2]. As such, received signal strength (RSS) is probably the most proper venue for source localization because of its low complexity and low cost [3, 4]. In this paper, we address the RSS based source localization problem.

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In the previous study, the outdoor RSS measurement model is widely adopted in the RSS based source localization literature. The most common method to solve the problem is the maximum likelihood (ML) estimator formulated based on the outdoor measurement model. Unfortunately, the ML problem is nonlinear and non-convex, implying that the traditional iterative algorithm may fail to converge to the global optimum without good initial estimates [5]. Due to this, the linearization methods and convex relaxation methods have been proposed to solve the problem. The linearization methods usually have closed-form solutions, however, their performance may significantly degrade as the shadowing effect becomes severe [6, 7]. The convex relaxation methods relax the ML or approximate problem into a convex semidefinite program (SDP). This kind of methods typically achieve better performance than the linearization methods, at the cost of higher complexity. Ouyang *et al.* [8] eliminated the logarithms firstly, and then relaxed the formulated problem based on MLE into an SDP. Wang *et al.* [9] proposed the least squares relative error (LSRE) method based on the least absolute relative error criterion, where the formulated problem was proven to be an approximation to the MLE. The formulated problem was also relaxed into an SDP. It was shown that this method performs better than several existing methods.

Traditionally, the model parameters, e.g., the transmit power and the path loss exponent (PLE), are estimated before performing localization, by a large amount of training data during the calibration phase. However, the calibration significantly increases the communication overhead and computational complexity. Instead of doing calibration, researchers proposed several methods which jointly estimate the model parameters and the source location. Wang *et al.* [10] formulated a non-convex weighted least squares (WLS) problem based on the unscented transformation (UT) to jointly estimate the source location and the transmit power. The method was extended to the case when the PLE is also unknown. Tomic *et al.* [11] proposed a three-step alternating estimation procedure to estimate the source location and the model parameters simultaneously, where the second-order cone relaxation was used to solve the non-convex problem.

In indoors and urban areas, the non-line-of-sight (NLOS) propagations may cause additional path losses. Hence, the outdoor RSS measurement model is not sufficient for modeling these environments. To cope with this, some additional path-loss terms are added to the original outdoor RSS measurement model [12, 13]. The additional path losses can be computed by using some prior information. For example, in [12, 13], the additional path losses are computed by assuming that the number of partitions and the attenuation of each partition in indoors are known. However, the prior information is generally difficult to obtain in practice. Even if this information can be obtained, it may not be accurate. To alleviate this problem, Tomic *et al.* [14, 15] assumed that the additional path-loss terms were totally unknown. In [14], the path-loss terms were replaced by their mean, which was jointly estimated with the source location by formulating a generalized trust region sub-problem (GTRS). In [15], a worst-case robust

method was proposed to mitigate the effect of the path-loss terms, where the path-loss terms were treated as nuisance parameters.

In a typical localization scenario, the additional path losses vary in a dramatic range. The additional path loss is very small if there is a line-of-sight (LOS) path between the sensor and the source, while it can be very large if there are many partitions/barriers in the path. In the worst-case robust method, e.g., [15], an accurate upper bound for the additional path losses is important. Obviously, the fixed upper bound does not fit all scenarios, which degrades the performance of the robust methods in the localization scenario with more line-of-sight (LOS) paths. To solve this problem, we propose to introduce a balancing parameter according to the additional path-loss terms, and express the additional path-loss terms as the sum of the balancing parameter and some other error terms. To alleviate the effect of the error terms, we formulate a worst-case robust weighted least squares (RWLS) problem with the balancing parameter and the source location as variables. The proposed RWLS problem is difficult minimax problem. To solve this problem, we need to eliminate the maximization part first. To do so, we employ the S-Lemma to eliminate the maximization part and then the semidefinite relaxation (SDR) technique to relax it into a tractable convex SDP.

*Notations:* Bold face lower case and bold face upper case letters are used to denote vectors and matrices, respectively.  $A_{i,j}$  denotes the element at the  $i$ th row and  $j$ th column of matrix  $\mathbf{A}$  and  $\mathbf{A}_{i:j,m:n}$  denotes the submatrix whose elements are the intersection of the  $i$ th to  $j$ th row and  $m$ th to  $n$ th column of matrix  $\mathbf{A}$ .

