The Recursive Spectral Bisection Probability Hypothesis Density Filter

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Abstract. Particle filter (PF) is used for multi-target detection and tracking, especially in the context of variable tracking target numbers, high target mobility, and other complex environments, it is difficult to detect, estimate and track targets in these situations. This paper discusses the probability hypothesis density (PHD) filtering which is widely used in the field of multi-target tracking in recent years. The PHD filter algorithm can estimate the number of targets effectively, however, existing algorithms does not make full use of particle information. This paper proposes a target state extraction method based on the recursive spectral bisection (RSB) node clustering algorithm, which focus on eigenvector centrality, algebraic connectivity, and the Fiedler vector from the established field of spectral graph theory (SGT). The method makes full use of the geometric distance relationship and the weight of particles to construct the particle neighborhood graph, then use the algebraic connectivity and Fiedler vector obtained by the eigenvalue decomposition of the Laplace matrix, finally extracts the target state from each class of particle group. Simulation results demonstrate that the new algorithm provides more accurate state estimations for multi-target detection and tracking.

Keywords: Multitarget tracking · Probability hypothesis density · Recursive spectral bisection · Fiedler vector · Algebraic connectivity

1 Introduction

In multi-target tracking problem, due to the need of considering the disappearance of targets, the emergence of new targets, the derivation and combination of existing targets, observation has problems such as miss detection, false alarm and so on, which leads to the uncertainty of target number and target trajectory. The traditional multi-target tracking technology needs to use data association and filtering. When the multi-target distance is very close, traditional methods such as MHT, JPDA and PF are difficult to determine the number of targets. For this tracking problem, it is generally processed based on the random finite set theory, which use the random finite set represents target. Then, the number of targets is the number of elements in the set, state of each target is represented by the set element, observation data are characterized by random finite sets. For a random finite set, estimation can also be performed as for a

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single variable, which can avoid the data association problem encountered in traditional methods. However, for the multi-target environment, it is very difficult to implement the integral of set function in the Bayes recursive formula. R. Mahler proposed an approximate estimation method using the first moment of multi-target posterior probability density function, namely PHD filtering algorithm based on random finite set [3–7, 18–21].

In this article, we review key concepts and basic techniques from the field of SGT theory. Then considering the particles themselves, can learn important information about the particle network topology and its topology-related properties, without relying on an external observer. The two important matrices in this theory are the adjacency matrix and the Laplace matrix. Constructing these two matrices by particles is the key to solve the problem in this paper. There are some important concepts in SGT theory, one of these concepts is eigenvector centrality, which forms the basis of the celebrated Google Pagerank algorithm [8]. Eigenvector centrality allows the identification of central particles but also allows the assignment of a measure of the network-wide influence of each node. Another important concept from SGT is the algebraic connectivity and the associated Fiedler vector [9–11]. The latter is able to identify densely connected particle clusters that only have a few cross links to other clusters, which also reveals the invalid particles in the particle network. As the number of particles increases, the classification of particles will be a big data processing problem [18]. It is also highlight the potential applicability of these techniques in several distributed signal processing tasks such as big signal processing data, distributed estimation, base station or cluster head selection, topology selection, resource allocation, and node subset selection [12, 13].

2 Background

2.1 Multi-target Tracking Model

This section provides a formulation of the PHD filter and its implementation based on particle filter for multi-target tracking. In multi-target tracking problem [16], the number of targets in surveillance region is usually time-varying and unknown; meantime, their states and observations evolve in time, too. Therefore, the states of N_k tracked targets at time k can be naturally represented as a random set $\Gamma_k = \{x_{k,1}, x_{k,2}, \ldots, x_{k,N_k}\}$, where $x_{k,i}$ is the state of an individual target. Similarly, m_k measurements can be given by a random set $Z_k = \{z_{k,1}, z_{k,2}, \ldots, z_{k,M_k}\}$, where $z_{k,j}$ is an observation from a target or due to clutter. Then the goal of multi-target filtering is to estimate the target states and the number N_k at time step k based on the observation collection $Z_{1:k} = \{Z_1, Z_2, \ldots, Z_k\}$.

