



Mechanical Vibration Analysis of Fiber Reinforced Polymer Composite Beams Using Analytical and Numerical Methods

Nigatu D. Tilahun^{1,2} and Hirpa G. Lemu³ (✉) 

¹ Addis Ababa Science and Technology University, Addis Ababa, Ethiopia

² Debre Berhan University, Debre Berhan, Ethiopia

³ University of Stavanger, Stavanger, Norway

Hirpa.g.lemu@uis.no

Abstract. In this study, the free and forced vibration analysis of fiber reinforced plastic composite beam has been conducted using numerical method (finite element analysis) and mathematical modeling in MATLAB. Mechanical and physical properties of the beam material are found using strength of material and semiempirical approach to get the equivalent properties. The beam is configured as cantilever beam with dimensions of 191 mm × 33 mm × 5.66 mm for all analyses of the vibration. For modal analysis of the vibrations, different effects such as fiber volume ratio, fiber materials, angle of orientation and stacking sequence of laminates are studied. From these effects, stacking sequence of laminates have highest influence on variation of stiffness and natural frequencies with unique fiber volume ratio and same fiber material. Forced vibration analysis was carried out using the same beam configuration as modal analysis, but with different stacking sequence of layers with harmonically exited loads. It has been observed that the vibration resonance occurred close to the natural frequencies of the beam.

Keywords: Beam of composites · Vibration analysis · Natural frequency · Numerical analysis · Resonance frequency

1 Introduction

There are several areas where fiber reinforced plastic (FRP) composites are dynamically loaded when structured as cantilevered beams such as those used in wind turbine blades, helicopter blades, fans, aeronautical and aerospace industries as well as in other fields of modern engineering technologies [1]. While designing such types of structures, it is necessary to make not only strength and deflection analysis, but also vibrational performance of the beams as initial step to determine their natural frequencies and resonance. Equivalent physical and mechanical properties like Young's modulus and density of FRP composite materials are influenced by different parameters such as fiber materials, fiber volume ratio, fiber angle and stacking sequence of layers. Mainly the

natural frequencies of a beam depend on effective modulus of elasticities and density of FRP composite beam.

The free vibration of a laminated composite beam is commonly studied using classical laminate theory for different boundary conditions and length-to-thickness ratios for several layers, for instance using MATLAB [2]. In a study reported by Murat [3], a new method was applied for numerical modelling of free vibration on cantilever composite beam having a series of open and non-propagating cracks. In the study, mass and stiffness matrices of the composite beam for vibration analysis purpose was established. The mode shapes and natural frequencies of a number of cantilever carbon fiber reinforced polymer composites (CFRPCs) and glass fiber reinforced polymer composites (GFRPCs) were numerically found using the commercial finite element analysis software (ANSYS) [1]. In this study, the vibration characteristics of elliptical cylinder shells made of laminated composite with general boundary conditions were examined. The hypothetical model was recognized by means of the improved variation principle and multilevel partition method based on the first-order shear deformation model [4]. Modal analysis of laminated functionally graded carbon nanotube (FG-CNT) reinforced composite plates by using kp-Ritz based method on the first-order shear deformation theory (FOSDT) was investigated by Lei et al. [5].

Furthermore, Mohandes and Ghasemi [6] have studied and reported that the free vibration of thin circular cylindrical shell laminated with fiber metal reinforced by single walled carbon nanotubes using different boundary conditions. Kumar et al. [7] presented the investigation of free vibrations of the composite by piezoelectric materials. A FOSDT was used to present the undamped natural frequencies of symmetrically cross-ply laminated beams of the stacking sequence (0/90/90/0) using finite element method (FEM) [8, 9]. Babuska et al. [10] presented the theory and derivation of an element stiffness matrix for bend-twist coupled composite laminated beams. The exactness of the stiffness matrix terms was compared with those generated by finite element model in ABAQUS of an idealized beam geometry.

