# Research on the Rising Phenomenons in the Bit Error Rate Performances of LT-Based UEP Codes

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**Abstract.** The LT-based UEP codes have attracted much attentions in the past decades, because which codes can be used to overcome the issue that some different parts of data have different reliability requirements. But all the existing LT-based UEP codes provide UEP properties at the cost of lower transmission efficiency. In this paper, we focus on the phenomenon that the BER curves of such code would not monotonically decreases as UEP property grows. By quantized the BER of RUEP codes, and compared through two manners to illustrate the BER performances of such code, a useful design principle is given to avoid too lower transmission efficiency and too higher encoding and decoding complexity.

**Keywords:** LT codes · Unequal error protection · Waterfall region · Error floor · BER

## 1 Introduction

Rateless codes is a type of capacity-approaching error-correcting codes in which the code rate are dynamically. The most well known rateless code such as LT codes [1], Raptor codes [2] and Spinal codes [3]. Luby Transform (LT) codes is the first class of rateless codes which designed to transmit data on binary erasure channel (BEC), and the encoding and decoding complexities of LT codes are both equal to $O(k \log k)$.

Rateless codes with the unequal error protection (UEP) property have attracted much attentions in the past decade, and many classes of rateless codes which can provide UEP properties have been invented [4–7]. But nearly all of these codes are LT-based, and all these codes provide UEP properties at the price of lower transmission efficiency. For this reason, we will figure out and analyze how to avoid to lower transmission efficiency by transmitting data using the LT-based UEP codes.


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As the codes proposed by Rahnavard *et al.* is most famous class of LT-based UEP code [5], and the authors have provided a useful quantized expression by using and-or tree technique [8]. Then we focus on this class of code. By compared the performances between BER versus overhead and BER versus UEP weight two manners, a useful design principle have been proposed.

The organize of this paper is as follows. Section 2 briefly review two classes of well known LT-based UEP codes, and the BER versus overhead of these codes are compared. Then the quantization analysis on RUEP codes is given in Sect. 3, and the phenomenon some BER curves are not monotonically is also been illustrated. In Sect. 4, we reveal the reason of the aforementioned phenomenon, and provide a disign principle to design the LT-based UEP codes proposed by [5]. And the concluding remarks of this paper is drawn in Sect. 5.

## 2 Related Work

As the LT-based UEP codes have attracted the attentions of much researchers, many classes coding schemes have been invented. All the existing LT-based UEP codes can be divided into two categories. The one is the LT-based UEP codes with only one encoder in each code. For the other one, there are several sub encoders in each LT-based UEP code.

The most well-known class of LT-based UEP codes is proposed by Rahnavard, *et al.* [5], which belongs to the first category. To distinguish with the other LT-based UEP codes, the codes in [5] are named as Rahnavard UEP (RUEP) codes in this paper. In a given RUEP code, the input bits can be divided into different blocks, by allocated the different selecting probabilities for input bits in different blocks, the input bits can be decoded to provide various BER performances. It is worth to given the definition of UEP weight of RUEP codes. Assuming there are $\alpha_1 k, \alpha_2 k, \ldots, \alpha_i k, \ldots$ input bits in the blocks $b_1, b_2, \ldots, b_i, \ldots$, and the selecting probabilities of the blocks are $q_1, q_2, \ldots, q_i, \ldots$, where $\sum_i \alpha_i = 1$ and $\sum_i q_i = 1$, then the UEP weight of input bits in $i$th block is given by $K_i = \frac{q_i}{\alpha_i}$. And a higher weight would leading to a better BER performance of the input bits in the related block.

The most well known LT-based UEP codes in the second category is Expanding Window Fountain (EWF) codes [6]. Different with the LT-based UEP codes in first category, there are several encoders existed in each EWF code. In a EWF code, the input bits also been divided into blocks $b_1, b_2, \ldots, b_i, \ldots$, and the input bits in block $b_i$ are same as which of the RUEP codes. The UEP wight of EWF codes are not given in [6], instead of the selecting probabilities $\rho_i$ of each window $w_i$. By allocates a sub encoder for each window $w_i$, the input bits in first $i$ windows are selected randomly to generate output bits. As the input bits in a fronter window have been selected in more windows than which of the later blocks, the input bits in fronter blocks can provide better BER performances than which of the later blocks, and the differences BER performances of different blocks can be determined by adjusted the corresponding selecting probabilities.

