A Unified Approach to Analyze the Performance of NOMA with Different Grouping Strategies in Downlink

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Abstract. This paper conducts an analysis and investigation of outage performances and ergodic sum capacity for different users within each cluster in downlink non-orthogonal multiple access (NOMA) systems. Analytical expressions for outage performance and ergodic sum capacity are derived and analyzed. The analysis and simulation demonstrate that NOMA systems with more users in one cluster exhibit higher performance with lower average channel gains, while NOMA systems with fewer users in one cluster exhibit higher performance with higher average channel gains. This suggests the need for different user grouping strategies under varying signal-to-noise ratio conditions, and the fixed user grouping schemes for NOMA systems are inadequate to adapt to complex environments and diverse services. Consequently, this paper brings in the mixed user grouping NOMA systems, which offers greater advantages comparing to orthogonal multiple access (OMA) and conventional NOMA. Finally, this study provides pathways for further research on mixed user grouping in downlink NOMA systems.

Keywords: NOMA · downlink · ergodic sum capacity · outage performance · multiple clusters · mixed user grouping

1 Introduction

Comparing with orthogonal multiple access (OMA), non-orthogonal multiple access (NOMA) [1–3] offers higher channel capacity, spectral efficiency and support more users. This fully meets the requirements of next-generation

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communication systems for enhanced mobile broadband (eMBB) high capacity and massive machine-type communication (MMTC) massive connectivity [4].

NOMA can make full use of the power domain, but due to the disadvantages of superposition coding (SC) at the transmitter and successive interference cancellation (SIC) at the receiver, The NOMA systems lead to error propagation and data delay issues [5]. Error propagation occurs when incorrectly decoded data from distant-end users may affect target user data integrity. As the number of cluster users increases, accumulated errors become intolerable, resulting in degraded system performance. Due to these reasons, current research on user grouping algorithms for NOMA systems generally focuses on clusters with only two users in order to limit decoding delays and error propagation within acceptable bounds.

However, this limitation contradicts the original intention behind NOMA's proposal (providing large capacity and massive connectivity) fails to fully exploit its potential. Existing technologies can be leveraged to address both shortcomings in NOMA systems. The nature of error propagation lies in practical limitations imposed by existing channel coding technologies [6], where Gaussian coding has not been achieved thus far. Recent studies have explored certain channel codes such as low-density parity-check codes (LDPC) [7], which approximate Gaussian coding approaches close to Shannon limits. By effectively utilizing channel coding, error propagation can be reduced within narrow margins. For delay-sensitive scenarios such as V2X communications [8,9] or video gaming where decoding delays are unacceptable under typical NOMA settings; however, future wireless communication systems will feature diverse data services: some insensitive towards latency like video streaming while others involve small datasets such as IoT [10,11] applications where NOMA could play a crucial role. Two-user NOMA setups fall short in meeting future demands for diversified business scenarios requiring extensive connections and high data rates. Consequently, we explore various user grouping strategies before proposing mixed user grouping [12] for further study.

2 System Model

There are K single-antenna users randomly distributed in the cell with radius D. The user k's channel gain is h_k, where $h_k = g_k d_k^{-\frac{\alpha}{2}}$, and d_k denotes the distance of the user from the base station, α is the path loss factor, and g_k is Rayleigh fading channel gain. h_k is arranged in descending order: $|h_1|^2 \geq \cdots \geq |h_K|^2$. According to NOMA principles, users with higher channel gains are allocated smaller power coefficients: $\beta_1 \leq \cdots \leq \beta_K$. Every user receives a signal containing all users' data transmitted by the base station at their receiving end. The received signal for user k is given by

$$y_k = h_k \sum_{k'=1}^{K} \sqrt{\beta_{k'} P_t} s_{k'} + n_k, \tag{1}$$

where n_k represents additive Gaussian white noise at user k's receiver. Following successive interference cancellation (SIC) principles, user k decodes user K's data firstly before subtracting it from its current signal, then decodes user $K-1$'s data, and subtract it from the remaining signal. This process continues until decoding its own data. When decoding its own data, the remaining users' signals are regarded as Gaussian noise. The data rate for user k is expressed as:

$$R_k = \log\left(1 + \frac{\beta_k \gamma_k}{1 + \sum_{k'=1}^{k-1} \beta_{k'} \gamma_k}\right), \tag{2}$$

where γ_k denotes each user's Signal-to-Noise Ratio (SNR), defined as

$$\gamma_k = \frac{P_t |h_k|^2}{N_0 B}, \tag{3}$$

where N_0 representing noise power spectral density and B denotes signal bandwidth.