## 2 System Model

Consider a  $k$ -dimensional ( $k = 2$  or  $3$ ) WSN composed of  $N$  sensor nodes and one source node that needs to be located. The locations of the sensors and the source are denoted by  $\mathbf{s}_1, \dots, \mathbf{s}_N$ , and  $\mathbf{x}$ , respectively. Assume that the source emits signals to the sensors, which are able to compute the power from the received signals, i.e., the RSS. In outdoor environments, it is known that the RSS measurement at the  $i$ th sensor node can be denoted by:

$$P_i = P_0 - 10\gamma \log_{10} \frac{\|\mathbf{x} - \mathbf{s}_i\|}{d_0} + n_i, \quad i = 1, \dots, N, \quad (1)$$

where  $P_0$  represents the received power at a reference distance  $d_0$ ,  $\gamma$  the PLE, and  $n_i$  the shadowing effect modeled by a zero-mean Gaussian random variable with variance  $\sigma_i^2$ , i.e.,  $n_i \sim \mathcal{N}(0, \sigma_i^2)$ .

However, the above model is not appropriate in indoors or urban areas due to the fruitful multipath fading and partitions/barriers, or, NLOS signal propagations, which cause additional path losses. To model the additional path loss, we add an additional term  $b_i$  into the above propagation model, yielding the following:

$$P_i = P_0 - b_i - 10\gamma \log_{10} \frac{\|\mathbf{x} - \mathbf{s}_i\|}{d_0} + n_i, \quad i = 1, \dots, N. \quad (2)$$

In the previous study [12, 13],  $b_i$  is computed by assuming that the number of partitions and the attenuation of each partition are known. However, it is difficult to obtain this prior information in practice since a large amount of training data are required. Even if it can be obtained, it may not be accurate, yielding performance loss. Due to these, we treat  $b_i$  as a completely unknown variable and develop a robust localization method in this paper. The only assumption on  $b_i$  is that  $b_i$  is upper bounded by a known constant  $\rho_i$ , i.e.,  $0 \leq b_i \leq \rho_i$ .

### 3 The RWLS Method

In this section, we detail the derivations of the proposed RWLS method. Without loss of generality, we consider the 2-D case in the following, i.e.,  $k = 2$ . Extension to the 3-D case is straightforward.

According to the discussion in the Introduction, a balancing parameter  $\bar{b}$  is introduced to the propagation model (2), giving:

$$P_i = P_0 - \hat{b}_i - \bar{b} - 10\gamma \log_{10} \|\mathbf{x} - \mathbf{s}_i\| + n_i, i = 1, \dots, N \tag{3}$$

where  $\hat{b}_i = b_i - \bar{b}$  and  $d_0 = 1$  without loss of generality.

In the following, we jointly estimate the source location  $\mathbf{x}$  and the balancing parameter  $\bar{b}$ , and simultaneously, develop a robust method to eliminate the effect of the path-loss terms  $\hat{b}_i$  ( $i = 1, \dots, N$ ). To this end, we first rewrite (3) into an equivalent form:

$$10^{\frac{P_0 - P_i}{10\gamma}} 10^{-\frac{\hat{b}_i}{10\gamma}} = 10^{\frac{\bar{b}}{10\gamma}} 10^{\log_{10} \|\mathbf{x} - \mathbf{s}_i\|} 10^{-\frac{n_i}{10\gamma}}, i = 1, \dots, N. \tag{4}$$

Applying the first-order Taylor-series expansion to the noise term on the right-hand side, we have

$$10^{\frac{P_0 - P_i}{10\gamma}} 10^{-\frac{\hat{b}_i}{10\gamma}} \approx 10^{\frac{\bar{b}}{10\gamma}} \|\mathbf{x} - \mathbf{s}_i\| \left(1 - \frac{\ln(10)}{10\gamma} n_i\right), i = 1, \dots, N. \tag{5}$$

