Downlink Power Allocation Strategy in Multi-antenna Ultra-dense Networks Based on Non-cooperative Game

Donglai Zhao[1(✉)], Gang Wang[1], Haoyang Liu[1], and Shaobo Jia[2]

[1] Communication Research Center, Harbin Institute of Technology, Harbin, China
gwang51@hit.edu.cn

[2] School of Information Engineering, Zhengzhou University, Zhengzhou, China
ieshaobojia@zzu.edu.cn

Abstract. This paper investigates the downlink power allocation strategy for a multi-antenna spectrum sharing ultra-dense small cell network in order to suppress the inter-cell interference and improve the system spectral efficiency (SE). The non-cooperative game is adopted to transform the system SE maximization problem into several convex subproblems which maximize the utility function of each user. By designing a dynamic pricing, each Nash equilibrium (NE) of the game is a stationary point of the original optimization problem. In addition, an interference power constraint is applied to guarantee the quality-of-service (QoS) of the key user. Under the game theory framework, an iterative dynamic pricing power allocation (DPPA) algorithm is designed, which is proved to be convergent to the NE of the game model. Furthermore, in order to reduce the signaling overhead and improve the resource utilization, an approximate dynamic pricing power allocation (ADPPA) algorithm is also proposed. Simulation results show that the proposed DPPA algorithm achieves a better performance than benchmark methods and the proposed ADPPA algorithm effectively reduces the signaling overhead with a little performance loss.

Keywords: Power allocation · Ultra-dense network · Non-cooperative game · Spectral efficiency (SE) · Quality-of-service (QoS)

1 Introduction

In the era of data explosion, data traffic in mobile wireless networks has grown explosively. Ultra-dense networks (UDNs) technology is emerging as one of the most promising solutions to meet the requirements of users' wireless traffic volume [1]. UDNs can efficiently improve the network coverage and boost the network capacity by densely deploying the various small cell base stations (SBSs)

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in the service area of macro cells. Actually, the SBS is a general term, it includes micro, pico, femto, relay and remote radio head (RRH).

Due to the intensive deployment of various types of SBSs, the inter-cell interference in the network is quite serious. Moreover, the interference scenarios are fairly complex with the flexible deployment of small cell base stations. Therefore, interference management for ultra-dense small cell networks becomes an important issue to be resolved. The authors in [2] summarized the advanced interference management techniques for UDNs, including coordinated multi-point, muliple access methods, parallel interference cancellation and power allocation. In [3], the authors designed a multi-domain interference management scheme which combined power control, multiple access scheduling and interference alignment (IA) technology in ultra-dense small-cell networks. Many existing works attempted to manage interference in the power domain. In [4], the authors proposed a contract-based traffic offloading and resource allocation mechanism for the software-defined heterogeneous UDN. Since the mechanism needs a centralized controller to manage the entire network globally, the information exchanged is very huge. To reduce the signaling overhead and computational complexity of the central management mechanism, the authors in [5] presented a local information-based iterative distributed power allocation algorithm by constructing the interfering domains. In [6], a novel energy efficient dynamic power allocation strategy applying a new receiver puncturing technique was proposed for the fifth generation systems. In [7], the authors proposed a cluster-based energy-effcient resource allocation scheme to mitigate the interference and boost energy effciency.

It has been investigated that efficient power allocation strategy can greatly mitigate interference and improve system throughput. As we know, power optimization problems are usually non-convex, so it is a challenging task to obtain the global optimal solution. Fortunately, we can use the game theory to transform non-convex problems into various convex problems. In addition, compared with the centralized power allocation scheme, the distributed algorithms based on game theory require less channel state information (CSI). The authors in [8] investigated a non-cooperative game with penalty factor for UDN, and proved the Nash equilibrium (NE) of the game is unique and Pareto-efficient. Under the game framework, a power allocation algorithm with virtual cell local information was poposed. In [9], a stackelberg game was established to study the joint utility maximum problem subject to a interference power constraint, then the power allocation strategy with non-uniform price was presented for densely deployed scenario. In [10], a distributed interference-aware power control scheme was designed to mitigate interference. In order to reduce the huge interference-related information exchange between the players, the authors formulated a robust mean field game. However, there are still some shortcomings in the above-mentioned research work. First of all, it remains unclear about the gap between the NE points of aforementioned game models and the globally optimal solutions of original optimization problems. Secondly, no interference power constraint has been adopted to guarantee the QoS of key users. Besides, the convergence of proposed iterative algorithms are not proved theoretically. Last but not least, the system models are too simple, it is assumed that the base stations are equipped with a single antenna.