Specially, one user per cluster in the NOMA system can be regarded as the OMA system; thus unifying both system types formally which facilitates subsequent research on mixed user grouping algorithms [13].

To eliminate transmit power influence on system performance analysis, we introduce normalized distance $R = D(P_t/N_0 B)^{-\frac{1}{\alpha}}$. Specially, when $R = 1$, at this time, the SNR $\gamma_k = |g_k|^2$ of the edge user in the cell, the large-scale fading effect is exactly cancelled out, which is the so-called"normalization". For different transmission powers, the real distance corresponding to the normalized distance is different, and the larger the transmission power, the larger the real distance. and the larger the normalized distance, and the smaller the average SNR of the users in the cell.

We aim to study system ergodic capacity and interruption performance which need SNR distribution information. Given random uniform distribution of users within cells CDF function for *unordered* SNR γ can be represented as [14]:

$$F(\gamma) = \frac{2}{R^2} \int_0^R (1 - e^{-z^\alpha \gamma}) z \mathrm{d}z, \tag{4}$$

where α represents path-loss factor

However in many wireless channels scenarios such as indoor environments $\alpha > 2$, it's difficult to obtain analytical solutions, hence we use of Gaussian-Chebyshev quadrature method to simplify the formula:

$$F(\gamma) \approx \sum_{l=1}^{L} g(\theta_l), \tag{5}$$

where $g(\theta_l) = \frac{\pi}{2L}\sqrt{1-\theta_l^2}(\theta_l + 1)(1 - e^{-c_l \gamma})$, $c_l = (R(\theta_l + 1)/2)^\alpha$,$\theta_l = \cos((2l - 1)\pi/2L)$, and L serves complexity accuracy trade-off purpose. Consequently PDF approximation becomes:

$$f(\gamma) = \sum_{l=1}^{L} \hat{b}_l e^{-c_l \gamma}, \tag{6}$$

with $\hat{b}_l = \frac{\pi}{2L}\sqrt{1-\theta_l^2}(\theta_l+1)c_l$.

For ease in future derivations, CDF needs rewriting into form:

$$F(\gamma) \approx \sum_{l=0}^{L} b_l e^{-c_l \gamma}, \tag{7}$$

where $b_l = -\frac{\pi}{2L}\sqrt{1-\theta_l^2}(\theta_l+1), \forall l = 1, \ldots L, b_0 = -\sum_{l=1}^{L} b_l, c_0 = 0$. Obviously, $F(\infty) = b_0 = 1$.

3 Multi-Cluster NOMA Systems in Downlink

3.1 Ergodic Sum Capacity

We define $\gamma_0 = -\infty, \gamma_{k+1} = +\infty, \pi_{m,0} = 0, \pi_{m,K+1} = N+1$, then the joint SNR PDF of the m-th NOMA cluster can expressed as [15]

$$\begin{aligned} f_{\pi_m}(\Gamma) &= f_{(\pi_{m,1})\ldots(\pi_{m,K})}(\gamma_1, \ldots, \gamma_K) \\ &= N! \prod_{k=1}^{K} f(\gamma_k) \prod_{k=0}^{K} \frac{[F(\gamma_{k+1}) - F(\gamma_k)]^{\pi_{m,k+1}-\pi_{m,k}-1}}{(\pi_{m,k+1}-\pi_{m,k}-1)!}, \end{aligned} \tag{8}$$

where $\mathbf{\Gamma} = (\gamma_1, \ldots, \gamma_K)$, K is the total number of the users in one NOMA cluster, denotes the total number of the users in the NOMA system, $f(\gamma_k)$denotes the *unordered*ed PDF of the channel gain, $F(\gamma_k)$denotes the *unordereded* CDF of the channel gain,$\pi_{m,k}$ denotes the number of the k-th user in the m-th cluster with the grouping scheme π, $\beta_{\pi_{m,k}}$ denotes the power allocation coefficient of the user $\pi_{m,k}$.

Therefore, the ergodic sum capacity is formulated as

$$C_{sum} = K \sum_{m=1}^{M} \int_{\mathcal{A}} f_{\pi_m}(\Gamma) \sum_{k=1}^{K} \log(1 + \frac{\beta_{\pi_{m,k}}\gamma_k}{1 + \sum_{k'=1}^{k-1} \beta_{\pi_{m,k'}}\gamma_k}) \mathrm{d}\Gamma, \tag{9}$$

where $\mathcal{A}$ denotes the multiple integral region, γ_k denote the SNR of the k-th user in the m-th NOMA cluster.

