SIR Meta Distribution Analysis for Multi-antenna Multi-user Networks with Interference Nulling

Tianming Feng(✉), Yuquan Shu, and Zhihui Yao

The 54th Research Institute of China Electronics Technology Group Cooperation, Shijiazhuang 050081, China
feng_tm@163.com

Abstract. This paper studies the signal-to-interference ratio (SIR) meta distribution (MD) for multi-antenna multi-user networks with interference nulling (IN) considered. Since it is difficult to calculate the expression for the SIR MD, we resort to a tight upper bound on the conditional success probability (CSP) and obtain its first moment and second moment; with further utilizing the beta distribution, we obtain an approximation on the SIR MD. With numerical simulations, the tightness of this expression is verified and the effects of system design parameters on the network performance from the perspectives of the distribution of individual link reliability are revealed.

Keywords: Multi-antenna networks · Interference nulling · Meta distribution

1 Introduction

Nowadays, deploying multiple antennas at the base stations (BSs) in the network is a viable solution to meet the growing demand for high data throughput and spectral efficiency, driven by the rapid increase in the number of mobile terminals and the emergence of new scenarios in fifth-generation (5G) networks [1,2]. In the multi-antenna networks, the spatial degrees-of-freedom (DoF) provided by multiple antennas can be used for: (1) serving multiple users simultaneously over the same resource block so as to enhance the network spectrum efficiency and increase network capacity [3,4]; (2) providing spatial diversity for these users to enhance the strength of the desired signals received at these users [5,6]; (3) conducting interference nulling technique to suppress the interference from out-of-cell users [4,7,8].

However, the aforementioned study of performance analysis is only restricted to the average network success probability (SP) of the overall network. The SP is defined as the complementary cumulative distribution function (CCDF) of the signal to interference ratio (SIR) experienced by a typical user. It only reflects the average performance of the network but provides no information on

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the difference between different user links. In order to comprehensively study the network performance, the meta distribution (MD) principle is proposed in [9], which characterizes the distribution of the conditional success probability (CSP), given the realization of the BS distribution point processes. With the meta distribution, the question such as "How are the link reliabilities distributed among users in the whole network?" [10] can be answered, which are of great interest to network operators for network facility deployment.

The lack of study on the MD in networks with multi-antenna equipped considering interference nulling (IN) motivates our study of this paper. In this paper, we consider the SIR MD performance of multi-antenna multi-user networks with IN. We first illustrate the network model and IN scheme studied in this paper. Then, we derive an approximation on the SIR MD of the network architecture. Since it is difficult to calculate the expression for the SIR MD, we derive an approximated expression for it based on the beta distribution approximation and the first moment and the second moment on a tight upper bound of the CSP. The tightness of the approximated expression is verified using numerical simulations. Moreover, the impacts of some system design parameters on the network performance from the perspectives of distribution of individual link reliability are revealed using the SIR MD.

The rest of this paper is organized as follows. Section 2 present the system model investigated in this paper. Section 3 presents the main results of this paper, i.e., the expression for the SIR MD. The numerical results are provided in Sect. 4, and the conclusions are drawn in Sect. 5.

2 System Model

2.1 Network and Channel Models

In this paper, we consider a multi-antenna network, where BSs equipped with multiple antennas are distributed according to a homogeneous Poisson point process (HPPP) Φ with density λ in $\mathbb{R}^2$. Each BS is equipped with M antennas, with transmit power P, and can simultaneously serve U users, where $U \leq M$, over one time-frequency block. Single-antenna users are also distributed as a HPPP Φ_u in $\mathbb{R}^2$ independent of Φ, with density λ_u considerably lager than that of the BSs, i.e., $\lambda_u \gg \lambda$, so that each BS always has at least U users connecting to it. We consider the distance-based association mechanism where each user will connect to its nearest BS, as in [11,12]. The distance between a user and its serving base station is called the *serving distance*. Based on the Slivnyak's theorem [13], we focus on the performance analysis of the origin point, at where we assume a typical user u_0 locates.