Letting  $d_i = 10^{\frac{P_0 - P_i}{10\gamma}}$ ,  $\tilde{c}_i = 10^{-\frac{\hat{b}_i}{10\gamma}}$ , and  $\alpha = 10^{\frac{\bar{b}}{10\gamma}}$ , we write (5) as a more concise form:

$$\begin{aligned} d_i \tilde{c}_i &\approx \|\alpha \mathbf{x} - \alpha \mathbf{s}_i\| - \|\alpha \mathbf{x} - \alpha \mathbf{s}_i\| \frac{\ln(10)}{10\gamma} n_i, \\ &= \|\mathbf{y} - \alpha \mathbf{s}_i\| - \|\mathbf{y} - \alpha \mathbf{s}_i\| \frac{\ln(10)}{10\gamma} n_i, i = 1, \dots, N, \end{aligned} \tag{6}$$

where  $\mathbf{y} = \alpha \mathbf{x}$ .

According to (6), we can formulate the following worst-case RWLS problem:

$$\begin{aligned} \min_{\mathbf{g}, \mathbf{y}} \quad & \max_{\tilde{\mathbf{e}} = [\tilde{e}_1, \dots, \tilde{e}_N]^T} (\mathbf{A}\tilde{\mathbf{e}} - \mathbf{g})^T \mathbf{Q}^{-1} (\mathbf{A}\tilde{\mathbf{e}} - \mathbf{g}) \\ \text{s.t.} \quad & \mathbf{g} = [\|\mathbf{y} - \alpha \mathbf{s}_1\|, \dots, \|\mathbf{y} - \alpha \mathbf{s}_N\|]^T, \end{aligned} \tag{7}$$

where  $\mathbf{A} = \text{diag}\{d_1, \dots, d_N\}$ ,  $\tilde{\mathbf{e}} = [\tilde{e}_1, \dots, \tilde{e}_N]^T$ , and  $\mathbf{Q} = \mathbf{D}\mathbf{R}\mathbf{D}^T$  with  $\mathbf{D} = \text{diag}\left\{\|\mathbf{y} - \alpha\mathbf{s}_1\| \frac{\ln 10}{10^\gamma}, \dots, \|\mathbf{y} - \alpha\mathbf{s}_N\| \frac{\ln 10}{10^\gamma}\right\}$  and  $\mathbf{R} = \text{diag}\{\sigma_1^2, \dots, \sigma_N^2\}$ .

The weighting matrix  $\mathbf{Q}^{-1}$  is unknown since it is related to the unknown variables  $\mathbf{y}$  and  $\alpha$ . Here, we replace the weighting matrix with an approximation denoted by  $\hat{\mathbf{Q}}^{-1}$ , which is obtained by replacing  $\|\mathbf{y} - \alpha\mathbf{s}_i\|$  with  $d_i$  for sufficiently small errors.

Problem (7) is a difficult non-convex minimax optimization problem. To solve this problem, we first eliminate the maximization part by employing the S-Lemma, and then use the SDR technique to relax it into a tractable convex SDP.

Assume that  $\bar{b}$  is upper bounded by a given constant  $\bar{\rho}$ , i.e.,  $0 \leq \bar{b} \leq \bar{\rho}$ , which implies that  $-\bar{\rho} \leq \hat{b}_i \leq \rho_i - \bar{\rho}$ . It follows from  $\tilde{e}_i = 10^{-\frac{\hat{b}_i}{10^\gamma}}$  that  $10^{-\frac{\bar{\rho}-\rho_i}{10^\gamma}} \leq \tilde{e}_i \leq 10^{-\frac{\bar{\rho}}{10^\gamma}}$ , from which we further obtain

$$\frac{10^{-\frac{\bar{\rho}-\rho_i}{10^\gamma}} - 10^{-\frac{\bar{\rho}}{10^\gamma}}}{2} \leq \tilde{e}_i - \frac{10^{-\frac{\bar{\rho}-\rho_i}{10^\gamma}} + 10^{-\frac{\bar{\rho}}{10^\gamma}}}{2} \leq \frac{10^{-\frac{\bar{\rho}}{10^\gamma}} - 10^{-\frac{\bar{\rho}-\rho_i}{10^\gamma}}}{2}. \tag{8}$$