2.2 PHD Particle Filter

The first-order moment of the multi-target posterior probability density is expressed by PHD [4, 7, 20], expressed as $D_{k|k}$, and the PHD filter also includes prediction and update steps. The prediction is expressed as follows:

$$D_{k|k-1} = \int \phi_{k|k-1}(x, x_{k-1}) D_{k-1|k-1} dx_{k-1} + \gamma_k(x) \tag{1}$$

$$\phi_{k|k-1}(x, x_{k-1}) = P_S(x_{k-1}) f_{k|k-1}(x, x_{k-1}) + b_{k|k-1}(x, x_{k-1}) \tag{2}$$

Where $\gamma_k(x)$ is the intensity function of the new target random set at time k, b_k stands for derived target PHD, P_S represents the target survival probability, $f_{k|k-1}(x|x_{k-1})$ is the state transition equation. The update process is shown as follows:

$$D_{k|k} = \left[v(x) + \sum_{z \in Z_k} \frac{\psi_{k,z}(x)}{\kappa_k(z) + \langle D_{k|k-1}, \psi_{k,z} \rangle} \right] D_{k|k-1} \tag{3}$$

$$\psi_{k,z(x)} = P_D(x) g(z|x) \tag{4}$$

$$\langle D_{k|k-1}, \psi_{k,z} \rangle = \int D_{k|k}(x_t|z_{1:t}) \varphi(x_k) d(x_k) \tag{5}$$

Where $v(x) = 1 - P_D(x)$, $P_D(x)$ represents target detection probability, $g(z|x)$ represents a single-target likelihood function, $\kappa_k(z)$ Is the intensity function of the random set of clutter at time k.

From what has been discussed above, PHD filtering algorithm is a bayesian approximation algorithm, which is realized by the first-order moment of multi-target joint posterior probability density. However, the calculation is difficult to achieve due to the integral operation, particle filter is exploited for the implementation of the PHD [14].

The target detection probability [19] is calculated from the unnormalized weight:

$$\begin{aligned} \overline{M}_b &= P_b[1 - \hat{P}_{k-1}] \sum_{p=1}^{L_{k-1}} \bar{w}_k^{*p} \\ \overline{M}_c &= [1 - P_d]\hat{P}_{k-1} \sum_{p=L_{k-1}+1}^{J_k} \bar{w}_k^{*p} \end{aligned} \tag{6}$$

$$M_c = \frac{\overline{M}_c}{\overline{M}_b + \overline{M}_c} \quad M_b = \frac{\overline{M}_b}{\overline{M}_b + \overline{M}_c} \tag{7}$$

$$\hat{P}_k = \frac{\overline{M}_b + \overline{M}_c}{\overline{M}_b + \overline{M}_c + P_d \hat{P}_{k-1} + [1 - P_b][1 - \hat{P}_{k-1}]} \tag{8}$$

Where P_b represents the target birth probability, $\hat{P}_{k-1}$ represents the target detection probability of time k-1, $L_{k\text{-}1}$ represents the number of surviving and derived target particles, $\bar{w}_k^{*p}$ represents the unnormalized particle weight, P_d represents the target death probability, $L_{k-1} + 1, \ldots, J_k$ represents the new born target particles.

2.3 RSB Clustering Algorithm

In accordance with WSN literature, the vertices of a network graph are referred to as "nodes" and the edges of the graph are referred to as "links" [8–12, 17]. We consider the network graph where the set of nodes is denoted by Q, containing $|Q|$ elements, and the set of links is denoted by E. We denote B_k as the set of neighbors of node k, and $|B_k|$ is referred to as the degree of node k. Two commonly used matrices in SGT are the adjacency matrix and the Laplace matrix. The entries of the adjacency matrix $A = [a_{mn}]_{J\times J}$ are defined as

$$a_{mn} = a_{nm} = \begin{cases} 1 & \text{if n} \in B_m \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

The entries of the laplacian matrix $L = [l_{mn}]_{J\times J}$ of an undirected graph are defined as

$$l_{mn} = l_{nm} = \begin{cases} \sum_{j\in \mathrm{K}} w_{mj}, \text{ if } m = n \\ -w_{mn}, \text{ if } n \in B_m \\ 0, \ otherwise \end{cases} \tag{10}$$

The algebraic connectivity λ_2 and the fiedler vector f contain important information about the connectivity and clustering properties of the network graph. The RSB-based algorithm is a top-down clustering algorithm, starting from the complete network graph and subsequently dividing it into smaller clusters. In this paper, the network graph Q composed of all nodes (particles) in the observation area at a certain time.