In this paper, we investigate the downlink power allocation problem for a spectrum sharing multi-antenna UDN. The main contributions of our work are: (i) We use the non-cooperative game theory to design a iterative dynamic pricing power allocation algorithm, which converges to the NE of the game as well as the stationary point of the original SE maximization problem. (ii) An interference power constraint is applied to guarantee the QoS of the key user. (iii) An approximate power allocation algorithm is also proposed, which aims to reduce the signaling overhead and improve the resource utilization.

The remainder of this paper is organized as follows. The system model is introduced in Sect. 2. Section 3 formulates the non-cooperative game. In Sect. 4, the power allocation algorithms are proposed. Numerical results and discussions are presented in Sect. 5. Finally, Sect. 6 concludes this paper.

2 System Model

Consider a spectrum sharing ultra-dense small cell network, where the SBSs are equipped with M antennas. The distribution of BSs with density λ is modeled as a homogeneous Poisson point processes (PPPs) $\Phi = \{1, 2, \cdots, N\}$, where N is the number of SBS in region $\mathcal{S}$. The users with single antennas are also modeled by an independent homogeneous PPP of density λ^{u}. Each user is associated with the closest SBS. We assume $\lambda^{\mathrm{u}} \gg \lambda$, so all SBS in the system are active. Each SBS serves one user at each time slot, as the time division multiple access (TDMA) is adopted. For the sake of brevity, we use user n to represent the user connected to SBS n. Although there is no intra-cell interference, the severe inter-cell interference limits the system capacity. The interference links are described in Fig. 1.

The power allocation strategy is divided into two levels. At the system level, the total transmit power p_n of SBS n is allocated under the limitation of the maximum transmit power $p_n^{\max}$; At the cell level, the SBS n allocates the transmit power for each antenna. In this paper, we focus on the system level. As for the cell level, the maximum ratio transmission is adopted.

The channel power gain between SBS i and user j is $h_{i,j}$, modeled as $d_{i,j}^{-\alpha}\beta_{i,j}$, where $d_{i,j}$ is the distance between the transmitter and the receiver, α is the path loss exponent, and $\beta_{i,j}$ is independent random fading coefficient. It should be noted that $\beta_{i,j} \sim \mathrm{Gamma}(1, 1)$ when $i \neq j$, otherwise, $\beta_{i,j} \sim \mathrm{Gamma}(M, 1)$. Under the above framework, the signal-to-interference-plus-noise ratio (SINR) of user of small cell n can be denoted by

$$\mathrm{SINR}_n = \frac{p_n h_{n,n}}{\sum\limits_{i \neq n} p_i h_{i,n} + w_n} \tag{1}$$

where w_n denotes the additive white Gaussian noise power. Then, the SE of small cell n is given by

$$R_n = \log_2(1 + \mathrm{SINR_n}) \tag{2}$$

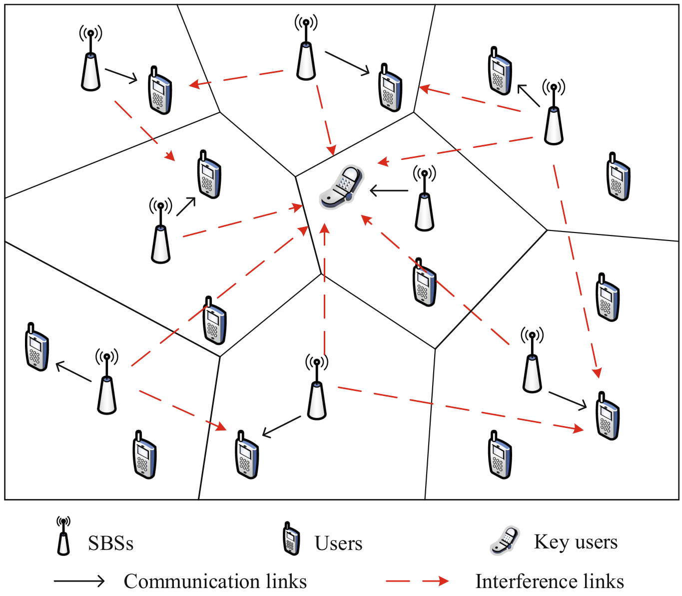

Fig. 1. System model. For clarity, only part of the interference links are plotted.