By defining  $\bar{v} = \frac{10^{-\frac{\bar{\rho}}{10^\gamma}}}{2}$  and  $\hat{v}_i = \frac{10^{-\frac{\bar{\rho}-\rho_i}{10^\gamma}}}{2}$ , (8) can be written into a more concise form:

$$|\tilde{e}_i - \bar{v} - \hat{v}_i| \leq \bar{v} - \hat{v}_i, \quad i = 1, \dots, N. \tag{9}$$

Collecting  $\hat{v}_i$  ( $i = 1, \dots, N$ ) and  $\bar{v} - \hat{v}_i$  ( $i = 1, \dots, N$ ) into vectors  $\hat{\mathbf{v}} = [\hat{v}_1, \dots, \hat{v}_N]^T$  and  $\tilde{\mathbf{v}} = [\bar{v} - \hat{v}_1, \dots, \bar{v} - \hat{v}_N]^T$ , respectively, we have

$$\|\tilde{\mathbf{e}} - \bar{v}\mathbf{1}_N - \hat{\mathbf{v}}\|^2 \leq \|\tilde{\mathbf{v}}\|^2, \tag{10}$$

where  $\mathbf{1}_N$  is an all-one column vector of length  $N$ .

Problem (7) can be equivalently written as the epigraph form:

$$\begin{aligned} \min_{\mathbf{g}, \mathbf{y}, \tau} \quad & \tau \\ \text{s.t.} \quad & \max_{\tilde{\mathbf{e}}} \{(\mathbf{A}\tilde{\mathbf{e}} - \mathbf{g})^T \hat{\mathbf{Q}}^{-1} (\mathbf{A}\tilde{\mathbf{e}} - \mathbf{g})\} \leq \tau, \\ & \mathbf{g} = [ \|\mathbf{y} - \alpha\mathbf{s}_1\|, \dots, \|\mathbf{y} - \alpha\mathbf{s}_N\| ]^T, \\ & 1 \leq \alpha \leq 10^{-\frac{\bar{\rho}}{10^\gamma}}, \end{aligned} \tag{11}$$

where  $\mathbf{Q}$  has been replaced by its approximation  $\hat{\mathbf{Q}}$ .

By invoking (10), the first constraint in (11) implies that

$$\forall \tilde{\mathbf{e}} \in \{\tilde{\mathbf{e}} \mid \|\tilde{\mathbf{e}} - \bar{v}\mathbf{1}_N - \hat{\mathbf{v}}\|^2 \leq \|\tilde{\mathbf{v}}\|^2\} \Rightarrow (\mathbf{A}\tilde{\mathbf{e}} - \mathbf{g})^T \hat{\mathbf{Q}}^{-1} (\mathbf{A}\tilde{\mathbf{e}} - \mathbf{g}) \leq \tau, \tag{12}$$

i.e.,

$$\begin{aligned} \begin{bmatrix} \tilde{\mathbf{e}} \\ 1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_N & -\bar{v}\mathbf{1}_N - \hat{\mathbf{v}} \\ (-\bar{v}\mathbf{1}_N - \hat{\mathbf{v}})^T & 4\bar{v} \sum_{i=1}^N \hat{v}_i \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{e}} \\ 1 \end{bmatrix} &\leq 0 \Rightarrow \\ \begin{bmatrix} \tilde{\mathbf{e}} \\ 1 \end{bmatrix}^T \begin{bmatrix} \mathbf{A}^T \hat{\mathbf{Q}}^{-1} \mathbf{A} & -\mathbf{A}^T \hat{\mathbf{Q}}^{-1} \mathbf{g} \\ -\mathbf{g}^T \hat{\mathbf{Q}}^{-1} \mathbf{A} & \mathbf{g}^T \hat{\mathbf{Q}}^{-1} \mathbf{g} - \tau \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{e}} \\ 1 \end{bmatrix} &\leq 0. \end{aligned} \tag{13}$$