3 Multi-target Detection and Tracking Based on RSB-PHD

In this section, We First introduce the traditional PHD particle filter, and propose a method which construct the adjacency matrix and Laplace matrix using the particle neighborhood information. Then estimate the number of target N_k according to the unnormalized particle weight using PHD particle filter, and use the RSB clustering algorithm to divide the particles into N_k classes. Finally, extract the target state from each class of particles.

3.1 Multi-target State Estimation

A single target PHD can be used for multi-target state estimation on the problem of multi-target detection and tracking, among which the number of targets estimated by the unnormalized PHD particle weights. The number of single target PHD will be bigger than the number of estimated target if the observation were drowned by noise at time k. This problem can be solved by calculating the weight sum of each observation, finding the single target PHD of the Nth target from the largest to the smallest according to the weight sum, and then extracting the target state.

Suppose there are N_{k-1} targets exist in the observation area at time k-1, the PHD $D_{k-1|k-1}(x|z_{1:k-1})$ represents by a set of particles and their weights $\{\tilde{x}^i_{k-1}, \bar{w}^i_{k-1}\}_{i=1}^{L_{k-1}+J_k}$.

If there are M_k observations and N_k targets in the observation area at time k, according to the PHD particle filter algorithm, the PHD at time k can be represented by particle set $\left\{\tilde{x}_k^i, \bar{w}_k^i\right\}_{i=1}^{L_{k-1}+J_k}$, where

$$\overline{w}_k^{(i)} = \left[v(\tilde{x}_k^{(i)}) + \sum_{z \in Z_k} \frac{\psi_{k,z}(\tilde{x}_k^{(i)})}{\kappa_k(z) + C_k(z)}\right] \overline{w}_{k|k-1}^{(i)} \tag{11}$$

However, the reality is that we can only get the observation based on all the targets at the same time, and cannot get the observation based on each target at that time alone. Therefore, it is difficult to implement the above algorithm when solving practical problems. To solve this problem, an RSB-PHD algorithm is proposed in this paper.

3.2 Construct the Particle Neighborhood Graph

A particle is a neighbor of any other particle if it lies within a fixed radius γ or is one of the K closest points to it, see Fig. 1. The neighborhood graph is constructed with edges equal to the distance between the particles, the edges of the graph are referred to as "links". The entries of the adjacency matrix $A = [a_{mn}]_{J \times J}$ are defined as

$$a_{mn} = a_{nm} = \begin{cases} 1 & \text{if } \mathrm{d_{mn}} < \gamma \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

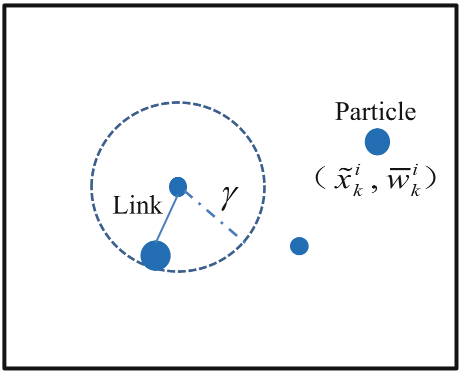

Fig. 1. Construct the particle neighborhood graph within a fixed radius

Where $\mathrm{d_{mn}}$ represents the distance between particle m and particle n.

The entries of the Laplacian matrix $L = [l_{mn}]_{J \times J}$ of an undirected graph $(w_{mn} = w_{nm})$ are defined as

$$l_{mn} = l_{nm} = \begin{cases} \sum_{j \in \mathrm{K}} w_{mj}, & \text{if } m = n \\ -w_{mn}, & \text{if } d_{mn} < \gamma \\ 0, & otherwise \end{cases} \tag{13}$$

Where K here represents all particles in the graph, for an unweighted network graph, L has the same definition where the weight for link (m, n) is set to $w_{mn} = 1$ such that $L = D - A$ where $D = diag(|B_1|, \ldots, |B_k|)$. We denote B_k as the set of neighbors of particle k, and $|B_k|$ is referred to as the degree of particle k.

