



# Detection Algorithm of Compressed Sensing Signal in GSM-MIMO System

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**Abstract.** For the generalized spatial modulation in the underdetermined system with the number of transmitting antennas larger than the number of receiving antennas, the activation of the antenna is small and inaccurate. The traditional MMSE algorithm and the ZF algorithm still perform the pseudo-inverse operation on the entire channel matrix, which results in a large number of redundancy. Although the ML algorithm has the best detection performance, the complexity is difficult to meet the actual requirements. In this paper, for the sparse characteristics of GSM signals, a detection algorithm is improved based on the compressed sensing recovery algorithm SWOMP. The algorithm first selects multiple or one active antenna sequences conforming to the spatial modulation to form an index set according to the situation, and then uses the backtracking principle to select the atomic column according to the threshold, rejects the unreliable sequence, and finally uses the minimum mean square error algorithm to detect the modulation symbol according to the activated antenna index. The pseudo-inverse operation of the entire channel matrix is avoided, and the bit error rate is lower than the ZF algorithm, the MMSE algorithm and the OMP algorithm, and the performance of the proposed algorithm is closer to the ML algorithm, which is a better way that balance between complexity and detection performance.

**Keywords:** GSM-MIMO · Compressed sensing · Signal detection

## 1 Introduction

The MIMO system not only makes the system's throughput increase in proportion, but also makes the system's reliability stronger by the diversity technology. However, while high-speed and large-scale multi-dimensional data brings high-efficiency transmission, the detection method and performance of the receiving end face higher challenges. For a traditional multi-antenna spatial multiplexing system, how many independent RF links are there in the system. With the surge in data demand, in order to further increase the data transmission rate, the MIMO system needs to increase the number of transmitting antennas, and the number of corresponding RF links becomes enormous, which not only brings a small challenge to the miniaturization of the transmitter. It also makes the hardware implementation cost high. Unlike spatial multiplexing, SM-MIMO maps information bit blocks into two information bearing units: spatial constellation symbols

and signal constellation symbols. Only one or a few antennas are activated when transmitting, which reduces the number of links and opens up the possibility of hardware cost reduction at the receiving end.

Spatial modulation technology is a relatively new multi-antenna transmission technology, except for the amplitude phase in the general real and imaginary fields. Amplitude and Phase Modulation (APM), which also introduces the spatial dimension as the third dimension, mines the serial number of the transmitting antenna for additional mapping, and establishes the mapping relationship between the antenna number and the input bits to complete the spatial modulation target. Since the number of receiving antennas of the user is small and the number of antennas of the BS is small, signal detection is a challenging large-scale uncertain problem. When the number of transmitting antennas becomes large, the optimal maximum likelihood (ML) signal detector suffers from excessive complexity [3], which is unacceptable in practical engineering. Since the number of active antennas is smaller than the total number of transmit antennas, the SM signal has inherent sparsity, and the sparsity of the signal can be improved by utilizing the compression sensing (CS) theory [1, 2]. That is, the signal detection scheme based on the compressed sensing theory can be used for detection at the receiving end. And CS-based signal detectors have been proposed for uncertain small SM-MIMO [4, 5]. However, compared with the best ML detectors, their bit error rate (BER) performance still has a large gap, and Gao et al. [6] proposed a packet transmission scheme suitable for large-scale SM-MIMO systems. A structured Subspace Pursuit (SSP) detection algorithm is presented. This algorithm is an improvement of the OMP algorithm. Each iteration selects multiple atoms and expands the signal search space. Therefore, its bit error rate is lower than that of the OMP algorithm, but the complexity is also increased accordingly. A CoSaMP-based Spatial Matching Pursuit (SMMP) detector is proposed in [7], which is applied in a multiple access channel with a large-scale antenna base station. These compression-sensing detectors have a large performance penalty. Therefore, the balance between detection performance and complexity of detection algorithms needs further discussion.