Madhu and Kumaraswamy [11] have also studied modal analysis by using finite element analysis software for laminated cantilever composite plates to find the modal frequencies. Teter and Gawryluk [12] presented a free vibration of a rotor with three active composite blades completed by different methods. The rotor blades structured using unidirectional laminate of glass-epoxy. Elshafei [13] presented finite element model that was established to examine the response of orthotropic and isotropic beams, structural element for aeronautics applications using MATLAB code. Ganesa and Thirumavalavan [14] offered free vibration of glass fiber reinforced composite. The glass fibers were treated with hydrochloric acid and sodium hydroxide solutions. The damping of fiber reinforced composites depends on the diameter, structure, and orientation of fiber in matrix. In the previous investigations, damping of structural composite beams had been investigated to estimate the natural frequencies and damping ratios [15, 16]. Tita et al. [17] proposed a method to estimate the dynamic damped behavior of fiber reinforced composite beams in flexural vibrations. A set of experimental dynamic tests were presented in order to study the modal shapes and natural frequencies.

Composite materials have vast variety of mechanical and physical properties such as density, modulus of elasticity and strength by alternating fiber volume ratio, fiber and matrix materials, angle of orientations and stacking sequence of layers. These properties have effects on the equivalent properties of composite beam, natural frequencies and resonance of the beam. The literature review indicates that there still exists a need to justify the effects of those properties on the natural frequency and stiffness of the beam.

The objective of the study reported in this article is to investigate the mechanical and vibrational performance of cantilevered FRP composite beam subjected to transverse and distributed loads. The beam configuration is selected because FRP composites are widely used in this type boundary conditions and loading modes in several applications such as helicopter rotors and wind turbine blades.

2 Materials and Methods

The study of vibration analysis of FRP composite beam reported in this article is conducted using both numerical and analytical approaches. The steps followed include (1) identifying fiber and matrix material properties, (2) formulating physical and mechanical properties of composite materials using strength of material and semi-empirical approach and (3) coding in MATLAB and (4) conducting numerical analysis of free and forced vibration of continuous fiber composite plates and beams using ABAQUS/CAE 2017.

The elastic properties such as the longitudinal (E_{11} or E_1), transversal (E_{22}/E_2 and E_{33}/E_3) and shear (G_{12}) modulus and major Poisson’s ratio (ν_{12}) of composite materials are expressed as a function of the fiber and matrix volume ratios and their respective elastic properties as follows:

$$E_{11} = V_f \times E_{1f} + V_m \times E_m \tag{1}$$

$$E_{22} = E_{33} = \frac{E_{2f} \times E_m}{V_f \times E_m + V_m \times E_{2f}} \tag{2}$$

$$G_{12} = G_m \frac{(1 + V_f)G_{12f} + V_m G_m}{V_m \times G_{12f} + (1 + V_f) \times G_m} \tag{3}$$

$$\nu_{12} = V_f \nu_{12f} + V_m \nu_m \tag{4}$$

Where: E_{1f} and E_m , are longitudinal fiber and matrix moduli, respectively, E_{2f} is the transverse modulus of the fiber, ν_{12f} and ν_m are the longitudinal Poisson’s ratio of the fiber and the matrix respectively; G_{12f} and G_m are the shear moduli of the fiber and matrix, respectively. Table 1 shows the physical and mechanical properties of typical fiber and matrix materials used in the analyses.

Table 1. Typical Properties of Fibers and matrix (adopted from [18]).

Materials	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	Density (kg/m ³)
Graphite	230	22	22	0.3	1800
E-Glass	85	85	35.42	0.2	2500
Aramid (Kevlar-49)	124	8	3	0.036	1400
Epoxy	3.4	3.4	1.308	0.3	1200

Modal analysis of composite is conducted on composite beam vibration model as per ASTM standards. According to ASTM E756–05 [19], the composite beams are modeled with dimension of 191 mm \times 33 mm \times 5.66 mm as shown in Fig. 1. For numerical analysis in ABAQUS/CAE, vibration analysis of the composite beam, an element size of 0.5 mm was selected.

For analysis of convergency, different values of the element size and number of elements are taken in order to get optimum output. The mesh sizes are 8 mm, 6 mm, 4 mm, 3 mm, 2 mm, 1.5 mm, 1 mm, 0.75 mm, 0.5 mm and 0.3 mm. The beam was simulated for quasi-isotropic stacking sequence consisting of layers having fiber orientations of 0°, +45°, –45°, and 90° symmetrically or [0°/+45°/–45°/90°]_s to show the convergences of natural frequencies of the cantilevered beam. From these element sizes, the natural frequencies for all analyzed modes converged after the element size of 0.5 mm. For other vibration analysis of composite beam, the element size is selected as 0.5 mm from the convergency analysis for all numerical analysis.

