# New Analytical Formulas for Coupling Coefficient of Two Inductively Coupled Ring Coils in Inductive Wireless Power Transfer System

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**Abstract.** In this paper, an analytical formula for the coupling coefficient (k) was introduced for two inductively coupled coils of ring configuration. The response surface methodology (RSM) was used as a tool to develop this formula. The k was tested as a function of the geometrical parameters which include the followings parameters: an air-gap (d) between inductively coupled coils; coils dimensions which include the inner ($r_1$) and outer ($r_2$) radii of the transmitter coil, inner ($R_1$) and outer ($R_2$) radii of the receiver coil; and misalignment parameters. Therefore, the introduced k formula is facilitating of a ring coil design, performance optimization of an IPT system, and prediction of system behaviour at normal or misalignment cases. The percentage effect of each parameter on the k was calculated. It was found that the d has the most considerable impact on the k among other geometrical parameters.

**Keywords:** Wireless power transfer · Inductive power transfer · Battery charging · Electric vehicles · Ring coil · Coupling coefficient · Box-Behnken design · Response surface methodology

## 1 Introduction

The The inductive power transfer (IPT) technology has a pivotal role in the widely spreading of contactless battery charging, such as electric vehicles (EVs), mobile phones, and laptops etc. This technology plays major role in addressing the issues of convenience and safety which are causes by the wired charging systems. The IPT technology based on the electromagnetic induction, where the electric power is transferred from the transmitter coil to the receiver coil wirelessly. These coils could take many geometrical configurations, such as ring, rectangular, square etc. It has been found that the geometrical


X. Jiang and P. Li (Eds.): GreeNets 2020, LNICST 333, pp. 117–127, 2020.
https://doi.org/10.1007/978-3-030-62483-5_13

configuration of the coils and their dimensions have a significant influence on the power transfer level and distance (air-gap) [1].

Recently, much researches have been focused on enhancing the IPT power transfer capability in terms of optimization the geometrical coils parameters of the inductively coupled coils, or using resonance topologies [2, 3]. However, the power transfer capability and efficiency of an IPT system are mainly affected by the magnetic coupling coefficient (k) between the inductively coupled coils; this parameter is geometrical. The optimizing of this parameter results in an optimization of the whole system performance. For example, Hiroya T. et al. have found that the optimum k of the H-shaped core transformer obtained at 1.4 of the winding width to the air-gap ratio [4]. On the other hand, it was observed that as the receiver area increases, the k increases [5]. Also, it has been noted that a misalignment between the inductively coupled coils has a negative effect on the IPT system performance [6].

One way to optimize a ring coil performance is to find a mathematical expression of k. For example, an analytical formula of mutual inductance (M) and self-inductances of the transmitter ($L_1$) and receiver ($L_2$) coils in [7] was proposed based on the Neumann's expression; then, k can be calculated due to Eq. (1). However, the ferrite plates effect is not considered; the ferrite plates are usually used behind each coil to enhance the magnetic coupling. In [8], it was mentioned an analytical formula for k. However, this formula does not considered the inner radius of the coils, the ferrite effect, and the misalignment parameters. In this paper, a new analytical formula of k has introduced for ring coil optimization, design, and performance prediction in an IPT system. The response surface methodology (RSM) is used as a tool to develop a formula for the k of two inductively coupled ring coils. Based on this approach, the k can be explored as function of geometrical ring coil parameters which include the coils dimensions, air-gap, and the misalignments parameters. The effect of the ferrite plate behind each coil was considered.

$$k = \frac{M}{\sqrt{L_1 L_2}}. \tag{1}$$

This paper is organized as follows: Sect. 2 presents the IPT system structure, coils configurations and misalignment between the inductively coupled coils. Section 3 describes the proposed the proposed methodology to find the optimization parameters of a ring coil. Section 4 presents the simulation results. Section 5 examines the validation of the introduced formulas. Finally, Sect. 6 shows the conclusion.