3.3 RSB-PHD Algorithm

The adjacency matrix and Laplacian matrix are constructed based on the particles at time k, from which we can compute the Fiedler vector and algebraic connectivity. According to the estimated number of targets N_k computed by the PHD filter, we use RSB clustering algorithm to divide particles into N_k classes and extract the target state. The complete RSB-PHD algorithm described as follow:

(1) Initialization: $For\ \ i = 1, \ldots, L_0$, particles $x_0^{(i)}$ obtained by sampling from the prior distribution $q_0(X_0)$, and $w_0^{(i)} = 1/L_0$

(2) Prediction: To survival and derivative targets, $For\ \ i = 1, \ldots, L_{k-1}$, sampling $\tilde{x}_k^{(i)} \sim q_k(\cdot|x_{k-1}^{(i)}, Z_k)$, Calculate the weights of the predicted particles:$\bar{w}_{k|k-1}^{(i)} = \frac{\phi_k(\tilde{x}_k^{(i)}, x_{k-1}^{(i)})}{q_k(\tilde{x}_k^{(i)}|x_{k-1}^{(i)}, Z_k)} w_{k-1}^{(i)}$. To new born targets, $For\, i = L_{k+1} + 1, \ldots, L_{k-1} + J_k$, sampling $\tilde{x}_k^{(i)} \sim p_k(\cdot|Z_k)$, Calculate the weight of the new particle: $\bar{w}_{k|k-1}^{(i)} = \frac{1}{J_k} \frac{\gamma_k(\tilde{x}_k^{(i)})}{p_k(\tilde{x}_k^{(i)}|Z_k)}$

(3) Update: For each observation $z \in Z_k$, compute $C_k(z) = \sum_{j=1}^{L_{k+1}+J_k} \psi_{k,z}(\tilde{x}_k^{(j)}) \bar{w}_{k|k-1}^{(j)}$, $For\, i = 1, \ldots, L_{k-1} + J_k$, Update the weight of the particle $\bar{w}_k^{(i)} = \left[v(\tilde{x}_k^{(i)}) + \sum_{z \in Z_k} \frac{\psi_{k,z}(\tilde{x}_k^{(i)})}{\kappa_k(z) + C_k(z)} \right] \bar{w}_{k|k-1}^{(i)}$.

(4) Estimate the number of targets and detection probability $\hat{P}_k$: in PHD particle filter, the sum of weights can be used to estimate the number of targets $\tilde{N}_k = \sum_{j=1}^{L_{k-1}+J_K} \bar{w}_k^{(i)}$, Round the target number $\hat{N}_k = round(\tilde{N}_k)$.

(5) RSB particle clustering: construct the adjacency matrix and Laplacian matrix according to the particles at time k, divide the particles into $\hat{N}_k$ classes by using RSB node cluster algorithm.

① Let V denote the set of clusters and initialize $V \leftarrow \{g_1\}$ where $g_1 = Q$

② $\forall g_i \in V, i = 1 \ldots |V|$, compute the cluster-level Fiedler vector $f(g_i)$ and algebraic connectivity $\lambda_2(g_i)$, where bridge links between clusters are ignored.

③ Find $g^* \leftarrow \arg\min_{g \in C} [(\lambda_2(g))/|g|]$

④ Partition $g_+^* g_-^*$ into two clusters g_-^* and g_+^*, where g_-^* contains the nodes which have negative entries in $f(g^*)$, and where g_+^* contains the nodes which have positive entries in $f(g^*)$.

⑤ Set $V \leftarrow \{g_-^*, g_+^*\} \cup V \backslash \{g^*\}$

⑥ If $|V| < N_k$, return to step 2.