To show the effects of orientation angle of fiber, the analysis was done with 10° interval from 0° to 90° (with $V_f = 0.6$ of glass-epoxy). In order to demonstrate the effects of the fiber volume ratio on natural frequency results for unidirectional laminates, glass-epoxy composite beam was used. Fiber volume ratios from 0.1 to 0.9 were employed with 0.1 interval. For the analysis of natural frequency under the effects of different fiber materials, E-glass, graphite and aramid (Kevlar-49) fibers with epoxy matrix of 0.5 fiber volume ratio and Quasi-Isotropic Laminates or [0/ \pm 45/90]_s are used. As given in Table 2, different staking sequences of laminates were employed to identify laminates that have higher value of natural frequency for first five number of modes. For this analysis, E-glass epoxy is used with $V_f = 0.6$.

Forced vibration of FRP composite beam under harmonic excitation uses the same composite beam configuration and element size. In the steady-state, the dynamics inputs are the lower and upper frequency, start mode, end mode and damping ratio. For this study, lower and upper frequency are 0 and 2000 respectively and structural average damping ratio value of 0.055 [15–17] was used. To study the effects of different loads on the resonance and corresponding frequencies of the glass-epoxy composite beams, end force in z (EFz), distributed load in z (DLz) and end force in y (EFy) directions are applied with the magnitude of 10 N (illustrated in Fig. 2).

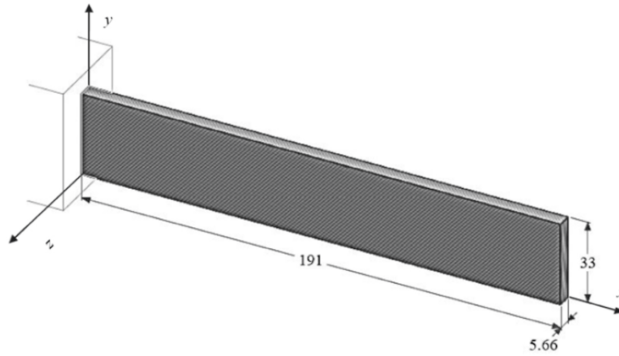


Fig. 1. Cantilever beam configuration.

Table 2. Laminates with different staking sequences.

	Laminates	Stacking sequence
SCAL	Symmetric Cross-Ply Laminates	[0/90/0/90]s
SAPL30	Symmetric Angle-Ply Laminates	[30/-30/30/-30/30]
SAPL45	Symmetric Angle-Ply Laminates	[45/-45/45/-45/45]
SBL	Symmetric Balanced Laminates	[30/-30/45/-45]s
ANBL	Antisymmetric Balanced Laminates	[30/45/-45/-30]
ASBL	Asymmetric Balanced Laminates	[30/45/-30/-45]
ANCPL	Antisymmetric Cross-Ply Laminates	[0/90/0/90/0/90/0/90]
ANAPL30	Antisymmetric Angle-Ply Laminates	[30/-30/30/-30/30/-30/30/-30]
ANAPL45	Antisymmetric Angle-Ply Laminates	[45/-45/45/-45/45/-45/45/-45]
QIL	Quasi-Isotropic Laminates	[0/ ± 45/90]s
CS		[0/ ± 30/90]s