Without loss of generality, we assume that the key user is user x_0. The interference power constraint Q is used to limit the aggregate interference received by the key user, which can be written as

$$\sum_{i \neq x_0} p_i h_{i,x_0} \leq Q \tag{3}$$

Then, the optimization problem of maximizing system SE can be expressed as

$$\text{Problem 1}: \max_{\mathbf{p}} R_{\text{sum}}(\mathbf{p}) = \sum_{n=1}^{N} R_n(\mathbf{p}) \tag{4a}$$

$$s.t. \quad 0 \leq p_n \leq p_n^{\max} \tag{4b}$$

$$\sum_{i \neq x_0} p_i h_{i,x_0} \leq Q \tag{4c}$$

where $\mathbf{p} = (p_1, p_2, \cdots, p_N)$ is the SBS transmit power vector. Obviously, problem 1 is non-convex, so it is a challenging task to obtain its global optimal solution. The model of non-cooperative game will be adopted to decouple the problem into several convex subproblems in next section.

3 Problem Formulation

In this section, the non-cooperative game model based on dynamic pricing is formulated.

3.1 Game Model

In order to obtain the maximum transmission rate, each SBS will continuously increase its own transmit power. This process can be regarded as a non-cooperative game. However, as each SBS constantly pursues its own utility maximization, it will also causes severe interference to other users. In order to limit this interference, it is necessary to introduce a pricing mechanism in the non-cooperative game model.

We model each SBS as a self-interested game player, the non-cooperative game can be denoted as $G = [C, \{\rho_n | n \in \mathrm{C}\}, \{U_n | n \in C\}]$. $C = \{1, 2, \cdots, N\}$ is the set of game players. $\rho_n = \{p_n | 0 < p_n < p_n^{\max},\ p_n h_{n,x_0} + \sum_{i \neq n} p_i h_{i,x_0} \leq Q\}$ is the set of available power strategy for player n. U_n is the utility function of player n, it is denoted by

$$U_n(p_n|\mathbf{p}_{-n}) = R_n(p_n|\mathbf{p}_{-n}) + \theta_n p_n \tag{5}$$

where $\mathbf{p}_{-n} = (p_1, \cdots, p_{n-1}, p_{n+1}, \cdots, p_N)$ is the transmit power of all the players except player n. θ_n can be regarded as a kind of dynamic price for the transmit power, it is given by

$$\theta_n = \sum_{i \neq n} \frac{\partial}{\partial p_n} R_i(\mathbf{p}) \tag{6}$$

Obviously, U_n is a convex function of p_n, so we can obtain a convex approximation of the system SE $R_{\text{sum}}(\mathbf{p})$ in (4a), it can be written as

$$\tilde{R}_{\text{sum}}(\mathbf{p}) \triangleq \sum_{n=1}^{N} U_n(p_n|\mathbf{p}_{-n}) \tag{7}$$

Under the above non-cooperative game model, problem 1 can be decomposed into N convex sub-problems as follows:

$$\text{Problem 2}: \max_{p_n} U_n(p_n|\mathbf{p}_{-n}),\ \forall n \in C \tag{8a}$$

$$s.t. \quad 0 \leq p_n \leq p_n^{\max} \tag{8b}$$

$$p_n h_{n,x_0} + \sum_{i \neq n} p_i h_{i,x_0} \leq Q \tag{8c}$$

So the system SE maximization problem can be equivalent to maximizing the SE of each small cell. We can use the convex optimization method to find the globally optimal solution of problem 2, which can be expressed as

$$b_n = \arg \max_{p_n \in \rho_n} U_n(p_n|\mathbf{p}_{-n}) \tag{9}$$

3.2 Analysis of Nash Equilibrium

Definition 1. *The transmit power vector* $\mathbf{p}^*$ *is the NE point of game* G *if and only if the transmit power of each player is the best strategy corresponding to the transmit power of others, i.e.,* $p_n^* = b_n, \forall n \in C$.

Once the NE is reached, no player will attempt to change its strategy, due to

$$U_n(p_n^*|\mathbf{p}_{-n}^*) \geq U_n(p_n|\mathbf{p}_{-n}^*), \ \ \forall n \in C \tag{10}$$

Each NE of game G (i.e., problem 2) is a stationary point of the non-convex problem 1 and vice versa [11]. As we know, the globally and locally optimal solutions must be stationary points. Therefore, by finding the NE of game G, we may obtain the locally optimal solutions of the original problem, even the globally optimal solutions if we are lucky enough.