The integral is very hard to calculate due to the multiple integral and complex expression of the SNR PDF, but the multi integral can be converted to the simple single integral since the users are decoupled.

Therefore, the ergodic sum capacity can be simplified as follows:

$$C_{sum} = K \sum_{m=1}^{M} \sum_{k=1}^{K} \int_{0}^{\infty} f_{\pi_{m,k}}(\gamma) \log\left(1 + \frac{\beta_{\pi_{m,k}}\gamma}{1 + \sum_{k'=1}^{k-1} \beta_{\pi_{m,k'}}\gamma}\right) \mathrm{d}\gamma. \tag{10}$$

where M is the total number of the clusters, K is the total number of the users in one cluster, $\pi_{m,k}$ denotes the number of the k-th user in the m-th cluster with the grouping shceme π, $\beta_{\pi_{m,k}}$ denotes the power allocation coefficient of the user $\pi_{m,k}$, γ denotes the normalized SNR, $f_{\pi_{\mathbf{m,k}}}(\gamma)$ denotes the SNR PDF of the user $\pi_{m,k}$.

Theorem 1. *Under the random uniform user distribution condition, the ergodic sum capacity of the NOMA system in the downlink can be approximately expressed as*

$$C_{sum} \approx \frac{N!K}{\ln 2} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{k'=0}^{\pi_{m,k}-1} \sum_{A(k_l,k_l')} \prod_{l=0}^{L} \frac{(-1)^{k'} b_l^{k_l+k_l'}}{k_l! k_l'!} \times \left(e^{\varpi_1}\mathrm{Ei}(-\varpi_1) - e^{\varpi_2}\mathrm{Ei}(-\varpi_2)\right), \tag{11}$$

where $\varpi_1 = \frac{\sum_{l=0}^{L}(k_l+k_l')c_l}{(1+\phi)\beta_{\pi_{m,k}}}$,$\varpi_2 = \frac{\sum_{l=0}^{L}(k_l+k_l')c_l}{\phi\beta_{\pi_{m,k}}}$, $A(k_l,k_l')$ satisfied $\sum_{l=0}^{L} k_l = K - k'$,$\sum_{l=0}^{L} k_l' = k'$,$k_0 \neq K$,$k_0' = 0$, Ei(.) denotes exponential integral, which is

$$\mathrm{Ei}(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t}\mathrm{d}t. \tag{12}$$

Proofs seen in **Appendix A**.

Lemma 1: in high SNR conditions, when $\gamma \to \infty$, the ergodic sum capacity of the NOMA system can be expressed as follows:

$$C_{sum} \approx \frac{N!K}{\ln 2} \sum_{m=1}^{M} \sum_{k'=0}^{\pi_{m,1}-1} \sum_{A(k_l,k_l')} \prod_{l=0}^{L} \frac{b_l^{k_l+k_l'}}{k_l! k_l'!} \mathrm{Ei}\left(-\sum_{l=0}^{L}(k_l+k_l')c_l\right), \tag{13}$$

Proofs seen in **Appendix B**.

Lemma 2: in low SNR conditions, when $\gamma \to 0$, the ergodic sum capacity of the NOMA system can be expressed as follows:

$$C_{sum} \approx N!K \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{k'=0}^{\pi_{m,k}-1} \sum_{A(k_l,k_l')} \frac{(-1)^{k'+1}\beta_{\pi_{m,k}}}{\sum_{l=0}^{L}(k_l+k_l')c_l} \prod_{l=0}^{L} \frac{b_l^{k_l+k_l'}}{k_l! k_l'!}, \tag{14}$$

Proofs seen in **Appendix C**.

Theorem 2. *Under the random uniform user distribution condition, the ergodic sum capacity of the OMA system in the downlink can be approximately expressed as*

$$C_{sum} \approx -\frac{N\pi}{2L\ln 2} \sum_{l=1}^{L} \sqrt{1-\theta_l^2}(\theta_l+1)e^{c_l}\mathrm{Ei}(-c_l), \tag{15}$$

where denotes the total number of the users, $f(\gamma)$ denotes the SNR PDF of the user. Proofs seen in **Appendix D**.