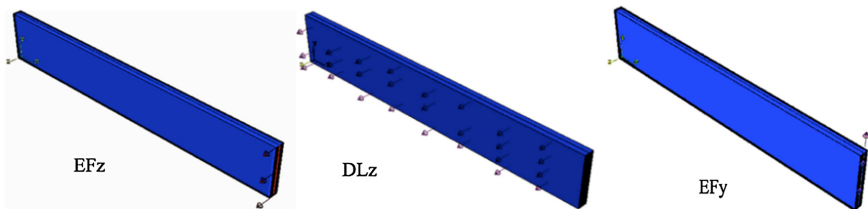


Fig. 2. Applied harmonic loads

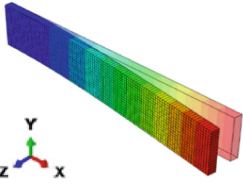
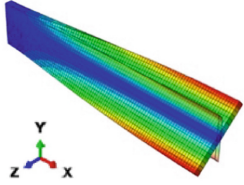
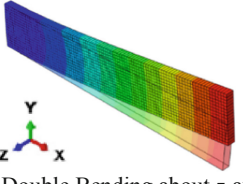
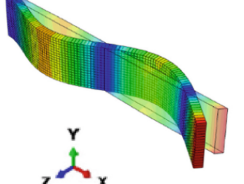
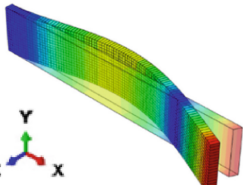
3 Results and Discussion

3.1 Results of Modal Analysis of Composite Beam

The results of modal analysis of FRP composite beams were considered using different parameters such as angle of orientation, fiber volume ratio, fiber materials and stacking sequence of laminates. For some of these parameters, the analysis results are done using both analytical and numerical analysis. Table 3 shows the mode shapes of the analysis of the FRP cantilever beam under those parameters conducted in ABAQUS/CAE.

Figure 3 shows the results of the effects of angle of orientation of fiber on natural frequency of fiber reinforced plastic composite (E-glass Epoxy with $V_f = 0.6$). Natural frequencies for Mode 1, Mode 2, Mode 3 and Mode 5 decreased from 0° to 70° and slightly increased from 70° to 90° angle of orientation due to value of axial modulus. But for Mode 4 or twisting mode the natural frequency has peak value at 30° angle of orientation.

Table 3. Mode shapes of cantilever FRP composite beam

Mode	Type and mode shape	Mode	Type and mode shape
1	Single Bending about z-axis 	4	Twisting 
2	Single Bending about y-axis 	5	Triple Bending about z-axis 
3	Double Bending about z-axis 		

As illustrates in Fig. 4, natural frequency is increased from fiber volume ratio of 0.1 to 0.9 for all modes. But graphs show that for bending modes (Mode 1, Mode 2, Mode 3 and Mode 5) concave downward and for twisting mode (Mode 4) concave upward directions.

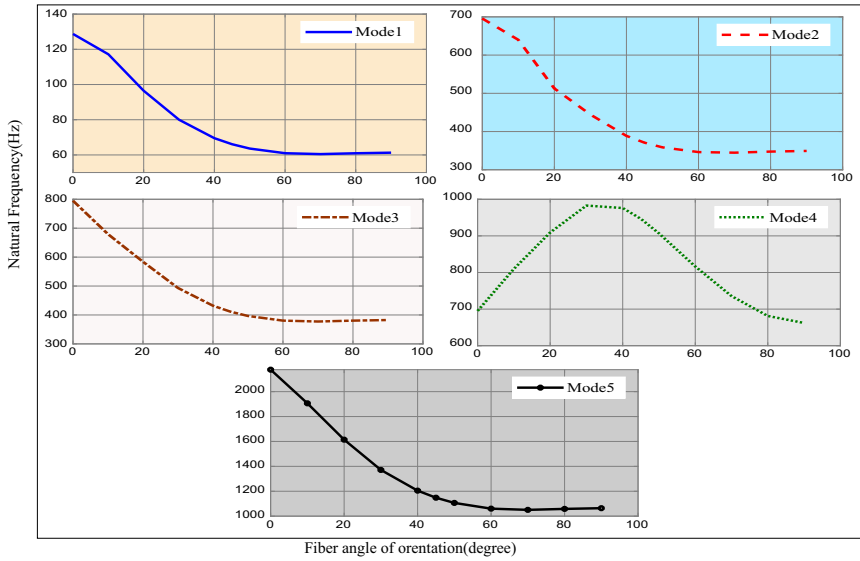


Fig. 3. Effects of angle of orientation for unidirectional laminates.

