



# Analysis of Chaotic Behavior in Single Mode NH<sub>3</sub> Molecular Laser

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**Abstract.** In this paper, a single-mode NH<sub>3</sub> molecular laser system is introduced. Firstly, the basic dynamic behaviors of the laser system such as bifurcation diagram, Lyapunov exponent and complexity C<sub>0</sub> are analyzed by numerical simulation using MATLAB, and the chaotic characteristics of the laser system and its sensitivity to parameters are described. At the same time, in order to further study the influence of its parameters, the relationship between the system period and the existence of chaos is studied by using the complexity C<sub>0</sub> under the change of two parameters.

**Keywords:** NH<sub>3</sub> molecular laser · Chaotic · Complexity

## 1 Introduction

Chaos is the internal randomness of deterministic systems, and Chaos is a universal phenomenon in nature. With people's understanding of nature, chaos phenomenon was discovered in meteorology in 1963. The first chaos model was established [1], which opened people's research on chaotic system and chaotic dynamics [2]. More than ten years ago, most researchers focused on the construction of chaotic systems. However, in recent years, it has been gradually discovered that chaotic phenomena exist widely in many engineering systems, such as motor systems [3], power system, DC/DC converter system [4] and laser system [5].

In the 1980s, many scientists have found chaos in a series of lasers such as CO<sub>2</sub> laser, XE laser, He-Ne laser, NH<sub>3</sub> laser and semiconductor laser. NH<sub>3</sub> laser system is a class of strong nonlinear and multi-coupling complex system. The chaotic behavior of NH<sub>3</sub> laser system has been paid much attention and research by engineers and researchers. In paper [5], a parameter mismatch scheme is adopted for a kind of complex laser system synchronized and the security of two laser chaotic systems are analysed and tested. Ref [6] reviewed the random number generators in laser chaos physics, pointed out the advantages and disadvantages of various random number generators, and described the new research results in this direction. The team of Southwest University studied the laser chaotic system [7–9] by using finite-time master-slave control, delay feedback control,

parallel control and series control methods respectively, and finally achieved stable control. Based on this, more attention should be paid to the study of laser chaos mechanism. The study of laser chaos mechanism is particularly important for the synchronous control of laser chaotic system [10–12]. Laser chaos is also studied in secure communication and optical fiber communication [13]. Image encryption based on traditional chaotic system is easy to be implemented in the circuit. However, it is also limited by the inherent problems of electronic circuits such as low speed, low complexity, high attenuation, high cost and so on [14]. Applying optical chaos generated by semiconductor lasers to image information security processing is expected to effectively solve the problems of low bandwidth and low transmission efficiency in traditional image information encryption and transmission.

The mathematical model of a class of single-mode lasers is studied in this paper. Firstly, the phase diagram is used to show the attractor of the whole system. Secondly, using bifurcation diagram, Lyapunov exponent and complexity CO analysis methods, the influence of parameters on the system is studied. Finally, in order to further expatiate the strength of the interaction between the parameters and the system, the system complexity under the change of two parameters is used to study it. The simulation results show that the parameter  $a$  has a great influence on the system. The study of laser chaos in this paper provides important value for its application in encryption and optical fiber communication.

## 2 Chaotic System Model of Single-Mode NH<sub>3</sub> Molecular Laser

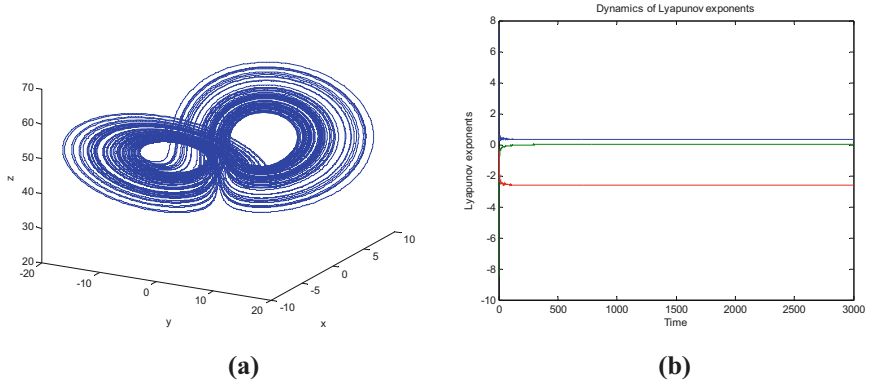
In the 1970s, the Maxwell-Bloch equation describing the uniformly broadened single-mode laser is derived by Haken. After mathematical derivation and simplification for the single-mode NH<sub>3</sub> molecular laser. It is called the Lorenz-Haken laser system. Through scale change, it is concluded that this equation is consistent with the classical Lorenz equation in form. And its simplified dynamic equation is:

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - y - xz \\ \dot{z} = xy - bz \end{cases} \quad (1)$$

In the formula,  $x$  represent the electric field intensity of the laser;  $y$  is macroscopic planned strength of laser working medium,  $z$  represent the reverse ion number density of atoms in the laser working material. Parameter  $a$  and  $c$  represent the pump power and  $b$  represents the pump parameter. The equilibrium points of system (1) can be easily calculated by setting the left-hand side to zero. The system (1) has three special equilibria  $E1(0, 0, 0)$ ,  $(\sqrt{b(c-1)}, \sqrt{b(c-1)}, c-1)$ ,  $E2(-\sqrt{b(c-1)}, -\sqrt{b(c-1)}, c-1)$ . The linearization analysis is carried out on the system at the equilibrium point  $E = (x_0, y_0, z_0)$ , and the Jacobian matrix is obtained as:

$$J = \begin{bmatrix} -a & a & 0 \\ c - z_0 & -1 & -x_0 \\ x_0 & y_0 & -b \end{bmatrix} \quad (2)$$

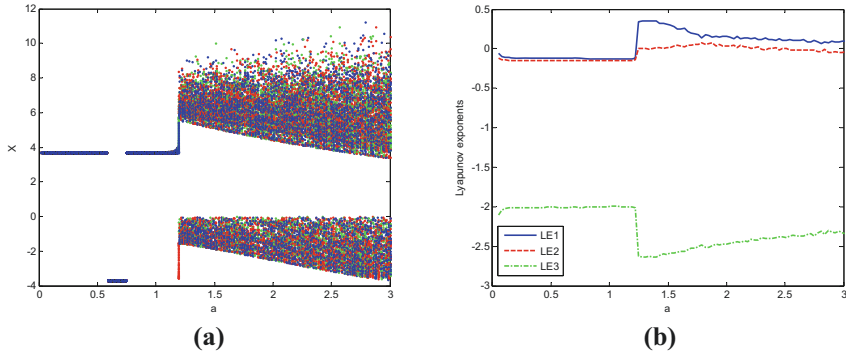
when  $a = 1.4253$ ,  $c = 50$ ,  $b = 0.2778$ , the eigenvalues at E1 are:  $\lambda_{11} = -9.6572$ ,  $\lambda_{12} = 7.2319 > 0$ ,  $\lambda_{13} = -0.27778$ ; The eigenvalues at E2 and E3 are  $\lambda_{21} = -2.7117$ ,  $\lambda_{22} = 0.0043 + 3.7828i > 0$ ,  $\lambda_{23} = 0.0043 - 3.7828i > 0$ , so system (1) that the Lorenz-Haken laser system enters a chaotic state. The three Lyapunov exponent of the system are  $LE1 = 0.33 > 0$ ,  $LE2 = 0$ ,  $LE3 = -2.6 < 0$ , which accord with the law of Lyapunov exponents of three-dimensional chaotic system (+ 0 -). And it is found through simulation that there is a typical chaotic attractor in system (1), as shown in Fig. 1(a). The Lyapunov exponent of the system is shown in Fig. 1(b).