4 Distributed Iterative Power Allocation Algorithms

In this section, the dynamic pricing power allocation (DPPA) algorithm is proposed, and the convergence analysis shows that it converges to the NE of game G. In order to reduce the signaling overhead, the approximate dynamic pricing power allocation (ADPPA) algorithm is also presented.

4.1 The Dynamic Pricing Power Allocation Algorithm

Since the individual utility function U_n and the constraint set ρ_n are convex, problem 2 is a typical convex optimization problem. The Lagrange function of problem 2 is given by

$$\begin{aligned} L_n(p_n, \mu_1, \mu_2, \mu_3) = & -R_n(p_n|\mathbf{p}_{-n}) - \theta_n p_n + \mu_1(p_n - p_n^{\max}) \\ & -\mu_2 + \mu_3(p_n h_{n,x_0} + \sum_{i \neq n} p_i h_{i,x_0} - Q) \end{aligned} \tag{11}$$

Then the Lagrangian dual function is denoted by

$$g_n(\mu_1, \mu_2, \mu_3) = \inf_{p_n} L_n(p_n, \mu_1, \mu_2, \mu_3) \tag{12}$$

It can be seen that the strong dual condition is satisfied, the duality gap is zero. By solving the Karush-Kuhn-Tucher (KKT) conditions, we obtain the optimal solution of the original problem, that is given by

$$b_n = \max\{0, \min\{p_n^Q, p_n^{\text{op}}, p_n^{\max}\}\} \tag{13}$$

where

$$p_n^Q = \frac{1}{h_{n,x_0}}(Q - \sum_{i \neq n} p_i h_{i,x_0}) \tag{14}$$

$$p_n^{\text{op}} = \frac{1}{\sum\limits_{i \neq n} \frac{h_{i,i} p_i h_{n,i}}{\sigma_i(\sigma_i + h_{i,i} p_i)}} - \frac{\sigma_n}{h_{n,n}} \tag{15}$$

with $\sigma_j, \forall j \in C$ representing the interference-plus-noise experienced by user j, it is denoted by

$$\sigma_j = \sum_{i \neq j} p_i h_{i,j} + p^{\text{m}} h_j^{\text{ms}} + w_j, \ \ \forall j \in C \tag{16}$$

The DPPA algorithm is designed based on above analytical solutions, as shown in Algorithm 1. In each iteration, given the current power vector $\mathbf{p}^t$, each SBS computes its optimal transmit power by (13). Then update the transmit power vector and step-size. The NE of game G is obtained until the termination condition is satisfied, i.e., $\left|\tilde{R}_{\text{sum}}(\mathbf{p}^{t+1}) - \tilde{R}_{\text{sum}}(\mathbf{p}^t)\right| < \delta$, where δ is an arbitrary small positive constant.

Algorithm 1. Dynamic pricing power allocation algorithm

Input: The transmit power vector $\mathbf{p}$, the step-size r, constant δ and ε.
Output: The optimal transmit power vector $\mathbf{p}^*$.

1: **Initialization:** Set the initial value $\mathbf{p}^0$, $r^0 \in (0,1]$, δ and $\varepsilon \in (0,1)$.
2: **while** $\left|\tilde{R}_{\text{sum}}(\mathbf{p}^{t+1}) - \tilde{R}_{\text{sum}}(\mathbf{p}^t)\right| \geq \delta$. **do**
3: As termination condition is not satisfied, set $t \leftarrow t+1$.
4: Given the transmit power vector $\mathbf{p}^t$, each SBS independently computes its optimal transmit power $b_n^t, n \in C$ by (13).
5: Update the transmit power vector by $\mathbf{p}^{t+1} = \mathbf{p}^t + r^t(\mathbf{b}^t - \mathbf{p}^t)$, where $\mathbf{b}^t = (b_0^t, b_1^t, \cdots, b_N^t)$.
6: Update the step-size by $r^{t+1} = r^t(1 - \varepsilon r^t)$.
7: Compute the value of $\tilde{R}_{\text{sum}}(\mathbf{p}^{t+1})$.
8: **end while**
9: **return** $\mathbf{p}^{t+1}$