According to the S-Lemma [16], there exists a  $\lambda \geq 0$  such that

$$\begin{aligned} & \begin{bmatrix} \mathbf{A}^T \hat{\mathbf{Q}}^{-1} \mathbf{A} & -\mathbf{A}^T \hat{\mathbf{Q}}^{-1} \mathbf{g} \\ -\mathbf{g}^T \hat{\mathbf{Q}}^{-1} \mathbf{A} & \mathbf{g}^T \hat{\mathbf{Q}}^{-1} \mathbf{g} - \tau \end{bmatrix} \\ & \preceq \lambda \begin{bmatrix} \mathbf{I}_N & -\bar{v} \mathbf{1}_N - \hat{\mathbf{v}} \\ (-\bar{v} \mathbf{1}_N - \hat{\mathbf{v}})^T & 4\bar{v} \sum_{i=1}^N \hat{v}_i \end{bmatrix}. \end{aligned} \tag{14}$$

Thus, the RWLS problem (11) can be rewritten into

$$\begin{aligned} & \min_{\mathbf{y}, \mathbf{g}, \tau, \lambda} \quad \tau \\ & \text{s.t.} \quad (14), \lambda \geq 0, \\ & \quad \mathbf{g} = [\|\mathbf{y} - \alpha \mathbf{s}_1\|, \dots, \|\mathbf{y} - \alpha \mathbf{s}_N\|]^T, \\ & \quad 1 \leq \alpha \leq 10^{\frac{\bar{p}}{10\bar{\gamma}}}, \end{aligned} \tag{15}$$

where the ‘‘max’’ part has been eliminated.

Problem (15) is still non-convex. We relax it as a tractable convex SDP in the following. By defining  $\mathbf{G} = \mathbf{g}\mathbf{g}^T$ ,  $\mathbf{z} = [\mathbf{y}^T, \alpha]^T$ , and  $\mathbf{Z} = \mathbf{z}\mathbf{z}^T$ , problem (15) can be equivalently written as

$$\begin{aligned} & \min_{\substack{\mathbf{g}, \tau, \lambda \\ \mathbf{Z}, \mathbf{G}}} \quad \tau \\ & \text{s.t.} \quad \begin{bmatrix} \mathbf{A}^T \hat{\mathbf{Q}}^{-1} \mathbf{A} & -\mathbf{A}^T \hat{\mathbf{Q}}^{-1} \mathbf{g} \\ -\mathbf{g}^T \hat{\mathbf{Q}}^{-1} \mathbf{A} & \text{tr}\{\hat{\mathbf{Q}}^{-1} \mathbf{G}\} - \tau \end{bmatrix} \\ & \quad \preceq \lambda \begin{bmatrix} \mathbf{I}_N & -\bar{v} \mathbf{1}_N - \hat{\mathbf{v}} \\ (-\bar{v} \mathbf{1}_N - \hat{\mathbf{v}})^T & 4\bar{v} \sum_{i=1}^N \hat{v}_i \end{bmatrix}, \end{aligned} \tag{16a}$$

$$\lambda \geq 0, \tag{16b}$$

$$\begin{aligned} & G_{i,i} = \text{tr}\{\mathbf{Z}_{1:2,1:2}\} - 2\mathbf{s}_i^T \mathbf{Z}_{1:2,3} + Z_{3,3} \|\mathbf{s}_i\|^2, \\ & i = 1, \dots, N, \end{aligned} \tag{16c}$$

$$1 \leq Z_{3,3} \leq 10^{\frac{\bar{p}}{5\bar{\gamma}}}, \tag{16d}$$

$$\mathbf{Z} \succeq 0, \tag{16e}$$

$$\text{rank}\{\mathbf{Z}\} = 1, \tag{16f}$$

$$\begin{bmatrix} \mathbf{G} & \mathbf{g} \\ \mathbf{g}^T & 1 \end{bmatrix} \succeq 0, \tag{16g}$$

$$\text{rank}\{\mathbf{G}\} = 1, \tag{16h}$$

where the following equivalences

$$\begin{aligned} & \mathbf{Z} = \mathbf{z}\mathbf{z}^T \Leftrightarrow \mathbf{Z} \succeq 0, \text{rank}\{\mathbf{Z}\} = 1, \\ & \mathbf{G} = \mathbf{g}\mathbf{g}^T \Leftrightarrow \begin{bmatrix} \mathbf{G} & \mathbf{g} \\ \mathbf{g}^T & 1 \end{bmatrix} \succeq 0, \text{rank}\{\mathbf{G}\} = 1, \end{aligned} \tag{17}$$

have been used.