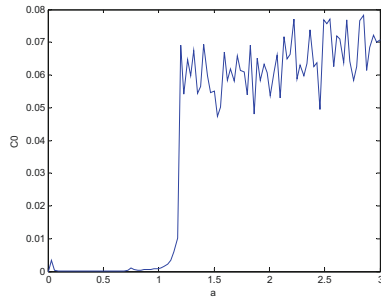
**Fig. 1.** Chaotic attractor and Lyapunov exponent of single-mode NH<sub>3</sub> molecular laser system. (a) Chaotic attractor, (b) Lyapunov exponent.

### 3 Influence of Parameters on Laser Chaotic System

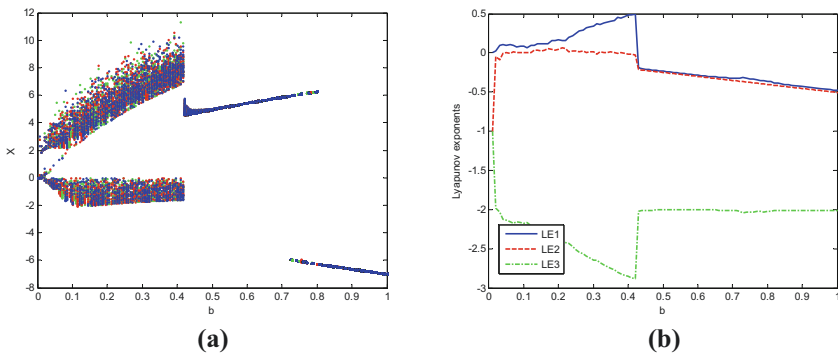
For laser chaotic system, the influence of parameters  $a$ ,  $b$  and  $c$  on the system is analyzed by bifurcation diagram, Lyapunov exponential spectrum (LE spectrum) and complexity C0. When  $a$  varies from 0 to 3 and other parameters keep unchanged, the bifurcation diagram of the state variable  $x$  and the corresponding Lyapunov exponent are shown in Fig. 2. The system is periodic when  $a \in [0, 1.2)$ , and largest Lyapunov exponent is negative. The system is chaotic when  $a \in [1.2, 3)$ , and largest Lyapunov exponent is positive. Thus it can be seen that the bifurcation diagram is consistent with the Lyapunov exponent. As can be seen from Fig. 3, when the system is in periodic state, the complexity C0 is small; when the system is in chaotic state, the complexity C0 of the system is large.



**Fig. 2.** Bifurcation diagram and Lyapunov exponent of the system changing with  $a$ . (a) Bifurcation diagram. (b) Lyapunov exponents.



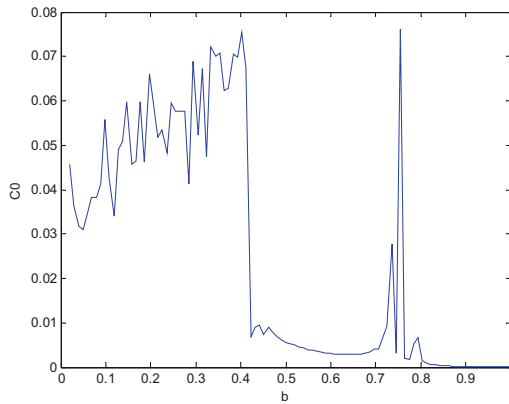
**Fig. 3.** System complexity  $C_0$  of  $a$ .



**Fig. 4.** Bifurcation diagram and Lyapunov exponent of the system changing with  $b$ . (a) Bifurcation diagram. (b) Lyapunov exponents.

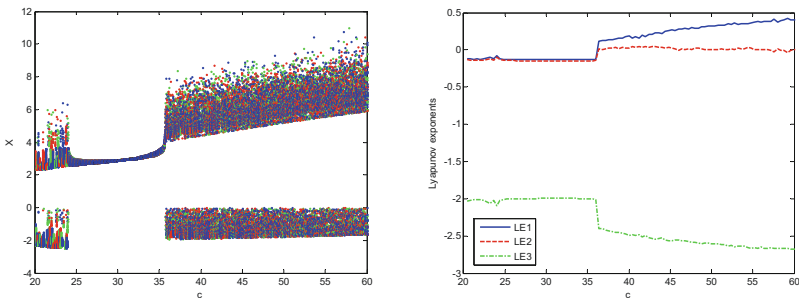
When  $b$  varies from 0 to 1 and other parameters keep unchanged, the bifurcation diagram of the state variable  $x$  and the corresponding Lyapunov exponent are shown in Fig. 4. The system is fixed point when  $b \in [0.45, 1]$ , and largest Lyapunov exponent is negative. The system is chaotic when  $b \in [0, 0.45)$ , and largest Lyapunov exponent is