## 2 System Model

In this paper, it is assumed that the number of input signal antennas and the number of output signal antennas in the MIMO system are respectively  $N_T$  and  $N_R$ , noting  $N_T \leq N_R$ . If the number of active antennas on the sender is  $N_A$ , there are  $C_{N_T}^{N_A}$  a total possible combination. There are a combination of spatial symbols  $N = 2^{\log C_{N_T}^{N_A}}$ . The system model for spatial modulation is (Fig. 1):

Assume that there is perfect channel information. The receiving model of the system is

$$y = Hx + \sqrt{N_T E_x / \rho n} \quad (1)$$

Where  $y$  is a column vector of  $N_R \times 1$ .  $S$  is a constellation symbol set,  $x$  is a column vector of  $N_T \times 1$ , and the sparsity is  $N_A$ . Which is  $x \in S^{N_T \times 1}$ .  $H$  is the channel matrix

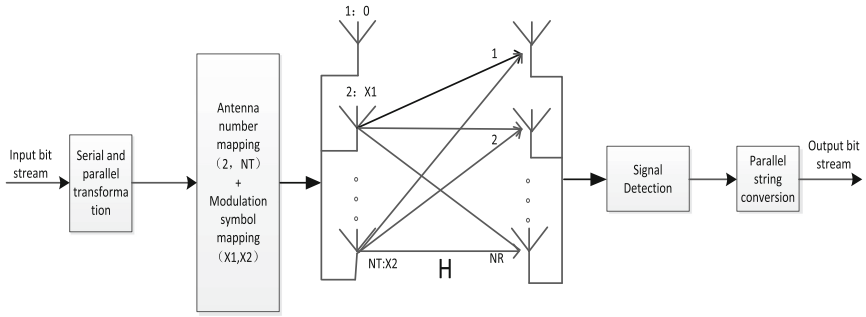


Fig. 1. Block diagram of GSM system

of  $N_T \times N_R$ .  $n$  is Gaussian white noise,  $\rho$  is signal to noise ratio,  $E_x = \frac{\sum_{i=1}^{N_T} \|x_i\|_2^2}{N_T}$  is the mean of the transmitted symbol energy.

### 3 Detection Algorithm Based on Compressed Sensing Reconstruction

According to the above GSM system model, the transmitted signal vector  $x$  is an  $N_A$  sparse signal, that is, there are only  $N_A$  non-zero elements in  $x$ , which is much smaller than the number of transmitting antennas  $N_T$ . For MIMO channels, if the channel matrix  $H$  satisfies the  $N$ -order RIP, then for any sparse signal  $x$ , the following formula is satisfied.

$$(1 - \delta_k) \|x\|_2^2 \leq \|Hx\|_2^2 \leq (1 + \delta_k) \|x\|_2^2 \tag{2}$$

It can be accurately recovered from the received signal, where is the  $K$ -order RIP parameter [5]. According to the research literature [6], it can be seen that when  $N_R \geq cN_A \cdot \log(N_T/N_A)$ ,  $\delta_{N_A} \leq 0.1$  satisfies the requirement of formula (2). Therefore, a compression-aware recovery algorithm can be used to solve the GSM detection problem.

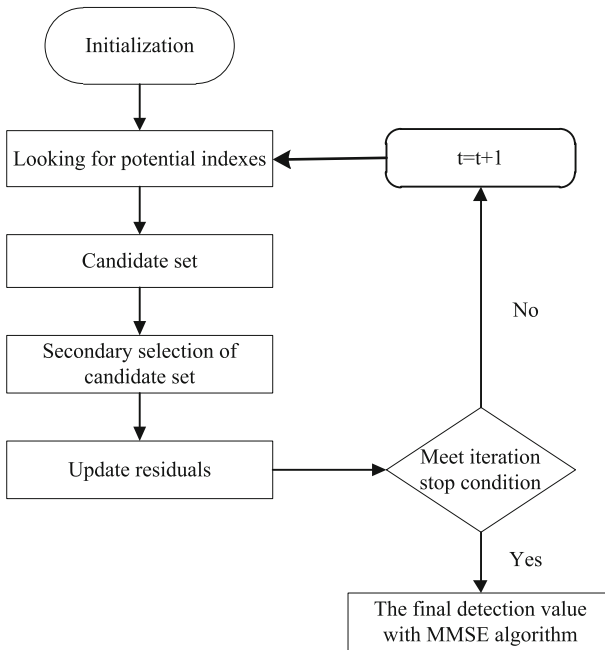
#### 3.1 GSM Detection Based on SWOMP Algorithm

In theory, the OMP algorithm can reconstruct the sparse signal  $x$  after iteration. SWOMP selects multiple atomic columns at a time to effectively reduce the number of iterations. However, in the wireless channel, since the received signal is affected by additive noise, only one inner product maximum or multiple is selected for each iteration, and it is very likely that the wrong activated antenna index is selected, so that the detection performance of the OMP algorithm is large. discount. Therefore, we use an improved flexible and

optional atomic selection strategy and backtracking ideas to make use of Eq. (3) to make an compromise between algorithm complexity and detection performance.