## 2 Fundamentals of the Inductive Per Transfer Technology

### 2.1 System Structure

The structure of the typical IPT system for contactless batteries charging is illustrated in Fig. 1. It composes of two unattached sides, which are the transmitter and receiver. The transmitter side is include the following components: a rectifier; an inverter is utilized to excite the transmitter coil ($L_1$) with a high AC voltage of high frequency through a resonance circuit topology. The receiver side is compose of the following components:

a receiver coil ($L_2$) to pick up the electromagnetic field from the transmitter coil and convert it to electric power with the same frequency of the inverter; a rectifier for purpose of AC/DC conversion to supply the battery load with a DC output voltage ($V_L$) [8].

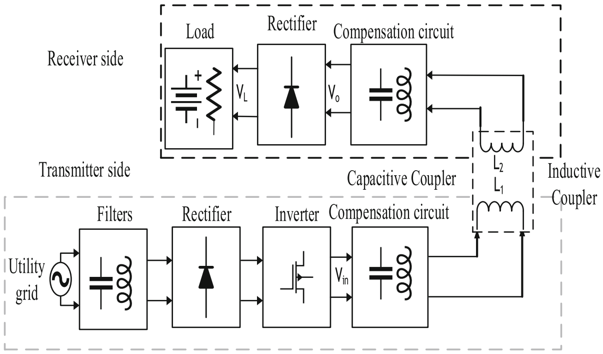


**Fig. 1.** Typical IPT structure for contactless charging system.

The inductive coupler is described by the k, M, $L_1$, and $L_2$ parameters. The k parameter is usually used to predict an IPT system performance, efficiency and power capability that could be transferred [2, 12, 13]. The power delivered to the load ($P_L$) in a compensated system is expressed by Eq. (2) [1]. The quality factor of the receiver coil is given by Eq. (3) [7].

$$P_L = \omega I_1^2 \frac{M^2}{L_2} Q_2 = V_{in} I_1 k^2 Q_2 \tag{2}$$

$$Q_2 = \begin{cases} \frac{\omega L_2}{R_L} & \textit{for series compensation} \\ \frac{R_L}{\omega L_2} & \textit{for parallel compensation} \end{cases} \tag{3}$$

## 2.2 Inductively Coupled Coils

Ithis paper, a ring configuration has been used to represent the inductively coupled coils. A disc of ferrite plate was added behind each coil. The ferrite plates are commonly added to the inductively coupled coils for the purpose of improving magnetic coupling [11]. However, the key aspects of the geometrical parameters of the inductively coupled ring coils are involve: inner radius ($r_1$) and outer radius ($r_2$) of the transmitter coil; inner radius ($R_1$) and outer radius ($R_2$) of receiver coil, (see Fig. 2a); and air-gap (d) between the coils (see Fig. 2b).

## 2.3 Misalignments

The misalignment is a deviation of a moveable receiver coil from its normal position with respect to the transmitter coil which is fixed. In this research, two misalignment cases were considered: lateral misalignment (L) and angular misalignment ($\alpha$) (see Fig. 3a and b).

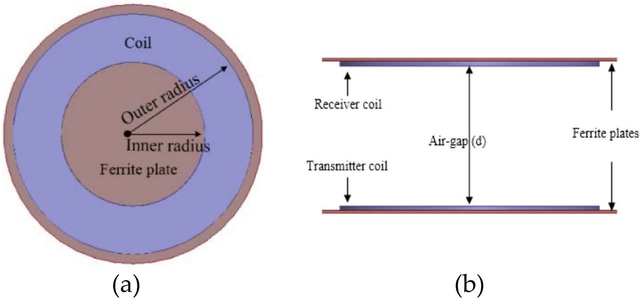


**Fig. 2.** (a) A ring coil with ferrite plate; (b) inductively coupled coils.

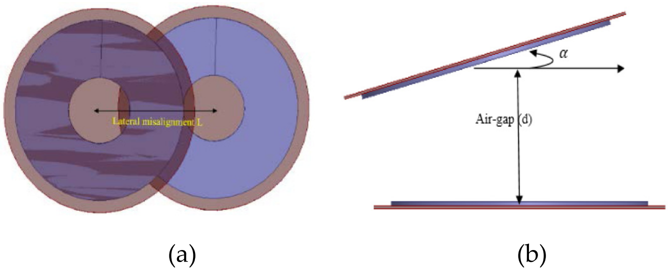


**Fig. 3.** Misalignment: (a) lateral; (b) angular.

# 3 Methods

This paper involves analysing two inductively coupled coils of ring configuration to develop an analytical formula for k. This parameter was explored as a function of the geometrical parameters. The geometrical parameters are including the following parameters: dimensions of the inductively coupled coils, air-gap, and misalignment.