Lemma 3: in high SNR conditions, when $\gamma \to \infty$, the ergodic sum capacity of the OMA system can be expressed as follows:

$$C_{sum} \approx \frac{N}{\ln 2}(\gamma_e + \frac{\alpha}{2} - \alpha \ln \mathcal{R} + \ln N). \tag{16}$$

Proofs seen in **Appendix E**.

Lemma 4: in low SNR conditions, when $\gamma \to 0$, the ergodic sum capacity of the OMA system can be expressed as follows:

$$C_{sum} \approx \frac{2N}{(\alpha-2)(R^2-R_0^2)} \left(\frac{1}{R_0^{\alpha-2}} - \frac{1}{R^{\alpha-2}} \right). \tag{17}$$

Proofs seen in **Appendix F**.

3.2 Outage Performance

To avoid the outage, the data rate must satisfy the minimum objective rate [16], which means

$$R_{\pi_{m,k'}}^{(\pi_{m,k})} = K \log(1 + \frac{\beta_{\pi_{m,k'}} \gamma_{\pi_{m,k}}}{1 + \sum_{l=1}^{k'-1}} \beta_{\pi_{m,l}} \gamma_{\pi_{m,k}}) > R_0, \tag{18}$$

$$\forall k' = k, \ldots, K, \quad R_0 < K \log(1 + \frac{1}{\delta}), \tag{19}$$

where $\pi_{m,k}$ denotes the number of the k-th user in the m-th cluster with the grouping shceme π,$R_{\pi_{m,k'}}^{(\pi_{m,k})}$denotes the data rate of the user $\pi_{m,k'}$ at the user $\pi_{m,k}$, since we need decode the user $\pi_{m,k'}$ signal before decoding the user $\pi_{m,k}$signal according to the SIC principle. $\beta_{\pi_{m,k'}}$ denotes the power allocation coefficient of the user $\pi_{m,k'}$, $\gamma_{\pi_{m,k}}$ denotes the normalized SNR of the user $\pi_{m,k}$, R_0 denotes the minimum objective rate.

Using the aforementioned optimal power allocation, we have

$$\gamma_{\pi_{m,k}} > \frac{2^{\frac{R_0}{K}} - 1}{(1 + \frac{1}{\delta} - 2^{\frac{R_0}{K}})\beta_{\pi_{m,k'}}}, \quad \forall k' = k, \ldots, K. \tag{20}$$

since we use the same power allocation algorithm in different NOMA clusters, we have

$$\gamma_{\pi_{m,k}} > \frac{2^{\frac{R_0}{K}} - 1}{(1 + \frac{1}{\delta} - 2^{\frac{R_0}{K}})\beta_{\pi_{m,k}}}, \quad \forall k = 2, \ldots, K. \tag{21}$$

Therefore the threshold $\theta_{\pi_{m,k}}$ for the user $\pi_{m,k}$ is given as

$$\varTheta_{\pi_{m,k}} = \frac{\delta(2^{\frac{R_0}{K}} - 1)}{(1 + \delta - \delta 2^{\frac{R_0}{K}})\beta_{\pi_{m,k}}}, \quad \forall k = 2, \ldots, K. \tag{22}$$

It can be easily proved that

$$\frac{2^{\frac{R_0}{K}}-1}{(1+\frac{1}{\delta}-2^{\frac{R_0}{K}})\beta_{\pi_{m,2}}} > \frac{2^{\frac{R_0}{K}}-1}{\beta_{\pi_{m,1}}}. \tag{23}$$

$$\Theta_{\pi_{m,1}} = \frac{2^{\frac{R_0}{K}}-1}{(1+\delta-\delta 2^{\frac{R_0}{K}})\beta_{\pi_{m,1}}}. \tag{24}$$

$$P_{\pi_{m,k}} = \int_0^{\Theta_{\pi_{m,k}}} f_{\pi_{m,k}}(\gamma)\mathrm{d}\gamma = F_{\pi_{m,k}}(\Theta_{\pi_{m,k}}), \tag{25}$$

The outage probability of the j-th user in the i-th cluster is

$$P_{\pi_{m,k}} \approx N! \sum_{k'=0}^{\pi_{m,k}-1} \sum_{A(k_l,k_l')} \prod_{l=0}^{L} \frac{b_l^{k_l+k_l'}}{k_l!k_l'!} e^{-\Theta_{\pi_{m,k}} \sum_{l=0}^{L}(k_l+k_l')c_l}, \tag{26}$$

where $\pi_{m,k}$ denotes the number of the k-th user in the m-th cluster with the grouping shceme π, θ_k denotes the threshold of the user k, $f_{\pi_{m,k}}(\gamma)$ denotes the SNR PDF of the user $\pi_{m,k}$, $F_{\pi_{m,k}}(\theta_k)$ denotes the SNR CDF of the user $\pi_{m,k}$.