To illustrate the BER performances of the aforementioned two classed of LT-based UEP codes, we compared the RUEP and EWF codes with two blocks, in

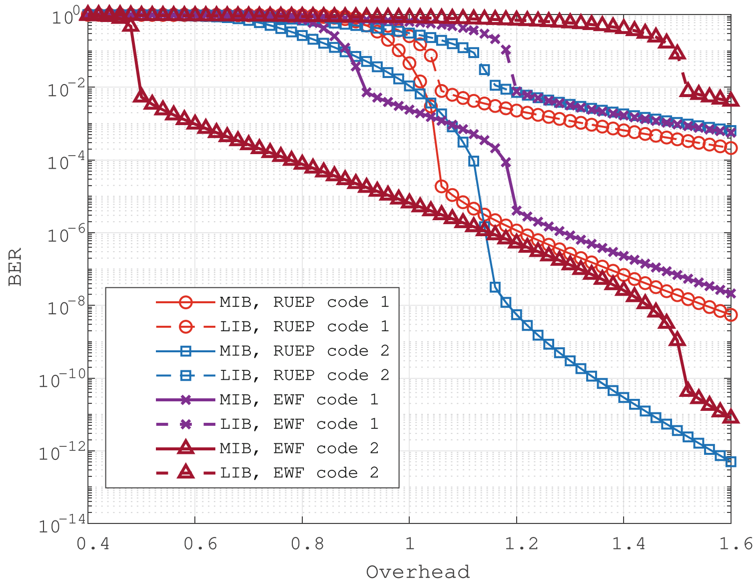


**Fig. 1.** The BER performances of RUEP and EWF codes.

which there are $0.1k$ and $0.9k$ input symbols in the first and second block, respectively. As there are only two blocks in each code, then the block in which the input symbol can provide better BER performances are dubbed More Important Block (MIB), and the other one is named as Less Important Block (LIB).

The coding parameters of RUEP and EWF codes in Fig. 1 are given as follows. The encoders in both RUEP and EWF codes share the same output degree distribution given by [2], which is given following.

$$\begin{aligned} \Omega(x) = \; & 0.007969x^1 + 0.493570x^2 + 0.166220x^3 + 0.072646x^4 \\ & +0.082558x^5 + 0.056058x^8 + 0.037229x^9 + 0.055590x^{19} \\ & +0.025023x^{64} + 0.003137x^{66}. \end{aligned} \tag{1}$$

For RUEP codes, the UEP weights are $K_{MIB,1} = 2$ and $K_{MIB,2} = 3$, respectively. And the selecting probabilities of MIBs of EWF codes are $\rho_{MIB,1} = 0.1$ and $\rho_{MIB,2} = 0.2$. By observed on Fig. 1, for both the RUEP and EWF codes, the MIBs can provide better BER performances than their pair LIBs. And one can find that for RUEP codes, the UEP property (the gap between the BER curves) of a pair MIB and LIB blocks would as larger as a higher UEP weight adopted. For EWF codes, the UEP property would increases with selecting probability of MIB block growth.

As for EWF codes, the Unequal Recovery Time (URT) properties are too larger, which means when most of the input bits are have already been decoded, the input bits in LIB are barely to be recovered, which property would dramatically impacted on the timeliness performances of communication systems, for which reason the RUEP codes have attracted much more attentions than EWF codes. And for this reason, in the follows of this paper, we would focus on the RUEP codes.

## 3 The Rising Phenomenon on BER of RUEP Codes

In this section, the BER performances of RUEP codes is concerned. By observed in Fig. 1, one can find that the BER performances of both RUEP and EWF codes, the BER of each block would monotonically decreasing as overhead increases, and the higher UEP weight or selecting probability would leading a better (lower) BER performance.

Then we focus on the BER performances of RUEP codes versus the Overhead. Firstly, we will given the function of BER, and the overhead $\gamma$ is considered as a variate. As shown in [5], the asymptotic iterative expression of RUEP codes is provided by using the and or tree analysis tool, which is expressed as

$$y_{l,i} = \lambda_i\Big(1 - \sum_d \omega_d\big(\sum_i q_i(1 - y_{l-1,i})\big)^{d-1}\Big), \tag{2}$$

where $\lambda(x) = \frac{\Lambda'(x)}{\Lambda'(1)}$ and $\omega(x) = \frac{\Omega'(x)}{\Omega'(1)}$, and $\Lambda(x)$ is the input degree distribution of input bits. It is worth to noting that when $l$ is enough large, $y_{l,i}$ would tends to the BER of the given RUEP code.