The finite-element analysis (FEA) as well as the response surface analysis (RSA) were applied to explore the k model. Firstly, series of FEA simulations were carried out in the ANSYS Maxwell software to analysis the inductively coupled coils at systematic variation of the geometrical parameters based on a design technique that utilized in the RSM. Secondly, the FEA results were inserted into JMP software to conduct the RSA. The RSA outputs include the quadratic k model; this model is function of the geometrical parameters [12].

## 3.1 Finite Element Analysis (FEA)

This study was conducted with two ring coils of Litz copper wire. A ferrite plate behind each coil was considered with a radius of 4 cm larger than the outer radius of transmitter/receiver coil. The FEA was carried out in the ANSYS Maxwell software to calculate the k.

### 3.2 Response Surface Analysis (RSA)

The RSM utilizes quantitative data from the pertinent experiment to define the regression model to optimize the output model which is affected by several input parameters. The RSM includes a group of statistical and mathematical techniques that are used to fit the empirical model to the experimental outcomes acquired from the relative to experimental design. A quadratic surface fitting is implied by the RSM, this helps to optimize the output model with a minimum number of experiments, furthermore, analysing the interaction effect between the input parameters [13]. Consequently, several linear and/or square polynomial terms are utilized to describe the behaviour of the studied system, and then explore the conditions of an experimental system until its optimization [14]. In this work, the FEA simulation data were utilized instead of experimental data.

The RSA of a model involves the followings three basics steps; multivariate design of experiment (DOE); design technique which utilized for prediction of the coefficients in the output quadratic model. Finally, analysis of variance (ANOVA) which is used for purpose of the analysis and evaluation of the resulted quadratic models. The RSA was achieved with aid of the utilizing the JMP statistical software package.

#### 3.2.1 Multivariate Design of Experiment (DOE)

The multivariate DOE based on the RSM was performed with the following aspects: k is the output response; the $r_1$, $r_2$, $R_1$, $R_2$, d, L, and $\alpha$ are the independent input parameters. In this research, these input parameters are varied with the ranges mentioned in Table 1. These ranges are usually used in electric vehicles applications. However, the maximum, average and minimum values are coded in the JMP software as +, 0, and −, respectively.

**Table 1.** The ranges of the geometrical parameters of two ring coils.

| Studied points | $r_1$ (cm) | $r_2$ (cm) | $R_1$ (cm) | $R_2$ (cm) | d (cm) | L (cm) | $\alpha$ (deg) |
|---|---|---|---|---|---|---|---|
| Minimum | 2 | 30 | 2 | 30 | 10 | 0 | 0 |
| Maximum | 20 | 60 | 20 | 60 | 50 | 20 | 7 |
| Average | 11 | 45 | 11 | 45 | 30 | 20 | 3.5 |

#### 3.2.2 Design Technique

There are many experimental design techniques based on the RSM [14]. We have used Box–Behnken to find the coefficients of the k analytical model. Box-Behnken has an efficient estimation of the coefficients in the mathematical models. This technique is more economical among the others, especially for a large number of input parameters [14]. However, based on the Box-Behnken design, a total of 62 simulated data are required for the geometrical seven input parameters [13]. These data were obtained using FEA.

### 3.2.3 Analysis of Variance (ANOVA)

The graphical analyses are used to define the interaction between the input variables and the output responses to estimate the statistical parameters influence. For this purpose, ANOVA was implemented by using the JMP statistical software package design for the regression analysis. The output model involves linear, quadratic and the interactions terms between every two input parameters. For example, for two input parameters ($x_1$ and $x_2$) the output model (y) can be described by Eq. (4) [14].

$$y = b_0 + b_1x_1 + b_2x_2 + b_{11}x_1^2 + b_{22}x_2^2 + b_{12}x_1x_2 \tag{4}$$

Where $b_0$ is a constant coefficient, $b_i$ is a coefficient represents the linear effect of $x_i$ on y, $b_{ii}$ is a coefficient reveals the quadratic impact of xi on y, and $b_{ij}$ is the effect factor due to interaction between $x_i$ and $x_j$ on y.