4.2 Convergence Analysis

Theorem 1. *The transmit power vector $\mathbf{p}^t$ converges to the NE of game G and $\tilde{R}_{\text{sum}}(\mathbf{p}^t)$ converges to a finite value via Algorithm 1.*

Proof. The approximate system SE function $\tilde{R}_{\text{sum}}(\mathbf{p})$ is Lipschitz continuous on feasible domain, so there exists a positive constant η such that

$$\left\|\tilde{R}_{\text{sum}}(\mathbf{p}^{t+1}) - \tilde{R}_{\text{sum}}(\mathbf{p}^t)\right\| \leq \eta \left\|r^t(\mathbf{b}^t - \mathbf{p}^t)\right\| \tag{17}$$

Based on Descent Lemma [12], we obtain the following inequality

$$\tilde{R}_{\text{sum}}(\mathbf{p}^{t+1}) \leq \tilde{R}_{\text{sum}}(\mathbf{p}^t) + r^t \tilde{R}_{\text{sum}}(\mathbf{p}^t)^{\mathrm{T}}(\mathbf{b}^t - \mathbf{p}^t) + \frac{\eta (r^t)^2}{2}\left\|\mathbf{b}^t - \mathbf{p}^t\right\|^2 \tag{18}$$

$\mathbf{b}^t - \mathbf{p}^t$ is a descent direction of function $\tilde{R}_{\text{sum}}(\mathbf{p})$ at $\mathbf{p}^t$ such that

$$\tilde{R}_{\text{sum}}(\mathbf{p}^t)^{\mathrm{T}}(\mathbf{b}^t - \mathbf{p}^t) \leq -\tau\left\|\mathbf{b}^t - \mathbf{p}^t\right\|^2 \tag{19}$$

where τ is a positive constant. Resorting to (18) and (19), we obtain

$$\tilde{R}_{\text{sum}}(\mathbf{p}^{t+1}) \leq \tilde{R}_{\text{sum}}(\mathbf{p}^t) - r^t(\tau - \frac{\eta r^t}{2})\left\|\mathbf{b}^t - \mathbf{p}^t\right\|^2 \tag{20}$$

Since $r^t \to 0$, we can find a positive constant ψ when t is sufficiently large, i.e., $t \geq \bar{t}$, such that

$$\tilde{R}_{\text{sum}}(\mathbf{p}^{t+1}) \leq \tilde{R}_{\text{sum}}(\mathbf{p}^t) - r^t \psi \left\|\mathbf{b}^t - \mathbf{p}^t\right\|^2 \tag{21}$$

Since $\tilde{R}_{\text{sum}}(\mathbf{p})$ is coercive, using the Robbins-Siegmund Theorem [13], we derive that $\tilde{R}_{\text{sum}}(\mathbf{p}^t)$ converges to a finite value and the following inequation is hold.

$$\sum_{t \geq \bar{t}} r^t \left\|\mathbf{b}^t - \mathbf{p}^t\right\|^2 < \infty \tag{22}$$

As $\sum_{t \geq \bar{t}} r^t \to \infty$, then $\lim_{t\to\infty} \|\mathbf{b}^t - \mathbf{p}^t\| = 0$, i.e., $\lim_{t\to\infty} \mathbf{b}^t = \mathbf{p}^t$. The proof is completed.

4.3 An Approximate Form

Using the DPPA algorithm, each SBS needs the transmit power of others and the channel power gains of signal and interference links to compute its optimal power strategy. The number of signaling required for each SBS is $\frac{1}{2}N^2 + \frac{3}{2}N - 1$ in each iteration, so the signaling overhead is enormous when the number of small cells increases.

In order to reduce the signaling overhead, the ADPPA algorithm is proposed. The iterative process of ADPPA is the same as the DPPA algorithm, but the calculation method for solving b_n^t in step 4 is slightly different. $h_{ij}(i \neq j)$ in (14)–(16) is replaced with $\bar{h}_{ij}(i \neq j)$, which is approximate channel power gain model, where the influence of Rayleigh fading is ignored and the distance between SBS i and user j is replaced with the distance between SBSs.

As the distance between SBSs is constant, it is unnecessary to repeatedly transmit $\bar{h}_{ij}(i \neq j)$ during the iterative process. Using the proposed ADDPA algorithm, each SBS only requires $2N - 1$ signaling including: (i) The transmit power of other SBSs; (ii) The channel power gain between each SBS and its own user.