The probability when the outage event happens in the system is given by

$$P \approx N! \prod_{n=1}^{N} \sum_{k'=0}^{n-1} \sum_{A(k_l,k_l')} \prod_{l=0}^{L} \frac{(-1)^{k'+1} b_l^{k_l+k_l'}}{k_l!k_l'!} e^{-\Theta_n \sum_{l=0}^{L}(k_l+k_l')c_l}, \tag{27}$$

where M is the total number of the cluster, K is the total number of users in one cluster, $P_{\pi_{m,k}}$ is the user $\pi_{m,k}$'s outage probability.

We refer system outage index (SOI) as the expected number of users in outage. The SOI should be normalized due to comparing performances with varied number of users. Lower SOI means higher system reliability, and vise versa. The SOI for the multi-cluster NOMA system can expressed as

$$I = \frac{1}{N} \sum_{n=1}^{N} n \sum_{A(l_{n'})} \prod_{n'=1}^{N} P_{n'}^{l_{n'}} (1-P_{n'})^{1-l_{n'}}, \tag{28}$$

where N is the whole number of users in the NOMA system, $\mathbf{A}^{(n)}$denotes the matrix that contains all the combination vectors which select n numbers from N numbers. It includes n rows and $\binom{N}{n}$columns, $\mathbf{A}_{n'}^{(n)}$denotes the n'-th column vector. P_l denotes the outage probability of user l, $P_{l'}$ denotes the outage probability of user l'.

4 Simulation Results and Discussions

4.1 Ergodic Sum Capacity Performance

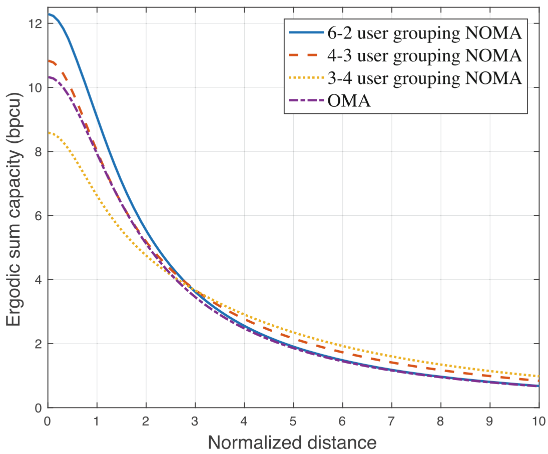

Fig. 1. Ergodic sum capacity for varied multi-cluster NOMA systems.

The simulation results indicate that under conditions of high average channel gains, NOMA systems of fewer users in one cluster exhibit more diversity and capacity. Initially, with low noise levels, interference becomes the primary limiting factor for system sum capacity, while multiplexing gains for multi-user NOMA systems are insufficient to counterbalance the drawback of significant users' interferences. Conversely, in scenarios with lower channel gains, NOMA systems of more users demonstrate larger channel capacities. This is attributed to interference becoming negligible compared to noise, which then becomes the principal limiting factor for system sum capacity at this stage. Additionally, multi-user NOMA systems can effectively leverage multiplexing gains to achieve higher diversity and capacity than OMA. Furthermore, the expanded range of channel gains provides multi-user NOMA systems with increased opportunities to identify suitable groupings of users and attain higher sum capacities (Figs. 1 and 2).

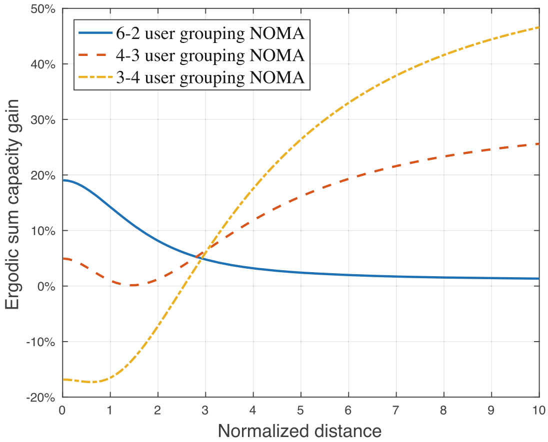

Fig. 2. Ergodic sum capacity gains for varied multi-cluster NOMA systems over OMA.