In the ANOVA, the significance of the statistical analysis can be examined. The parameters standard deviation (SD) and $R^2$ are an indication of the degree of the model fit. However, the significance of each parameter/term in the polynomial model can be evaluated based on the F-value or probability value (P-value) at a confidence interval of 95%.

The ANOVA table contains the following columns: source, the degree of freedom (DF), the sum of squares, mean square, F-value, and P-value. The F-value and P-value are utilized for assessment the response surface of the output model. The significant factor is described by F-value or P-value with a 95% confidence level. This means only the parameters/terms of P-value less than 5% or 0.05 have a significant effect on the output model. A large F-value or the smaller P-value of a parameter/term points out the significance effect of the corresponding parameter/term [12].

## 4 Results and Discussion

In this section, an analytical formula for k of inductively coupled two ring coils is introduced. The k was tested as a function of seven parameters with their ranges as were mentioned earlier in Sect. 3.2.1. According to the Box-Behnken design technique, a total of 62 experimental data are needed for the seven input parameters [12]. In this work, these data were acquired utilizing the FEA in ANSYS Maxwell software, then entered into the JMP software, as listed in Table 2.

### 4.1 ANOVA for the k Model

The terms that have P-value greater than 0.05 were removed from the ANOVA table of the k model because of their negligible effects. The ANOVA table of the miniature k model is detailed in Table 3. For this model, the F-value is as much as 558.023. This points out that this model is statistically significant. The following parameters terms are considered significate in the k model since they have a P-value less than 0.05: $r_2$, $R_2$, d, L, $r_2$ * $R_2$, $r_2$ * d, d * L, $r_2^2$, $R_2^2$, $d^2$, and $L_2$.

**Table 2.** Output response, k for seven input variables.

| No. | Pattern | Independent input variables | | | | | | | Output responses due FEA |
|---|---|---|---|---|---|---|---|---|---|
| | | $r_1$ (cm) | $r_2$ (cm) | $R_1$ (cm) | $R_2$ (cm) | d (cm) | L (cm) | α (deg) | k |
| 1 | − −0−000 | 2 | 30 | 11 | 30 | 30 | 10 | 3.5 | 0.12726 |
| 2 | − −0+000 | 2 | 30 | 11 | 60 | 30 | 10 | 3.5 | 0.18519 |
| 3 | −0−000− | 2 | 45 | 2 | 45 | 30 | 10 | 0 | 0.27112 |
| 4 | −0−000+ | 2 | 45 | 2 | 45 | 30 | 10 | 7 | 0.25728 |
| .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. | .<br>.<br>. |
| 61 | +0+000+ | 20 | 45 | 20 | 45 | 30 | 10 | 7 | 0.25789 |
| 62 | ++0−000 | 20 | 60 | 11 | 30 | 30 | 10 | 3.5 | 0.19057 |

**Table 3.** ANOVA details for the reduced k model.

| Source | DF | Sum of Squares | F-value | P-value (Prob > F) |
|---|---|---|---|---|
| Model | 11 | 1.9056906 | 558.0230 | |
| $r_2$ | 1 | 0.1118922 | 360.4062 | <.0001* |
| $R_2$ | 1 | 0.0882226 | 284.1661 | <.0001* |
| d | 1 | 1.4131795 | 4551.869 | <.0001* |
| L | 1 | 0.0625464 | 201.4626 | <.0001* |
| $r_2$ * $R_2$ | 1 | 0.0094160 | 30.3292 | <.0001* |
| $r_2$ * d | 1 | 0.0077811 | 25.0630 | <.0001* |
| d * L | 1 | 0.0332267 | 107.0236 | <.0001* |
| $r_2^2$ | 1 | 0.0062627 | 20.1724 | <.0001* |
| $R_2^2$ | 1 | 0.0056681 | 18.2570 | <.0001* |
| $d^2$ | 1 | 0.1482140 | 477.3992 | <.0001* |
| $L^2$ | 1 | 0.0027515 | 8.8627 | 0.0045* |
| Error | 50 | 0.0155231 | | |
| Lack of fit | 13 | 0.01255379 | 13.1078 | |
| Pure error | 37 | 0.00276928 | | |
| Correlated total | 61 | 1.9212136 | | |