$$|\theta_i| \geq T_g = g * \max |\Phi^T r| \tag{3}$$

Where,  $\forall i \in J$ , set J is the set of serial numbers of  $\Phi$  in the first selection sensing matrix, with coefficient  $g \in (0, 0.5]$ . The threshold standard of pruning is related to the residual error of this iteration, which can determine the correct atomic scolumn better. In order to adapt the recovery algorithm to the system, we will use the sparse signal to take the characteristics of the constellation symbol in the modulation constellation. In the recovery algorithm, we will select the point in the constellation as the estimated value of the signal instead of the original algorithm. The value obtained by direct calculation. Finally, the MMSE is used to detect the antenna number that retains the closest and satisfies the spatial symbol as the spatial symbol detection value (Fig. 2).



**Fig. 2.** Flow chart of improved swamp detection algorithm

The algorithm steps are as follows:

**Input:** Received signal  $y$ , Channel matrix  $A=H$ ;

**Output:** Input signal  $x$  estimate  $\theta_t$

**Initialization:**  $\theta_t = 0$ , Residual  $r_0 = y$ , Support set sequence  $\Lambda_0 = \phi$ ,

Support set  $A_0 = \phi$ ,  $t=1$ ,  $F=0$ .

**While**  $t < \max\_iter$  or  $\|r_t\|_2 \leq \varepsilon_1$  **do**

$$r_t = y - A_{\Lambda_t} \theta_t, \quad t=t+1 \quad \{\text{Update residual}\}$$

$$u = \left| A_{\Lambda_t}^T r_{t-1} \right| \quad \{\text{Correlation test}\}$$

**If**  $F=0$  **do**

$$j_a = \text{find} (u \geq T_h)$$

$$\text{else do } j_a = \text{find} (u = \max |A^T r|) \quad \{\text{First selection}\}$$

**end if**

$$\Lambda_t = \Lambda_{t-1} \cup j_a \quad \{\text{Merge supports}\}$$

$$\theta_t = (A_{\Lambda_t}^T A_{\Lambda_t})^{-1} A_{\Lambda_t}^T y \quad \{\text{Calculate a new estimate}\}$$

$$\Lambda_t = \text{find} \left( \left| \theta_t \right| \geq T_g = g * \max |A^T r| \right) \quad \{\text{Second selection}\}$$

**end while**

$$\theta_t = \theta_{A_{\Lambda_t}} = Q \left( \left( (H_{A_{\Lambda_t}})^H H_{A_{\Lambda_t}} + \sigma^2 I \right)^{-1} (H_{A_{\Lambda_t}})^H y \right), \quad \{\text{Final estimate}\} \quad (4)$$

Where  $Q$  is the quantization function. Each vector is quantized to an integer.  $\sigma^2$  is the statistical information of noise. The above iterative process can ensure that the final antenna combination is a spatial symbol.

Transform selection criteria:  $\|r_t\|_2 \geq \|r_{t-1}\|_2$ ; According to the comparison of the residual energy values of the adjacent stages, if the current stage is larger than the

previous stage, it means entering the small step long stage, then switch the selection criteria. The maximum number of iterations is the sampling value  $M$ . if the iteration stop condition is met, the output is direct. The stop iteration condition is set to the following two cases: in the first case  $\|rt\|_2 \leq \varepsilon_1$ ,  $\varepsilon_1$  is the noise measurement value under ideal condition; in the second case, there is no qualified atomic column in step (5), when there is no suitable selection column for the first time, it also means that there is no atom matching with the Acolumn of the sensing matrix in the residual error, and the result can be directly output; the above two conditions are full Any one of them will exit the cycle and output the result.

### 3.2 Complexity Analysis

As a measure, the number of floating-point operations (only considering multiplication, one complex number multiplication equals to four floating-point operations) needed to realize a GSM signal detection is used to compare and analyze the computational complexity of ML algorithm, ZF algorithm, OMP algorithm and the algorithm proposed in this paper.