(6) Target state extraction: we can compute the eigenvector centrality to identify the central particle of each class particle group; or compute the target state using $\left(\tilde{x}_k^{(i)}, \bar{w}_k^{(i)}/\tilde{N}_k\right)_{i=1}^{L_{k-1}+J_K}$

(7) Resampling particles: resampling the particle set $\left(\tilde{x}_k^{(i)}, \bar{w}_k^{(i)}/\tilde{N}_k\right)_{i=1}^{L_{k-1}+J_K}$ to new particle set $\left(x_k^{(i)}, w_k^{(i)}/\tilde{N}_k\right)_{i=1}^{L_k}$, the weight of particles times $\tilde{N}_k$ to get the particles $\left(x_k^{(i)}, w_k^{(i)}\right)_{i=1}^{L_k}$ which will be used at the next prediction step.

4 Simulation

In this paper, we use optimal subpattern assignment (OSPA) [15] metric to evaluate the performance of multi-target tracking. OSPA is still based on a Wasserstein construction, but completely eliminates most of the problems of the OMAT metric. Denote by $d^{(c)}(x,y) = \min(c, d(x,y))$ the distance between $x, y \in W$ cut off at $c > 0$, and by $\prod_k$ the set of permutations on $\{1, 2, \ldots, k\}$ for any $k \in \mathbb{N} = \{1, 2, \ldots\}$. For $1 \le p < \infty$, $c > 0$, and arbitrary finite subsets $X = \{x_1, \ldots, x_m\}$ and $Y = \{y_1, \ldots, y_n\}$ of W, where $m, n \in \mathbb{N}_0 = \{0, 1, 2, \ldots\}$, define

$$\overline{d}_p^{(c)}(X,Y) = (\frac{1}{n}(\min_{\pi \in \prod_n} \sum_{i=1}^{m} d^{(c)}(x_i, y_{\pi(i)})^p + c^p(n-m)))^{1/p} \tag{14}$$

If $m \le n$, and $\overline{d}_p^{(c)}(X,Y) = \overline{d}_p^{(c)}(Y,X)$, if $m > n$, moreover

$$\overline{d}_\infty^{(c)}(X,Y) = \begin{cases} \min\limits_{\pi \in \Pi_n} \max\limits_{1 \le i \le n} d^{(c)}(x_i, y_{\pi(i)}), & \text{if } m = n \\ c, & \text{if } m \ne n \end{cases} \tag{15}$$

in either case set the distance to zero if m = n = 0. We call the function $\overline{d}_p^{(c)}$ the OSPA metric of order p with cut-off c.

Consider multi-target tracking, the target motion model described as:

$$s_k = F s_{k-1} + v_k \tag{16}$$

Where, $F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $s = [x, \dot{x}, y, \dot{y}]$, represents the position and speed of the x direction, and the position and speed of the y direction, v_k is gaussian noise, and $v_k \sim N(0, 0.1)$. The target observation model described as:

$$z_k = Hs_k + w_k \tag{17}$$

Where, $H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $z = [x \quad y]$ represents the location of the target, w_k is gaussian noise, and $w_k \sim N(0, 2.5)$.

Suppose there are three targets in total, and the initial state of each target is X1 = [0, −0.5, 0, −0.5], X2 = [−5, 0.5, 5, 0.5], X3 = [5, 0.4, 0, −0.3], assume that both distance and velocity units are normalized. Observation data in each frame includes 3 targets and 2 clutter points. The simulation duration is 100 s, assuming the existence time of target 1 and 2 is 1–100 s, and the existence time of target 3 is 21–100 s. Considering the OSPA distances for $p = 2$ and $c = 20$.

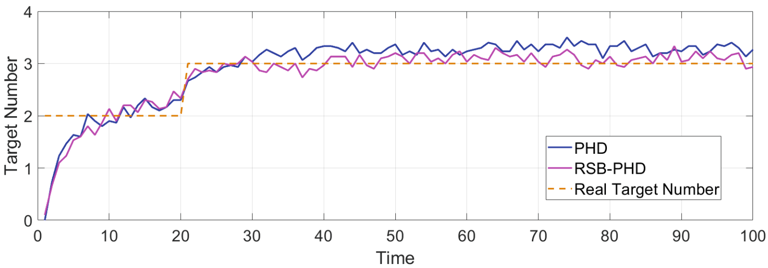

Fig. 2. 100 MC run average target number versus time, contrast of RSB-PHD and PHD algorithm