Let $P_{released}$ denote the probability that an arbitrary selected output bits can be decoded to recover a input bits, then which probability can be expressed by

$$P_{released} = \sum_d \omega_d\big(\sum_i q_i(1 - y_{l-1,i})\big)^{d-1}, \tag{3}$$

and the BER probability is given by

$$P_{BER} = \lambda_i(1 - P_{released}). \tag{4}$$

As shown in [5], when $k \to \infty$, $\lambda_i(x)$ can be computed by

$$\lambda_i(x) = \frac{\Lambda_i'(x)}{\Lambda_i'(1)} = \frac{(\bar{d}_i\gamma)e^{\bar{d}_i\gamma(x-1)}}{(\bar{d}_i\gamma)e^{\bar{d}_i\gamma(x-1)}\mid_{x=1}} = e^{\bar{d}_i\gamma(x-1)}, \tag{5}$$

where $\bar{d}_i$ is the average of input bits in $b_i$, and which is

$$\bar{d}_i = \frac{\gamma k q_i \Omega'(1)}{\alpha_i k} = \frac{\gamma q_i \Omega'(1)}{\alpha_i}. \tag{6}$$

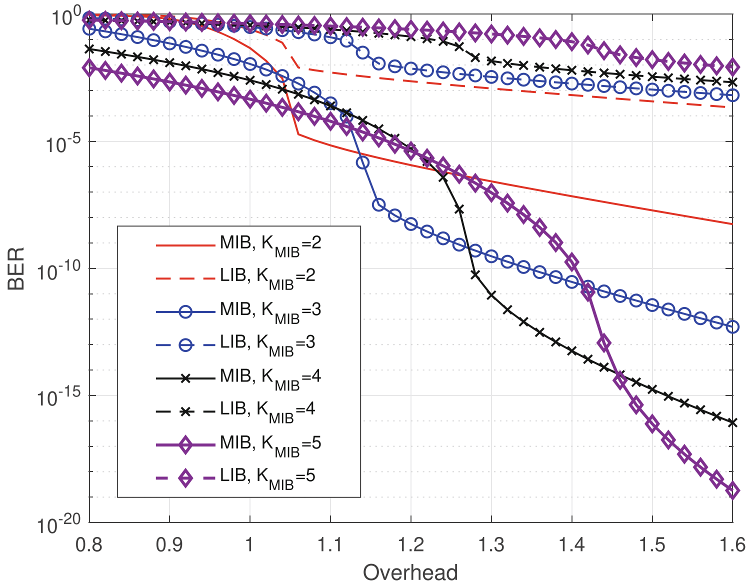


**Fig. 2.** The BER performances of RUEP codes versus overheads.

Then Eq. (4) can be rewritten as

$$P_{BER} = \exp\left\{\frac{\gamma^2 q_i \Omega'(1)}{\alpha_i}(-P_{released})\right\}, \tag{7}$$

as there is only one variate $\gamma$ in the right part of Eq. (7), then the BER is a function of $\gamma$, and BER is monotonicity decreases as $\gamma$ increases. By observed on Fig. 2, it is can be found that the BER performances of each blocks is goes better with overhead grows, whatever the UEP weight is adopted.

By given a fixed overhead $\gamma$, and let the $q_i$ is a variate, then we can arbitrary say that BER is also monotonicity decreases as $q_i$ increases. But by consider on the existed LT-based UEP codes, which is a paradox, actually in all the existing LT-based UEP codes, when UEP weight grows, the BER performances are not monotonicity decreases. By focus on RUEP codes, which phenomenon can be illustrated by observed on Fig. 3.

In Fig. 3, the overheads are set as constants, and the UEP weights are considered as variates. Then it is easily to see that the BER curves are not monotonously, and in some region, the BER performance would turned to goes worse as UEP weight $K_{MIB}$ grows.

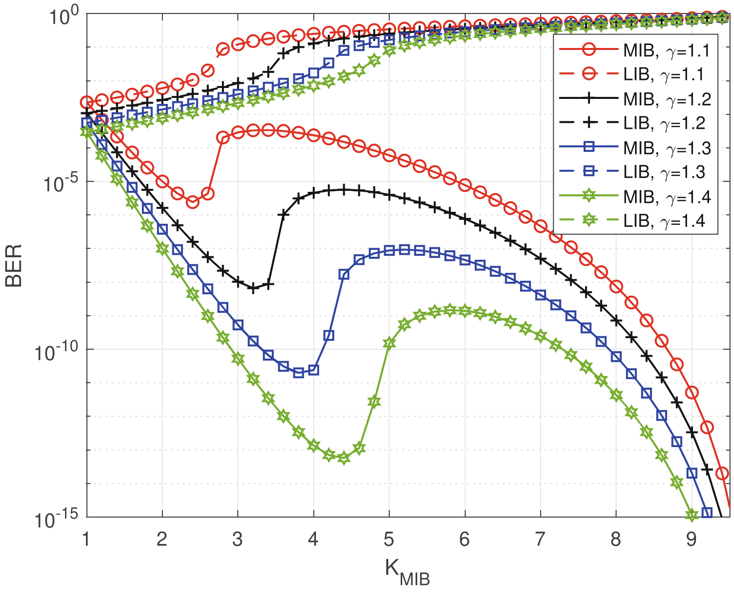


**Fig. 3.** The BER performances of RUEP codes versus UEP weight.

## 4 Discussion on Rising Phenomenon on BER Performances of RUEP Codes

To find out the reason about the phenomenon in which the BER performances rises as $K_{MIB}$ increases, we given the compare in Fig. 4 and Fig. 5. By setting three pair of points $a1$, $a2$ and $a3$ both in these two figure, one can easily find that the points which are figure outed are strictly same. By observed at the points of $a1$, as which points in Fig. 4 belongs to the error floor region, which points belongs to the region in which the BER of MIB are monotonicity decreases, and for the points of $a3$, as the point are belongs to the waterfall region, then its comparison points in Fig. 5 belongs to the region in which the BER of MIB are rising.