For MMSE algorithm, floating point operand required [8]:

$$C_{MMSE} = 4N_T^3 + 12N_T^2N_R + 7N_T^2 + 6N_TN_R \quad (5)$$

For ZF algorithm, floating point operand required [9]:

$$C_{ZF} = 4N_T^2N_R + \frac{4}{3}N_T^3 + 8N_RN_T + 11N_T^2 \quad (6)$$

For ML algorithm, floating point operand required [10]:

$$C_{ML} = (6N_A + 3)N_RM^{N_A}N \quad (7)$$

For the OMP algorithm, the required floating-point operands are:

$$C_{OMP} = 4N_TN_RN_A + 4N_R \sum_{t=1}^{N_A} (t)^2 + \frac{4}{3} \sum_{t=1}^{N_A} (t)^3 + 8N_R \sum_{t=1}^{N_A} t + 11 \sum_{t=1}^{N_A} (t)^2 \quad (8)$$

The algorithm proposed in this paper: the number of iterations is less than or equal to, and the operation amount of each iteration is mainly concentrated in two parts: inner product operation part and generalized inverse operation part.

The inner product operation part is similar to OMP,  $t$  represents the total number of iterations,  $t \leq N_A$  in the proposed algorithm.

$$C_a = 4N_TN_Rt \quad (9)$$

The generalized inverse operation part can be expressed as:

$$C_b = 4(G_{avg}^k)^2N_R + \frac{4}{3}(G_{avg}^k)^3 + 8G_{avg}^kN_R + 11(G_{avg}^k)^2 \quad (10)$$

Finally, MMSE algorithm is used to estimate symbols:

$$C_c = 4(G_{avg}^k)^3 + 12G_{avg}^k N_R + 7(G_{avg}^k)^2 + 6G_{avg}^k N_R \tag{11}$$

Where  $G_{avg}^k$  represents the number of indexes contained in the selected set in iteration K.

From the above analysis, the complexity of the algorithm can be expressed as follows:

$$C_{proposed} = 4N_T N_R t + 4(G_{avg}^k)^2 N_R + \frac{4}{3}(G_{avg}^k)^3 + 8G_{avg}^k N_R + 11(G_{avg}^k)^2 + 4(G_{avg}^k)^3 + 12G_{avg}^k N_R + 7(G_{avg}^k)^2 + 6G_{avg}^k N_R \tag{12}$$

From the above analysis, it can be seen that ML increases exponentially with the number of active antennas and modulation order, and the complexity is the highest. Compared with MMSE and ZF algorithm, because of the inverse of the whole channel matrix, the improved compressed sensing detection algorithm in this paper firstly uses the uncertain number of active antennas to obtain the sequence, which is equivalent to reducing the number of transmitting antennas, and then uses MMSE algorithm to obtain the signal modulation symbols. Although the complexity of the algorithm is higher than that of the OMP algorithm, the other advantages of compressed sensing are guaranteed and the bit error rate is reduced. See Table 1 for details. When the number of transmitting antennas  $N_T = 10$  and the number of receiving antennas  $N_R = 16$ , the specific complexity is:

**Table 1.** Complexity comparison

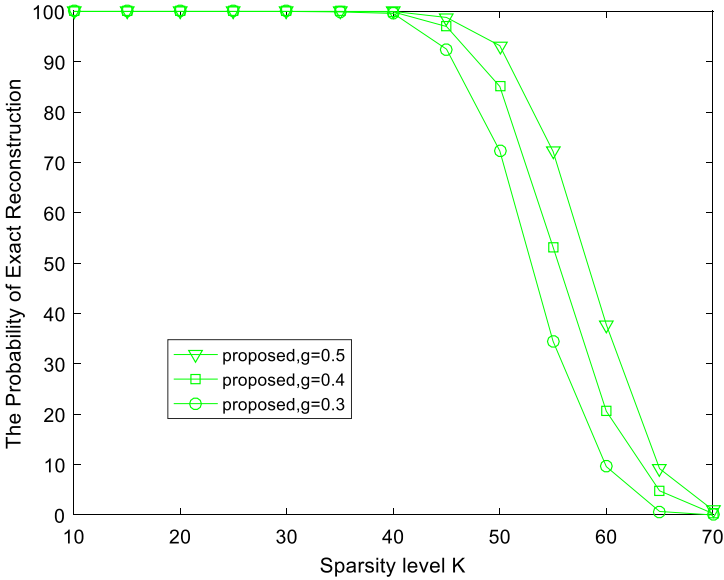
Algorithm	Average complexity	
	NA = 2	NA = 3
ML	122880	172032
MMSE	24860	24860
ZF	10113	10113
OMP	2052	2692
proposed	4562	4577

## 4 Simulation Analysis

In this paper, the bit error rate (BER) is used as the detection performance index to evaluate the BER performance of the above algorithms. The simulation parameters are configured as follows: in the single symbol GSM system, the number of transmitting antennas  $N_T = 10$ , the number of receiving antennas  $N_R = 16$ , the observed SNR range

[0,10], and the step size is 2. The channel adopts quasi flat Rayleigh fading channel, assuming that the receiver knows the exact channel state information.