5 Numerical Results

In this section, the performance of proposed algorithms are compared with state-of-art algorithms. For simplicity, it is assumed that $w_n = w, \forall n \in C$, $p_n^{\max} = p^{\max}$, $\forall n \in C$, and the key user is located at the origin of the Euclidean plane. The main simulation parameters are shown in Table 1.

The performance of proposed algorithms are compared with the penalty factor based power allocation (PFPA) algorithm in [8] and the non-uniform pricing based power allocation (NPPA) algorithm in [9]. We set the average SE of each small cell as the indicator.

The results in Fig. 2 are obtained by the Monte Carlo method, and the number of simulations is 10,000. We can see that the average SE of each small cell achieved by the proposed DPPA algorithm is the highest, and the performance

Table 1. Parameters setting

Parameter	**Value**
Pass loss exponent α	3.7
Thermal noise density	−174 dBm/Hz
Bandwidth	10 MHz
Maximum transmit power of each SBS $p^{\max}$	150 mW
Region $\mathcal{S}$	1 Km^2
Interference power constraint Q	−35 dBm

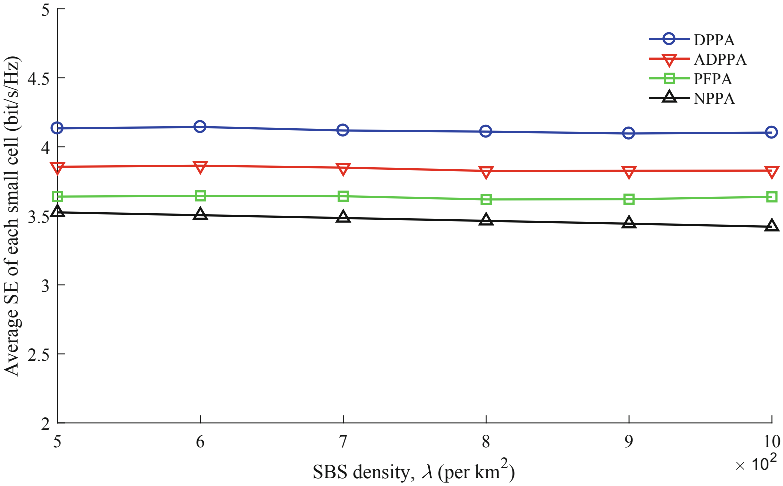

Fig. 2. Average SE of each small cell versus the SBS density. ($M = 4$.)

of the proposed ADDPA algorithm is better than the benchmark. Moreover, the performance of algorithms hardly change with the density of SBSs, which is counter-intuitive. The dense deployment of SBSs makes the average coverage area of each SBS smaller, and the distance between the SBS and the user is closer. Therefore, the negative effect caused by the severe interference is cancelled by the stronger signal.

Figure 3 shows the convergence performance of different algorithms. Although the convergence speed of the DPPA algorithm is slightly slower than the PFPA algorithm due to dynamic pricing, the average SE of each small cell obtained at the convergence time is higher. The ADPPA algorithm has similar convergence performance with the DPPA algorithm. Since the solving process of the NPPA algorithm does not require iteration, it is a horizontal line in the figure.

Figure 4 shows the effects of the number of antennas M on the performance of power allocation algorithms. As it shows, the average SE of each small cell achieved by algorithms is improved with the increase of M. The reason is obvious,

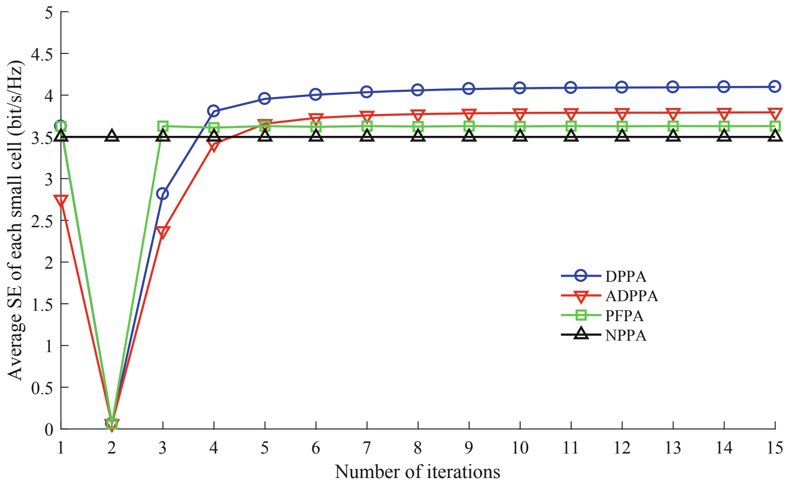

Fig. 3. Convergence performance of the proposed algorithms with benchmark algorithms. ($\lambda = 5 \times 10^2$ per km^2, $M = 4$.)