As can be seen from Table 3, the d has the most significant effect on k among other parameters, since it has the largest F-value. The quadratic terms that have a significant effect on k are: d, $r_2$, $R_2$, and L. On the other hand, the parameters that have a negligible effect on k are: $r_1$, $R_1$, and $\alpha$. However, only the parameters that have a major impact on k are appeared in its analytical model which can be written by Eq. (5). The absolute brackets were added to avoid the negative value of k due to fitting error. However, the SD for the k model is only 0.177469. This means a good accuracy of the k analytical model was obtained with a $R_2$ as much as 0.99.

$$
\begin{aligned}
k = \Bigg| & 0.265045 + 0.068280125.\left(\frac{r_2 - 45}{15}\right) + 0.06062954.\left(\frac{R_2 - 45}{15}\right) - 0.242657.\left(\frac{d - 30}{20}\right) \\
& - 0.05104995.\left(\frac{L - 10}{10}\right) + 0.034307.\left(\frac{r_2 - 45}{15}\right).\left(\frac{R_2 - 45}{15}\right) \\
& - 0.0311871.\left(\frac{r_2 - 45}{15}\right).\left(\frac{d - 30}{20}\right) + 0.0644463.\left(\frac{d - 30}{20}\right).\left(\frac{L - 10}{10}\right) \\
& - 0.0209289.\left(\frac{r_2 - 45}{15}\right)^2 - 0.0199105.\left(\frac{R_2 - 45}{15}\right)^2 \\
& + 0.1018143.\left(\frac{d - 30}{20}\right)^2 - 0.0138724.\left(\frac{L - 10}{10}\right)^2 \Bigg|
\end{aligned} \tag{5}
$$

## 4.2 Impact of the Input Parameters on the k Model

The graphical analyses are utilized to determine the impact of the interaction between the input variables on the output model (i.e. k) to estimate the statistical impact of the input parameters based on Eq. (5). Figure 4(a) details the interaction impact of ($r_2$ and $R_2$) on k; Fig. 4(b) explains the interaction effect of the (d and L) on the k.

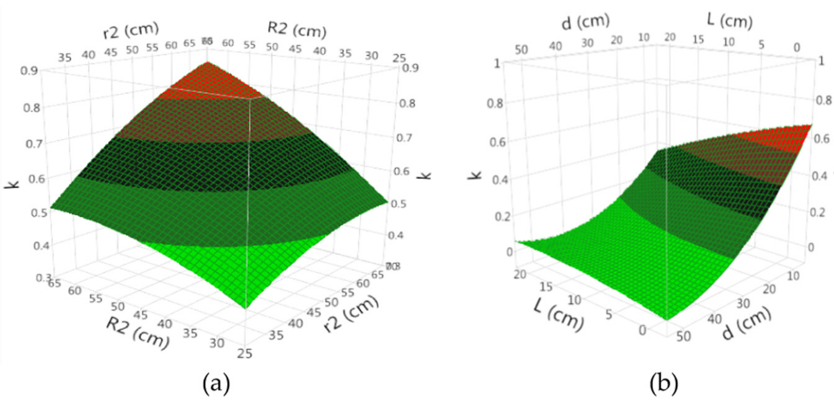


**Fig. 4.** Interaction effect of the input parameters on k: (a) $r_2$ and $R_2$ at d = 10 cm, L = 10 cm; (b) L and d at $R_2$ = 30 cm, $r_2$ = 30 cm.

As can be seen from Fig. 4, the k is considerably declining as d increases. Likewise, as the L increases, the k is slightly diminishing within the studied range of L. However, the increasing of d or L have a negative impact on the k value. On the other hand, as the r2 and/or R2 increasing this result in slightly increasing of k. However, the effect of the d and L on the k slightly decreases as the $r_2$ and/or $R_2$ increasing.