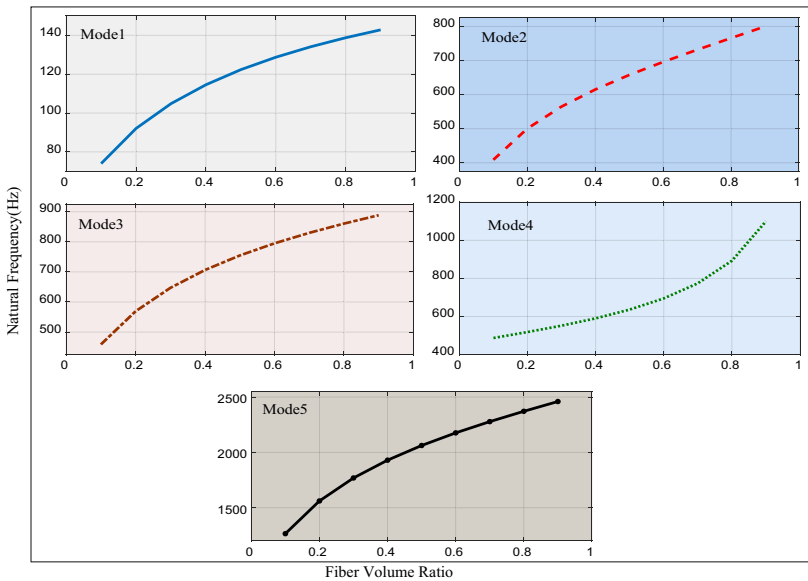


Fig. 4. Effect of fiber volume ratio on natural frequency

Fiber materials (E-glass, graphite and aramid or Kevlar-49) with epoxy matrix effects on natural frequency to be discussed which is done using both numerical and analytical solutions. The results in Table 4 shows that Graphite-Epoxy has greatest value of natural frequency for all mods compared to Glass-Epoxy and Aramid-Epoxy due to its

higher axial modulus. The analytical and numerical results have good agreement except for Mode 2 (bending about y-axis) the natural frequency has greater difference due to consideration of thin beam theory for analytical analysis.

Table 4. Effects of fiber materials on natural frequency

Mode	Natural frequency (Hz)								
	Glass-Epoxy			Graphite-Epoxy			Aramid-Epoxy		
	Num.	Analyt	%diff	Num	Analyt	%diff	Num	Analyt	%diff
1	102.83	102.192	-0.6244	176.76	175.94	- 0.467	140.73	139.91	-0.588
2	478.27	595.819	19.7289	775.66	1025.8	24.385	620.99	815.72	23.87
3	636.96	640.43	0.5412	1075.2	1102.6	2.485	859.52	876.79	1.969
4	832.22	807.96	-3.0023	1203.5	1225.5	1.7952	977.23	983.99	0.687
5	1755.4	1793.2	2.10796	2887.7	3087.3	6.4652	2321.5	2455	5.438

Tables 5 depicts numerical and analytical results of natural frequencies together with their difference in percentage as a function of the effects of stacking sequence of layers. The results show that SAPL45 and ANAPL45 have lower value natural frequency in bending modes (Mode 1, Mode 2, Mode 3 and Mode 5) because of their lower effective flexural modulus. This indicates that these materials have low stiffness in lateral direction. CS ([0/ ± 30/90]s) composite laminate beam has higher values of natural frequency in bending mode vibrations. However, ANCP and SCAL have lower values of natural frequency and ANAPL45 has greater values in torsional vibration (Mode 4).

The differences in percentage of the two results for most natural frequencies are minimum, while in some of the considered cases, higher percentage in natural frequency differences are registered which can be attributed to the assumptions considered in analytical solution.

3.2 Forced Vibration Results Under Harmonic Excitation Loads

In addition to modal analysis, it is necessary to demonstrate resonance due to applied harmonic loads on composite beam with different load conditions. Different stacking sequence layers have different natural frequencies with the same material and fiber volume ratio. So, it is important to show the effects of different loads in different directions on the resonance and corresponding frequencies quantitatively with in different stacking sequences layers and which layers have better characteristics.

Figure 5 (a) shows graphs of amplitude vs frequency of [0/±30/90]s laminate composite beam with end force along z-axis (EFz). The results in the figure indicate that the resonance of the vibration occurs close to the first mode natural frequency of the composite beam for both analytical and numerical solutions. The numerical results of the amplitude have values in both positive and negative direction because they consist of real and complex solutions. Furthermore, slight vibration resonance is observed in the numerical analysis results close to 3rd mode natural frequency.