In (16), the only non-convex constraints are the rank-1 constraints. Dropping the rank-1 constraints, we can relax (16) as the following SDP

$$\begin{aligned}
 & \min_{\substack{g, \tau, \lambda \\ \mathbf{Z}, \mathbf{G}}} \tau \\
 & \text{s.t.} \quad (16\text{a}), (16\text{b}), (16\text{c}), \\
 & \quad \quad (16\text{d}), (16\text{e}), (16\text{g}).
 \end{aligned} \tag{18}$$

Solve the SDP problem (18) and denote the SDP solution of  $\mathbf{Z}$  as  $\mathbf{Z}^*$ . The final estimate of the source location  $\mathbf{x}^*$  can be extracted from  $\mathbf{Z}^*$ :  $\mathbf{x}^* = \mathbf{Z}_{1,2,3}^*/\mathbf{Z}_{3,3}^*$ .

Finally, we give a hint for choosing the value of  $\bar{\rho}$ . Under dense NLOS conditions, a small  $\bar{\rho}$  (and large  $\rho_i - \bar{\rho}$ ) is required to make  $\rho_i - \bar{\rho}$  be a proper upper bound for  $\hat{b}_i$ . On the other hand,  $\bar{b}$  is small under sparse NLOS conditions, and hence, a small  $\bar{\rho}$  is sufficient. Thus, we conjecture that a small  $\bar{\rho}$  is proper for both dense and sparse conditions, which is consistent with the previous study in [17]. In the simulations, we set  $\bar{\rho}$  as  $\bar{\rho} = 0.4 \frac{1}{N} \sum_{i=1}^N \rho_i$ , i.e., a value smaller than half of the average value of  $\rho_i$ .

## 4 Simulation Results

In this section, simulations are conducted to show the performance of the proposed method (denoted by ‘‘RWLS’’). For comparison, we also include the performance of the recently proposed GTRS method [15] and the LSRE method [9]. Note that LSRE is originally developed based on the outdoor RSS measurement model (1), where the additional attenuations  $b_i$  are not taken into account. The SDP (18) and that in [9] are solved using CVX [18], where the solver is SDPT3 [19] and the precision is ‘‘best’’.

We use eight sensor nodes ( $N = 8$ ) to locate the unknown source. The sensors and the source are assumed to be randomly deployed in a square region with length 50 m. The root mean square error (RMSE), defined by  $\text{RMSE} = \sqrt{\frac{1}{M} \sum_{i=1}^M \|\hat{\mathbf{x}}_i - \mathbf{x}_i\|^2}$ , is used to evaluate the localization performance. Here,  $\mathbf{x}_i$  and  $\hat{\mathbf{x}}_i$  are the true source location and the source location estimate in the  $i$ th Monte Carlo (MC) run, respectively, and  $M = 5000$  is the number of MC runs. The propagation model (2) is used to generate the RSS measurements, where  $P_0 = 30$  dBm,  $d_0 = 1$ , and  $\gamma = 3$ . For simplicity, we assume that  $\sigma_i = \sigma$  for  $i = 1, \dots, N$ . Without loss of generality, we further assume that  $b_i = 0$  if there is an LOS path between the sensor and the source, and  $b_i$  follows the exponential distribution with fixed mean  $\mu = 4$  otherwise. The upper bound of  $b_i$ , i.e.,  $\rho_i$ , is set to  $\mu \ln(10)$ , i.e.,  $\rho_i = \mu \ln(10)$  for  $i = 1, \dots, N$ , which implies that the probability of  $0 \leq b_i \leq \rho_i$  is higher than 90%.

We first examine the scenario when the magnitude of the shadowing effect, i.e., the standard deviation (STD), varies. Assume that there are four randomly chosen NLOS paths (and also four LOS paths). Figure 1 shows the results. As expected, the performance of all the methods degrades as the STD increases.