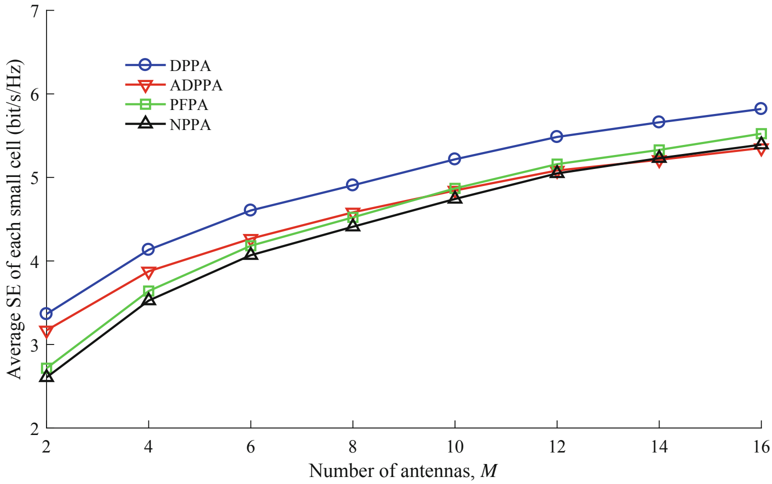

Fig. 4. Average SE of each small cell versus the number of antennas. ($\lambda = 5 \times 10^2$ per km^2.)

because a larger antenna number increases the diversity gain. In addition, it is shown that the proposed DPPA algorithm has the best performance, and the ADPPA algorithm outperforms the benchmark algorithms when M is less than 10, which means the proposed approximate algorithm is more suitable for the case with a small number of SBS antennas.

Figure 5 shows the average signaling overhead required for each SBS per iteration of the proposed algorithms. It is assumed that transmitting a floating

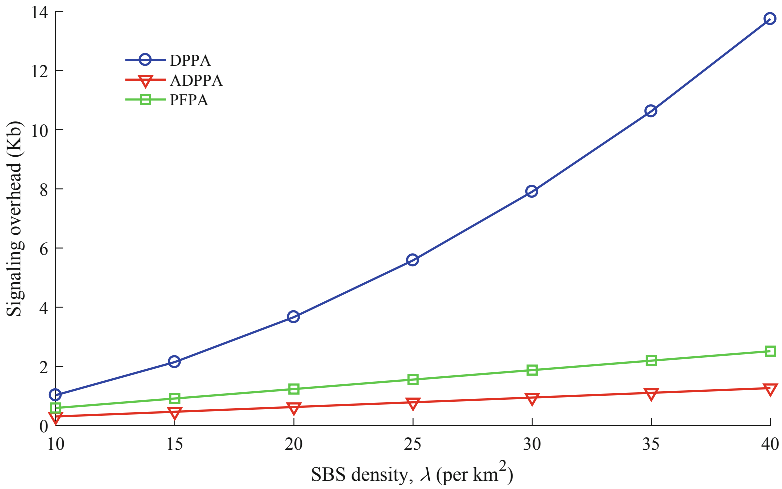

Fig. 5. Signaling overhead analysis.

number needs 16 bits. It can be seen that with the increase of SBS density, the signaling overhead of the proposed DPPA algorithm increases significantly, while it increases slowly for the ADPPA algorithm.

6 Conclusion

In this paper, distributed iterative downlink power allocation strategies were investigated for a spectrum sharing UDN with multi antennas resorting the game theory. The non-cooperative game model was adopted to decouple the non-convex system SE maximization problem into several convex subproblems. The dynamic pricing designed guarantees that each NE of the game is a stationary point of the original optimization problem. Moreover, the interference power constraint was adopted for the key user. Under the game theory framework, we proposed the DPPA algorithm required global information and the ADPPA algorithm which needs limited information. Simulation results show that the DPPA algorithm outperforms the benchmark, and the ADPPA algorithm with a little performance loss can effectively reduce the signaling overhead compared with the DPPA algorithm.

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