Table 5. Numerical analysis results of stacking sequence effect on natural frequency

Layup	Results	Natural frequency (Hz)				
		Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
SCAL	Numerical	112.35	560.62	696.4	687.88	1917
	Analytical	112.491	655.867	704.97	637.061	1973.9
	%Diff	0.12552	14.5224	1.2157	- 7.9772	2.88262
SAPL30	Numerical	91.258	533.96	565.04	1013.7	1569.1
	Analytical	88.268	514.637	553.166	964.21	1548.9
	%Diff	-3.3874	- 3.7548	- 2.1466	- 5.1327	- 1.3042
SAPL45	Numerical	69.664	394.99	434.21	1056.9	1215.8
	Analytical	66.9851	390.55	419.789	1062.3	1175.4
	%Diff	- 3.9992	- 1.137	- 3.4353	0.50833	- 3.4371
SBL	Numerical	91.868	473.25	570.3	1032.1	1585.5
	Analytical	89.2672	520.463	559.428	1024	1566.4
	%Diff	- 2.9135	9.07126	- 1.9434	- 0.791	- 1.2194
ANBL	Numerical	80.001	444.4	493.28	982.79	1371.3
	Analytical	82.4821	480.903	516.906	862.885	1447.4
	%Diff	3.00805	7.59044	4.57073	- 13.896	5.2577
ASBL	Numerical	74.854	448.54	464.65	849.3	1268.2
	Analytical	73.8364	430.495	462.725	904.172	1295.6
	%Diff	- 1.3782	- 4.1917	- 0.4161	6.06873	2.11485
ANCPL	Numerical	100.11	560.31	621.42	680.28	1714.3
	Analytical	100.223	584.34	628.088	637.061	1758.7
	%Diff	0.11285	4.1123	1.06159	- 6.7842	2.52459
ANAPL30	Numerical	95.141	535.26	590.57	1019.1	1638.7
	Analytical	92.7393	540.706	581.187	1030.1	1627.3
	%Diff	- 2.5897	1.0072	- 1.6144	1.06786	- 0.7005
ANAPL45	Numerical	70.756	395.3	441.21	1097.2	1236.8
	Analytical	67.9381	396.106	425.761	1132.7	1192.1
	%Diff	- 4.1477	0.20343	- 3.6285	3.1341	- 3.7497
QIL	Numerical	108.5	509.24	674.14	893.53	1860
	Analytical	108.096	630.243	677.428	865.886	1896.8
	%Diff	- 0.3735	19.1995	0.48535	- 3.1926	1.94011
CS	Numerical	115.96	569.6	716.2	857.37	1971.9
	Analytical	115.307	672.286	722.618	816.492	2023.4
	%Diff	- 0.5661	15.2742	0.8882	- 5.0065	2.54522

As illustrate in Fig. 5 (b), the resonance of $[0/\pm 0/90]_s$ laminate composite beam due to distributed load in z-axis (DLz) is similar to that of the end load (EFz) given in Fig. 5 (a), though the magnitude of the amplitudes are different. Similarly, the resonance

response of [45/-45/45/-45/45] laminate composite beam (Fig. 6) has been conducted with end force in y-axis direction (EFy). The results of the amplitude indicate that the vibration resonance occurs close to the 2nd mode or natural frequency of the composite beam in both the analytical and numerical solutions.

For the sake of comparison, results for different stacking sequences are analyzed and plotted in a single figure (Fig. 7). It is observed that the resonance amplitude of the beams with end force in z direction is higher for [45/-45/45/-45/45] laminate compared to the other laminates. Furthermore, lower resonance values are observed for [0/±30/90]_s laminate with the same fiber volume ratio, load and beam configuration. This indicates that [45/-45/45/-45/45] laminate is easily exposed to failure due to higher resonance near to its 1st mode natural frequencies and it is less stiff compared with the other stacking sequences.