4.2 Outage Performance

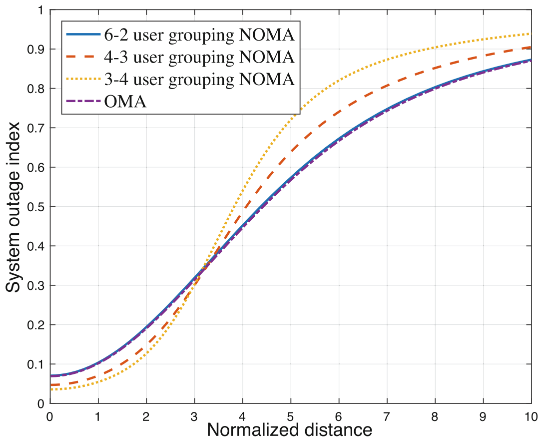

Fig. 3. SOI for varied multi-cluster NOMA systems vs normalized distance.

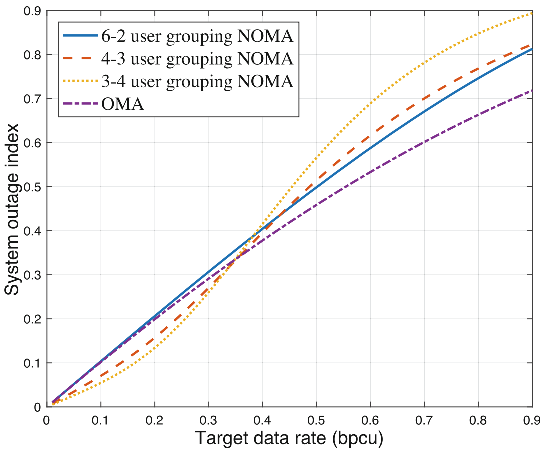

Fig. 4. SOI for varied multi-cluster NOMA systems vs target data rate.

It can be observed from the figure that when the target data rate increases, the system reliability deteriorates. However, the performance of NOMA schemes for different numbers of users varies. The performance of four-user NOMA is superior to that of three-user and two-user NOMA with lower target data rates, while it runs the opposite with higher target data rates. Both figures exhibit a consistent trend: multi-user NOMA system has better performance with favorable conditions, whereas two-user NOMA shows the opposite pattern (Figs. 3 and 4).

5 Conclusions

This paper investigates the system performance of multi-cluster NOMA systems with varying number of users, including outage performances and ergodic sum capacity. Analytical expressions are provided. Under conditions of lower channel gains, NOMA systems with more users in one cluster performs better, whereas under conditions of higher channel gains, NOMA systems with fewer users in one cluster performs better. The analysis suggests the need to adapt the user grouping based on different signal-to-noise ratio conditions. Consequently, static user grouping NOMA systems with fixed number of users in one cluster are unable to accommodate complicated and dynamic scenarios These points towards future research directions where dynamic user grouping NOMA systems could address the limitations of conventional NOMA system and cater to increasingly different service scenarios.

A Appendix

Appendix A

Since we have $\sum_{k'=1}^{k-1} \beta_{k'} = \phi\beta_k$, equation can be written as

$$C_{sum} = K \sum_{m=1}^{M} \sum_{k=1}^{K} (\underbrace{\int_0^\infty f_{\pi_{m,k}}(\gamma) \log\left(1 + (1+\phi)\beta_{\pi_{m,k}}\gamma\right) \mathrm{d}\gamma}_{T_1} - \underbrace{\int_0^\infty f_{\pi_{m,k}}(\gamma)(1+\phi\beta_{\pi_{m,k}}\gamma)\mathrm{d}\gamma}_{T2}). \tag{29}$$

for T_1, we use integration by parts

$$T_1 = \frac{(1+\phi)\beta_{\pi_{m,k}}}{\ln 2} \int_0^\infty \frac{1 - F_{\pi_{m,k}}(\gamma)}{1 + (1+\phi)\beta_{\pi_{m,k}}\gamma} \mathrm{d}\gamma. \tag{30}$$

since $F_k(0) = 0$, $F_k(\infty) = 1$, then can be rewritten as

$$T_1 \approx -\frac{(1+\phi)\beta_{\pi_{m,k}} N!}{\ln 2} \sum_{k'=0}^{\pi_{m,k}-1} \sum_{A(k_l,k_l')} \prod_{l=0}^{L} \frac{(-1)^{k'} b_l^{k_l+k_l'}}{k_l! k_l'!} \int_0^\infty \frac{e^{-\gamma \sum_{l=0}^{L}(k_l+k_l')c_l} \mathrm{d}\gamma}{1 + (1+\phi)\beta_{\pi_{m,k}}\gamma}. \tag{31}$$