positive. The system is in periodic state, the complexity  $C_0$  is small; when the system is in chaotic state, the complexity  $C_0$  of the system is large, as shown in Fig. 5.



**Fig. 5.** System complexity  $C_0$  of  $b$ .

When  $c$  varies from 20 to 60, and other parameters keep unchanged, the bifurcation diagram of the state variable  $x$  and the Lyapunov exponent are shown in Fig. 6. The system is fixed point when  $c \in [20, 35]$ , and largest Lyapunov exponent is negative. The system is chaotic when  $c \in [35.5, 60]$ , and largest Lyapunov exponent is positive. The system is in periodic state, the complexity  $C_0$  is small; when the system is in chaotic state, the complexity  $C_0$  of the system is large, as shown in Fig. 7.



**Fig. 6.** Bifurcation diagram and Lyapunov exponent of the system changing with  $c$ . (a) Bifurcation diagram. (b) Lyapunov exponents.

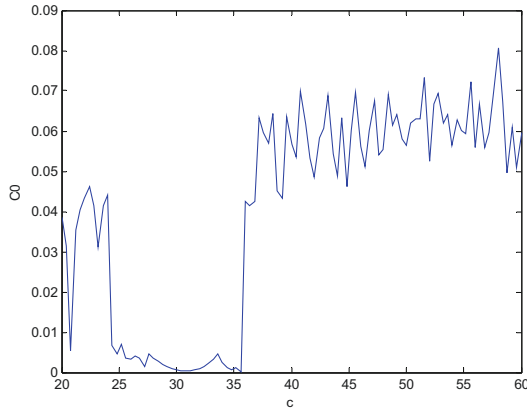


Fig. 7. System complexity  $C_0$  of  $c$ .

#### 4 The Influence of Two Parameters on Chaotic System

This paper describes the bifurcation space of the system, taking the complexity  $C_0$  and the maximum Lyapunov exponent as standard quantities. As can be seen from Fig. 8, the parameters of the system are mutually restricted. But the influence of the parameters of the system shown in above research, is a case of only one parameter changing. In order to further study the influence of parameter changes on the system, we give the bifurcation graph with two parameter changing together.

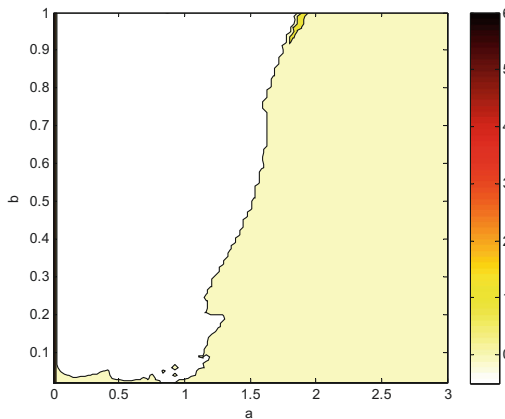


Fig. 8. The situation when the system changes with  $a$  and  $b$  simultaneously

## 5 Conclusions

In this paper, the chaotic dynamic characteristics of the system are analyzed by bifurcation diagram, Lyapunov exponent, and complexity analysis. The bifurcation space of the system is studied by using the complexity  $C_0$  under the change of two parameters. The simulation results show that the system has chaotic motion parameter range. The study of laser chaos dynamics in this paper provides a theoretical basis and reference value for the study of laser chaos synchronization control and optical fiber communication.

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**Conflicts of Interest.** The authors declare that there is no conflict of interest regarding the publication of this paper.

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