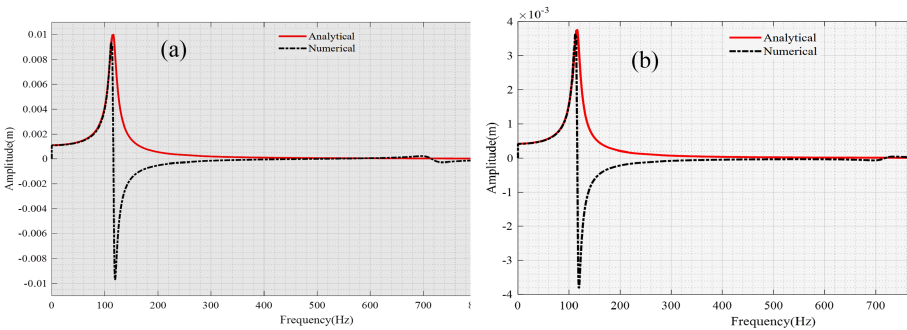


Fig. 5. Resonance of CS ([0/±30/90]_s) composite beam due to (a) end force along z, EFz and (b) distributed load along z, DLz.

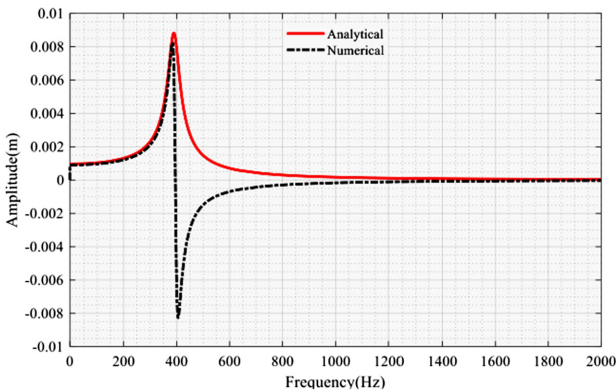


Fig. 6. Resonance of SAPL45 ([45/-45/45/-45/45]) composite beam due to EFy.

This analysis demonstrates that the resonance for specified design of applied cyclic load frequency on the composite beam can be controlled by alternating different stacking sequences, fiber volume ratios and using different fiber materials. The results also show

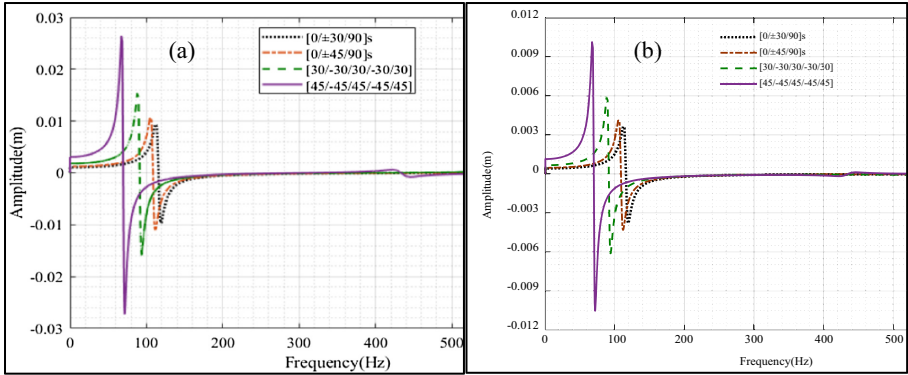


Fig. 7. Resonance of different laminate composite beam due to (a) EFz load and (b) EFy load.

that applied loads on different directions have different resonance frequencies on the same beam configurations because of different modes of natural frequencies. So, fiber reinforced plastic composite beams are better in controlling vibrations compared to isotropic beams because of fiber reinforced plastic beams that have variety of mechanical and physical properties and better opportunity to select the appropriate properties. As a result of resonances, the first mode natural frequencies of beams lead to failure of beams when designed with frequencies close to the natural frequencies due to same direction applied load compared to other modes.

4 Conclusions

This article presents analytical and numerical results of free and forced vibration analysis of FRP composite beam. The results of natural frequencies were obtained using different parameters such as fiber volume ratio, fiber materials, fiber angle and stacking sequence of layers for the first five modes. The results indicate that as the fiber volume ratio is increased, the natural frequencies also increases for all modes. It is also observed that graphite-epoxy composite beam has greater value of natural frequencies compared to aramid-epoxy and glass-epoxy. In general, the natural frequencies for bending modes mainly depend on the equivalent flexural modulus in longitudinal directions (E_{1f}) of composite beams. Comparison of analytical and numerical solutions of natural frequencies have shown best agreement for Mode 1, Mode 3, Mode 4 and Mode 5 for all fiber materials and stacking sequences of laminates. According to numerical results, the first mode of natural frequencies in each direction are more critical for resonance. Furthermore, it has been observed that distributed and end concentrated harmonic loads with unique direction and magnitude have similar impact on the amplitude of the beam, but concentrated load increases the magnitude of amplitude and resonance.