Similarly, we get T_2

$$T_2 \approx \frac{\phi\beta_{\pi_{m,k}} N!}{\ln 2} \sum_{k'=0}^{\pi_{m,k}-1} \sum_{A(k_l,k_l')} \prod_{l=0}^{L} \frac{(-1)^{k'} b_l^{k_l+k_l'}}{k_l! k_l'!} \int_0^\infty \frac{e^{-\gamma \sum_{l=0}^{L}(k_l+k_l')c_l} \mathrm{d}\gamma}{1 + \phi\beta_{\pi_{m,k}}\gamma}. \tag{32}$$

let $\varpi_1 = \frac{\gamma \sum_{l=0}^{L}(k_l+k_l')c_l}{1+(1+\phi)\beta_{\pi_{m,k}}}$ and $\varpi_2 = \frac{\gamma \sum_{l=0}^{L}(k_l+k_l')c_l}{1+\phi\beta_{\pi_{m,k}}}$, then

$$T_1 \approx \frac{N!}{\ln 2} \sum_{k'=0}^{k-1} \sum_{A(k_l,k_l')} \prod_{l=0}^{L} \frac{(-1)^{k'} b_l^{k_l+k_l'}}{k_l! k_l'!} (e^{\varpi_1} \mathrm{Ei}(-\varpi_1)). \tag{33}$$

$$T_2 \approx \frac{N!}{\ln 2} \sum_{k'=0}^{k-1} \sum_{A(k_l,k_l')} \prod_{l=0}^{L} \frac{(-1)^{k'} b_l^{k_l+k_l'}}{k_l! k_l'!} (e^{\varpi_2} \mathrm{Ei}(-\varpi_2)). \tag{34}$$

Thus,

$$C_{sum} \approx \frac{N! K}{\ln 2} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{k'=0}^{\pi_{m,k}-1} \sum_{A(k_l,k_l')} \prod_{l=0}^{L} \frac{(-1)^{k'} b_l^{k_l+k_l'}}{k_l! k_l'!} \times (e^{\varpi_1} \mathrm{Ei}(-\varpi_1) - e^{\varpi_2} \mathrm{Ei}(-\varpi_2)), \tag{35}$$

Theorem 1 is proved.

Appendix B

At high SNR, i.e., $\gamma \to \infty$, $\frac{\beta_{\pi_{m,k}}\gamma}{1+\sum_{k'=1}^{k-1}\beta_{\pi_{m,k'}}\gamma} \approx \frac{\beta_{\pi_{m,k}}}{\sum_{k'=1}^{k-1}\beta_{\pi_{m,k'}}} = \delta$, then we have

$$C_{sum} \approx K\sum_{m=1}^{M}\left(\sum_{k=2}^{K}\int_0^\infty f_{\pi_{m,k}}(\gamma)\log(1+\delta)\mathrm{d}\gamma + \int_0^\infty f_{\pi_{m,1}}(\gamma)\log(\beta_{\pi_{m,1}}\gamma)\mathrm{d}\gamma\right). \tag{36}$$

Then,

$$C_{sum} \approx K\sum_{m=1}^{M}\int_0^\infty f_{\pi_{m,1}}(\gamma)\log(\gamma)\mathrm{d}\gamma \approx \frac{K}{\ln 2}\sum_{m=1}^{M}\int_1^\infty \frac{1-F_{\pi_{m,1}}(\gamma)}{\gamma}\mathrm{d}\gamma. \tag{37}$$

then

$$C_{sum} \approx -\frac{N!K}{\ln 2}\sum_{m=1}^{M}\sum_{k'=0}^{\pi_{m,1}-1}\sum_{A(k_l,k_l')}\prod_{l=0}^{L}\frac{b_l^{k_l+k_l'}}{k_l!k_l'!}\int_1^\infty \frac{e^{-\gamma\sum_{l=0}^{L}(k_l+k_l')c_l}}{\gamma}\mathrm{d}\gamma. \tag{38}$$

Integrate the above formula, Lemma 1 is proved.