As for the threshold coefficient  $g$  of the middle backtracking of the algorithm proposed in this paper, when the signal-to-noise ratio is 0, the influence of the parameters on the algorithm is as follows:



**Fig. 3.** Influence of coefficient  $g$  on algorithm under different sparsity

It can be seen from Fig. 3 that under the same sparsity  $K$ , with the increase of coefficient  $g$ , the recovery probability of the algorithm is improved, and the attenuation of the increase of sparsity is also slow. This is because the increase of coefficient  $g$  will make the backtracking filtering more accurate. However, when the coefficient  $g$  is greater than 0.5, the overall recovery probability will be affected due to the small number of selected atomic column sets. Therefore, the coefficient  $g$  is taken as 0.5 in the later simulation.

Figure 4 shows the trend of the BER of ML, ZF, MMSE, OMP algorithm and the improved algorithm with the signal-to-noise ratio when the number of active antennas is unknown and QPSK modulation is adopted. The simulation results show that the BER of the proposed algorithm is lower than that of ZF algorithm, MMSE algorithm and OMP algorithm, and more close to the performance of ML algorithm. Among them, the method of selecting atomic columns in OMP algorithm is fixed and cannot eliminate the wrong sequence. Although the initial error rate is lower than ZF, MMSE algorithm, but with the improvement of SNR, the error rate has not improved significantly. Floor effect exists in the detection algorithms of compressed sensing [11]. The algorithm proposed in this paper can reduce the error rate and the convergence floor at the same time.

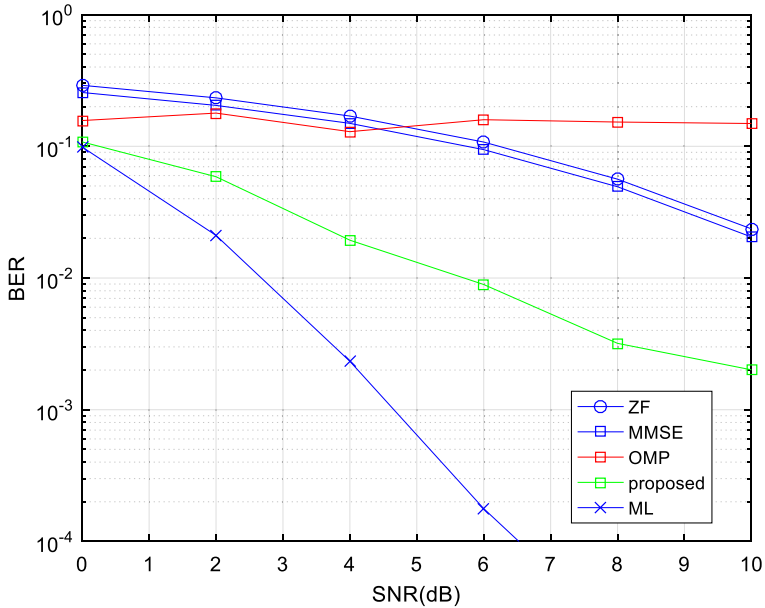


Fig. 4. Performance comparison of ML, ZF, MMSE, OMP and improved algorithm

## 5 Conclusion

In this paper, a low complexity signal detection algorithm based on sparse reconstruction theory is proposed by using the sparsity of GSM signal. Simulation results show that compared with the traditional OMP algorithm, the threshold value is used to select multiple atoms for fast approximation, and the switch selection strategy is used to accurately approximate the sparse when the sparse degree of the original signal may be overestimated. At last, we trace back the support set, prune the error columns which may be caused by multiple atom selection, and effectively reduce the system error rate at the expense of a small amount of complexity. Although there is some performance loss compared with ML algorithm, it effectively solves the problem of high complexity of ML algorithm and to some extent, the balance between complexity and detection performance is achieved, so it has certain application significance.

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