References

1. Pingulkar, P., Suresha, B.: Free vibration analysis of laminated composite plates using finite element method. *Polym. Polym. Compos.* **24**(7), 529–538 (2016)
2. Balci, M., Nalbant, M.O., Kara, E., Gündoğdu, Ö.: Free vibration analysis of a laminated composite beam with various boundary conditions. *J. Autom. Mech. Eng. (IJAME)*. **9**(1), 1734–1746 (2014)
3. Kisa, M.: Free vibration analysis of a cantilever composite beam with multiple cracks. *Compos. Sci. Technol.* **64**(9), 1391–1402 (2004)
4. Zhao, J., Choe, K., Shuai, C., Wang, A., Wang, Q.: Free vibration analysis of laminated composite elliptic cylinders with general boundary conditions. *Compos. B* **158**, 55–66 (2018)
5. Lei, Z.X., Zhang, L.W., Liew, K.M.: Free vibration analysis of laminated FG-CNT reinforced composite rectangular plates using the kp-Ritz method. *Compos. Struct.* **127**, 245–259 (2015)
6. Mohandes, M., Ghasemi, A.: A new approach to reinforce the fiber of nanocomposite reinforced by CNTs to analyze free vibration of hybrid laminated cylindrical shell using beam modal function method. *Eur. J. Mech./A Solids* **73**, 224–234 (2018)
7. Kumar, G.A.Y., Kumar, K.M.S.: Free vibration analysis of smart composite beam. *Mater. Today: Proc.* **4**(2), 2487–2491 (2017)
8. Osman, M.Y., Suleiman, O.M.E.: Free vibration analysis of laminated composite beams using finite element method. *Int. J. Eng. Res. Adv. Technol.* **3**(2), 1–9 (2017)
9. Torabizadeh, M.A., Fereidoon, A.: A numerical and analytical solution for the free vibration of laminated composites using different plate theories. *Mech. Adv. Compos. Struct.* **4**(1), 75–87 (2017)
10. Babuska, P., Weibe, R., Motley, M.R.: A beam finite element for analysis of composite beams with the inclusion of bend-twist coupling. *Compos. Struct.* **189**, 707–717 (2018)
11. Madhu, S., Kumaraswamy, M.: Experimental investigation and free vibration analysis of hybrid laminated composite beam using finite element method. *Int. J. Res. Appl. Sci. Eng. Technol. (IJRASET)*, **5**(VI), 40–53 (2017)
12. Teter, A., Gawryluk, J.: Experimental modal analysis of a rotor with active composite blades. *Compos. Struct.* **153**, 451–467 (2016)
13. Elshafei, M.A.: FE modeling and analysis of isotropic and orthotropic beams using first order shear deformation theory. *Mater. Sci. Appl.* **4**, 77–102 (2013)
14. Ganesa, P., Thirumavalavan, S.: Free vibration behaviour of glass fiber reinforced polymer composite. *Middle-East J. Sci. Res.* **20**(6), 734–737 (2014)
15. Naghipour, M., Taher, F., Zou, G.P.: Evaluation of vibration damping of glass-reinforced polymer-reinforced glulam composite beams. *J. Struct. Eng.* **131**(7), 1044–1050 (2005)
16. Kulkarni, P., Bhattacharjee, A., Nanda, B.K.: Study of damping in composite beams. *Mater. Today: Proc.* **5**(2), 7061–7067 (2018)

17. Tita, V., de Carvalho, J., Lirani, J.: A procedure to estimate the dynamic damped behavior of fiber reinforced composite beams submitted to flexural vibrations. *Mater. Res.* **4**(4), 315–321 (2001)
18. Kaw, A.K.: *Mechanics of composite materials*. CRC Press, Taylor & Francis Group (2006)
19. Duffy, K.P., Lerch, B.A., Wilmoth, N.G. Kray, N., Gemeinhardt, G.: Mechanical and vibration testing of carbon fiber composite with embedded piezoelectric sensors, **8341**, 1–14 (2012)