Appendix C

At low SNR, i.e., $\gamma \to 0$, $\frac{\beta_{\pi_{m,k}}\gamma}{1+\sum_{k'=1}^{k-1}\beta_{\pi_{m,k'}}\gamma} \to \beta_{\pi_{m,k}}\gamma$, the ergodic capacity turns to be

$$C_{sum} \approx K\sum_{m=1}^{M}\sum_{k=1}^{K}\beta_{\pi_{m,k}}\int_0^\infty \gamma f_{\pi_{m,k}}(\gamma)\mathrm{d}\gamma. \tag{39}$$

then,

$$C_{sum} \approx K\sum_{m=1}^{M}\sum_{k=1}^{K}\beta_{\pi_{m,k}}\int_0^\infty 1-\sum_{k'=0}^{\pi_{m,k}-1}\binom{N}{k'}F^{N-k'}(\gamma)\left(1-F(\gamma)\right)^{k'}\mathrm{d}\gamma. \tag{40}$$

then,

$$\begin{aligned} C_{sum} \approx & N!K\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{k'=0}^{\pi_{m,k}-1}(-1)^{k'+1}\beta_{\pi_{m,k}} \\ & \times\int_0^\infty \sum_{A(k_l,k_l')}\prod_{l=0}^{L}\frac{b_l^{k_l+k_l'}}{k_l!k_l'!}e^{-\gamma\sum_{l=0}^{L}(k_l+k_l')c_l}\mathrm{d}\gamma. \end{aligned} \tag{41}$$

Integrate the above formula, Lemma 2 is proved.

Appendix D

In the OMA system, all users have the same uniformity, and the probability distribution of signal-to-noise ratio is the same, i.e., $f_k(\gamma) = f(\gamma), \forall k = 1, \ldots, K$, so the performance of each user is also the same. Therefore, the ergodic sum capacity of the OMA system can be expressed as

$$C_{sum} = K \int_0^\infty f(\gamma) \log(1+\gamma) \mathrm{d}\gamma, \tag{42}$$

where $f(\gamma)$ denotes the *unordered* SNR PDF. Then we have

$$C_{sum} \approx K \sum_{l=1}^{L} \hat{b}_l \int_0^\infty e^{-c_l \gamma} \log(1+\gamma) \mathrm{d}\gamma, \tag{43}$$

Integrate the above formula, we get

$$C_{sum} \approx -K \sum_{l=1}^{L} \frac{\hat{b}_l}{c_l} e^{c_l} \mathrm{Ei}(-c_l). \tag{44}$$

Theorem 2 is proved.

Appendix E

Under the high SNR conditions, i.e., $\gamma \to \infty$, we have

$$C_{sum} \approx N \int_0^\infty f(\gamma) \log(\gamma) \mathrm{d}\gamma. \tag{45}$$

Then,

$$\begin{aligned} C_{sum} &\approx N \sum_{l=1}^{L} \hat{b}_l \int_0^\infty e^{-c_l \gamma} \log(\gamma) \mathrm{d}\gamma \qquad (46) \\ &\overset{z=c_l\gamma}{=} N \sum_{l=1}^{L} \frac{\hat{b}_l}{c_l} \left(\int_0^\infty e^{-z} \log(z) \mathrm{d}z - \log(c_l) \int_0^\infty e^{-z} \mathrm{d}z \right) \\ &= \frac{N}{\ln 2} \sum_{l=1}^{L} \frac{\hat{b}_l}{c_l} (\gamma_e - \ln(c_l)) \\ &= \frac{N}{\ln 2} (\gamma_e + \frac{\alpha}{2} - \alpha \ln \mathcal{R} + \ln N). \end{aligned}$$

Lemma 3 is proved.

Appendix F

Under the high SNR conditions, i.e., $\gamma \to 0$, we have

$$C_{sum} \approx N \int_0^\infty \gamma f(\gamma) \mathrm{d}\gamma. \tag{47}$$

Then,

$$C_{sum} \approx N \sum_{l=1}^{L} \hat{b}_l \int_0^{\infty} e^{-c_l \gamma} \gamma \mathrm{d}\gamma$$
$$= N \sum_{l=1}^{L} \frac{\hat{b}_l}{c_l^2}$$
$$= \frac{2N}{(\alpha-2)(R^2 - R_0^2)} \left(\frac{1}{R_0^{\alpha-2}} - \frac{1}{R^{\alpha-2}} \right). \tag{48}$$

Lemma 4 is proved.

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