The numerical representation was applied to measure the percentage impact of the input parameters on k, the outcomes are presented in Fig. 5.

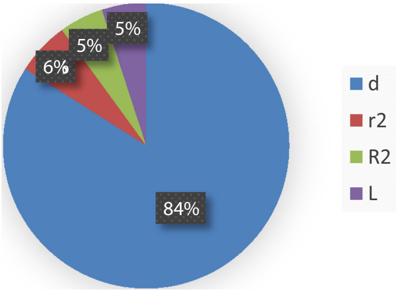


**Fig. 5.** The percentage impact of the significant input parameters on k.

According to Fig. 5, the d has a considerable impact on k which is as many as 84%, the $r_2$ and $R_2$ have a close effect on k i.e. only 6, and 5, respectively. The L has a slight effect on k i.e. 5%. The maximum value of k i.e. 0.864 occurs at maximum $r_2$ (i.e. 60 cm), minimum d (i.e. 10 cm), maximum $R_2$ (i.e. 60 cm), and minimum L (i.e. 0 cm).

## 5 Validation

To verify the analytical formula of k, its results data is compared with the FEA data. Since d has the most impact on the k among all other input parameters, it was chosen as a variable parameter to validate the analytical k formula with the following considerations: $r_2 = R_2 = 30$ cm, and $L = 0$ cm. The results due to the presented analytical formula and FEA are explained in Fig. 6.

As can be seen from Fig. 6, the data obtained using the introduced formula of k are in the line with that obtained by FEA especially when the d within the range specified in Table 1, at $d = 5$ cm the k formula is still working but the error increases. This points out that the introduced k formula has a very good accuracy with specified range, which listed in Table 1. However, the error increasing gradually as the input parameters are get far from the investigated ranges.

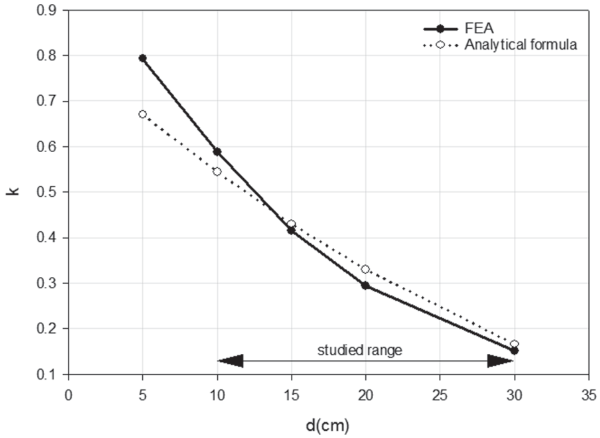


**Fig. 6.** Validation of the analytical k formula with FEA data.

## 6 Conclusion

In this paper, an analytical formula for magnetic coupling coefficient (k) of the two inductively coupled ring coils was introduced. The RSM has been used as a tool to find this formula as function of the geometrical parameters.

In this work, due to our knowledge, it was for the first time to calculate the percentage impact level of the geometrical parameters (i.e. air-gap, each design and misalignment parameter) on the k. This calculation has an importance in an IPT system design and optimization; which helps to specify which parameter has thoroughly to be designed or has a priority to be optimized. We found out that the d has the most impact on k among other geometrical parameters. On the other hand, the k is slightly sensitive to lateral misalignment and has low sensitivity to angular misalignment. This indicates that a ring coil has good misalignment tolerance; this is a desirable feature in the batteries charging of the electric vehicles.

The outcomes due to the introduced analytical model were consistent with the FEA data. This is conclusive evidence that the introduced k model is applicable for any two inductively coupled ring coils. However, the most we can talk about that k model has a very good accuracy within the investigated ranges of the input parameters. This model is still working outside these ranges but the error increases gradually as the parameters get far from their specified range, as given in Table 1.

**Acknowledgments.** This work funded by CTS – Centre of Technology and Systems of the UNL and the Portuguese Foundation of Science and Technology FCT (Project No. funds this work UID/EEA/00066/2019).

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