The comparison results in Fig. 4 and Fig. 5 illustrated that the beginning points of waterfall and error floor regions would as larger as UEP weight $K_{MIB}$ bigger, and the higher UEP properties would leading to lower transmission efficiencies.

As the higher UEP property would leading to lower transmission efficiency, and the larger overhead will leading to higher encoding and decoding complexities, which means for practice, the RUEP codes should be carefully designed. As in waterfall region the BER performances went better much quickly than which of the error floor region, an appropriated RUEP code should satisfies that the required BER performances just emerged near by the point between the waterfall and error floor regions.

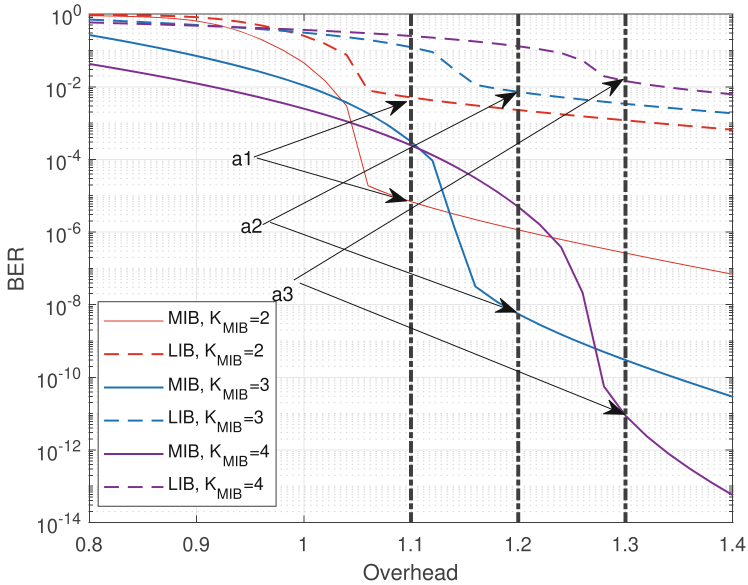


**Fig. 4.** The compared BER performances of RUEP codes versus overhead.

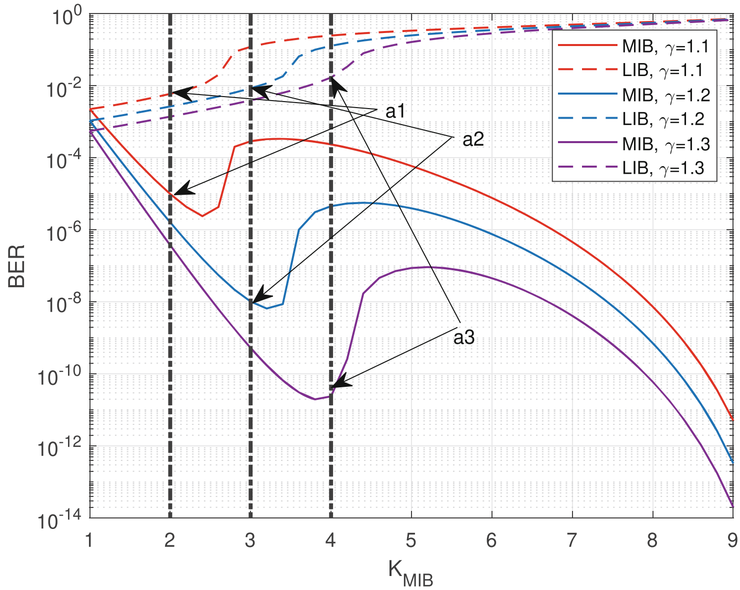


**Fig. 5.** The compared BER performances of RUEP codes versus UEP weight.

## 5 Conclusion

In this paper, we propose a design principle for RUEP code to avoid low transmission efficiency and higher computing complexity. By focus on the phenomenon that the BER curves of such code would not monotonically decreases as UEP property grows, and provide the BER function of overhead $\gamma$ and UEP weight $K_{MIB}$, the reason of this phenomenon have been revealed. Furthermore, based on the revealed reason, the design principle is given as a result.

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