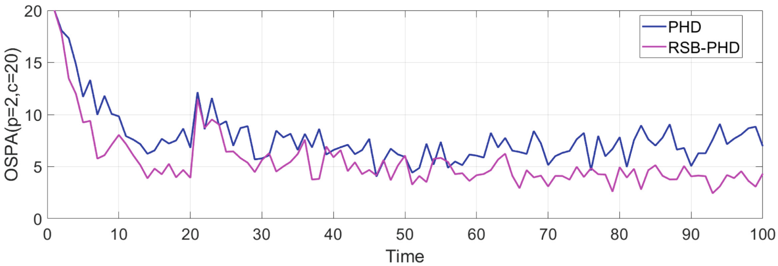

Fig. 3. 100 MC run average OSPA versus time, contrast of RSB-PHD and PHD algorithm

To capture the average performance, we run 100 Monte Carlo (MC) trials for each filter with the same target tracks but independently generated measurements. Figure 2 shows the estimate number of targets by RSB-PHD and PHD filter, these results confirm that the RSB-PHD filter provide more accurate result. Figure 3 shows the average OSPA distance for p = 2 and c = 20, it is obviously that RSB-PHD filter perform better for multi-target tracking. Figure 4 shows the detection probability of each target, since RSB-PHD filter still using unnormalized weight of particles to compute this probability, there is not much difference in performance between the two algorithms.

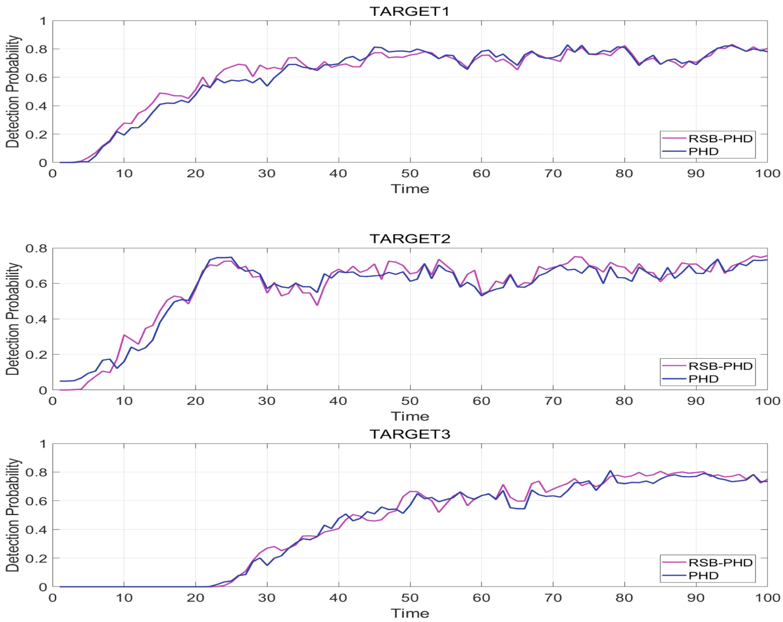

Fig. 4. 100 MC run average detection probability versus time for target 3, contrast of RSB-PHD and PHD algorithm

5 Conclusions

This paper has identified the limitations of PHD particle filter in full use of particle information in multi-target systems. And proposes a target state extraction method based on the recursive spectral bisection (RSB) node clustering algorithm. By setting the distance radius and particle position information, the particle adjacency matrix and Laplace matrix are constructed. Through the RSB algorithm based on the Laplace matrix eigendecomposition, the Fiedler vector is calculated to cluster the particles effectively and eliminate the redundant invalid particles, so that the estimation is more accurate. Simulation results show that the RSB-PHD filtering algorithm requires more computation, and does not affect the target detection probability, but it is more accurate in the estimation of target number. Moreover, smaller OSPA distance error is of great significance in multi-target environment. As the number of particles increases, the classification of particles will be a big data processing problem. In the next part, research on the application of RSB-PHD in TBD will be carried out, and analyze the influence of particle distance radius which can be used to construct the adjacency matrix and the Laplace matrix.

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