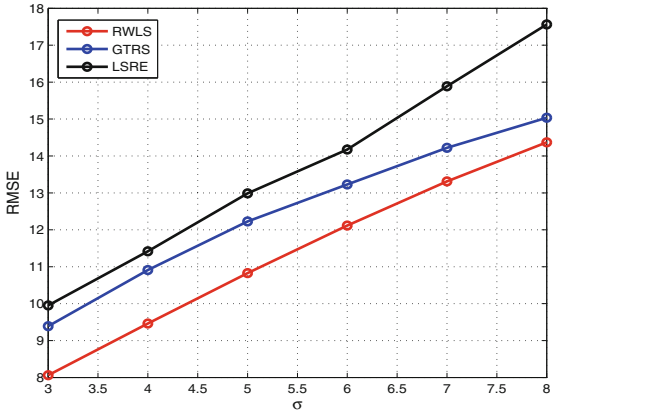


Fig. 1. RMSE versus the STD of the shadowing effect  $\sigma$ .

LSRE performs worst due to the fact that it does not deal with the additional attenuations from the NLOS propagations. Both GTRS and RWLS perform better since the effect of the additional attenuations is mitigated. The proposed RWLS method has the best performance by introducing an additional balancing parameter which improves the performance in mixed LOS/NLOS environments.

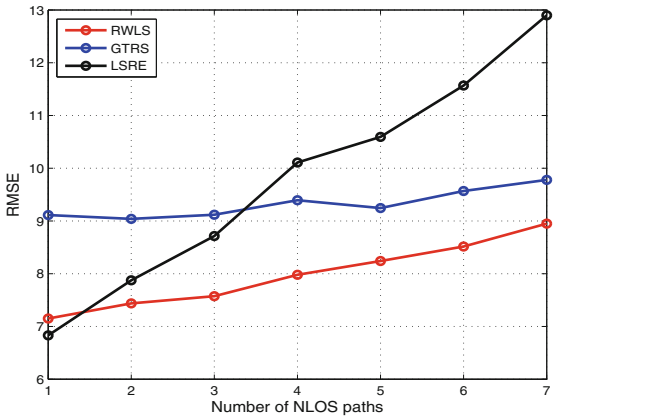
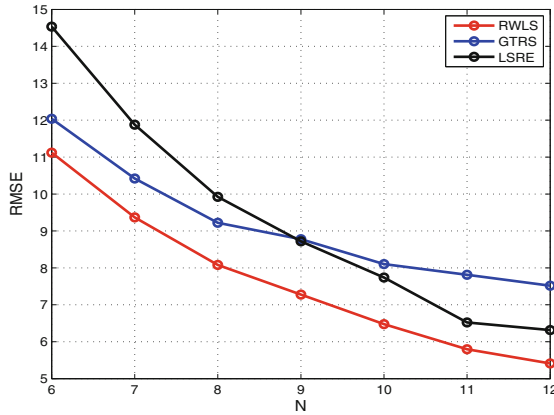


Fig. 2. RMSE versus the number of NLOS paths.

Next, we fix the STD of the shadowing effect  $\sigma = 3$  and change the number of NLOS paths from 1 to 7. The results are shown in Fig. 2. Generally, LSRE performs well under very sparse NLOS conditions; however, its performance deteriorates dramatically as the number of NLOS paths grows. In comparison, the GTRS and RWLS methods perform stably regardless of whether the NLOS

conditions are sparse or dense. RWLS still performs better than GTRS in this scenario, and performs only slightly worse as compared to LSRE in very sparse NLOS environments.



**Fig. 3.** RMSE versus the number of sensors.

Finally, we consider the scenario when the number of sensors varies, where  $\sigma = 3$  and the number of NLOS paths is 4. Figure 3 shows the results. When the number of sensors is small, LSRE performs much worse than the other two methods. However, when the number of sensors is large enough, it performs better than GTRS. In comparison, the proposed RWLS method always performs best regardless of the number of sensors.

## 5 Conclusion

In this paper, a novel method has been proposed for RSS based source localization in indoors and urban areas, where some additional path losses are included. Under the condition of knowing the upper bounds on the additional path-loss terms, we have formulated an RWLS problem to mitigate the effect of the additional path-loss terms. Simulation results have confirmed the superior performance of the proposed method over the existing methods.

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