For the wireless channel, we consider both path loss and small-scale fading. Consider a randomly chosen user i served by its serving BS located at x. The path loss is modeled by the power-law model, and can be written as $Z^{-\frac{\alpha}{2}}$, where Z denotes the serving distance of the user i, and $\alpha > 2$ denotes the path loss exponent. We assume Rayleigh fading channels, so that the small-scale fading between the user i and its serving BS x can be written as $\mathbf{h}_{ix} \in \mathcal{CN}(\mathbf{0}_{M\times 1}, \mathbf{I}_M)$.

In the multi-user multi-antenna system considered in this paper, we reserve $L \in \mathbb{N}$, $L \leq M - U$ spatial DoF for interference nulling. Each BS endeavors to cancel its interference at users that send IN requests to it. For a BS, the number of interference nulling requests it receives is denoted by Θ_r. Then, due to the limited antenna resource, over a given time-frequency resource block, a BS will adopt zero-forcing beamforming (ZFBF) to serve U users. In the meanwhile, its interference at other $\min\{\Theta_r, L\}$ users is also suppressed. For each BS, we assume perfect channel state information (CSI) is available. Each of the U associated users is allocated with equal power. With ZFBF, considering a BS located at x_0, the channel gain of the desired signal from x_0 to its served user i is denoted by g_{i0}, which follows $g_{i0} \overset{d}{\sim} \Gamma(M - U + 1 - \min\{\Theta_r, L\}, 1)$ [12]. Denote by g_x the interfering channel between user i and its interfering BS x, which follows $g_x \overset{d}{\sim} \Gamma(U, 1)$ [12].

In this work, the interference-limited scenario is considered, so that the additive white noise is neglected. Therefore, the received SIR at a user i the BS x_0 serves can be given by

$$\Upsilon = \frac{g_{i0} Z^{-\alpha}}{\sum_{x \in \Phi'} g_x \|x\|^{-\alpha}}, \tag{1}$$

where Z is the serving distance of the user i, and Φ' denotes the set consisting of all BSs that exert interference at user i.

2.2 The Interference Nulling Scheme

In this paper, a multi-antenna BS simultaneously serves U users over one time-frequency resource block while suppressing its interference at other $\min\{\Theta_r, L\}$ users. To achieve interference nulling, a user sends IN requests to all nearby base stations that are within the *IN range*, denoted as R_c. Once a BS receives the IN requests, it will uniformly randomly select up to L users to mitigate its interference. The remaining DoF are reserved to enhance spatial diversity for the U served users, thereby boosting their desired signal power. We refer to L as the *maximum IN DoF*.

Based on the aforementioned IN scheme, we observe that for the typical user u_0, the relation between its serving distance Z and R_c impacts the distribution of its interfering BSs. Specifically, if $R_c \geq Z$, the interfering base stations of u_0 can be categorized into two types: 1) those located within the annular region between radii Z and R_c, these interfering BSs have received the IN request from u_0, but are unable to satisfy it; and 2) the interfering base stations located outside the circle with radius R_c. Whereas, if $R_c < Z$, no interfering BS will receive the IN request from u_0, since the IN range is smaller than the serving distance of u_0, in this case, all the base stations located outside the circle of radius Z will exert interference to u_0. For the ease of notation, we define two sets Φ_a, Φ_b as

$$\Phi_i \triangleq \{x | x \in \Phi \backslash \{x_0\}, \|x\| \in \Omega_i\}, \quad i \in \{a, b\}, \tag{2}$$

corresponding to the above two cases, where x_0 denotes the serving BS of u_0, and

$$\begin{aligned} \Omega_a &= [\min\{Z, R_c\}, \max\{Z, R_c\}), \\ \Omega_b &= [\max\{Z, R_c\}, +\infty), \end{aligned} \tag{3}$$

are two distance intervals. Note that if $R_c < Z$, the set Φ_a is empty.

3 The SIR Meta Distribution Analysis of Multi-antenna Networks

3.1 Auxiliary Results

From the described system model, we observe that the network performance is influenced by the number of IN requests received by a user's serving base station. Similar to [7,8,14], we assume that: 1) all served users in the network form a homogeneous Poisson point process Φ'_u, which is derived from Φ_u, and is independent of the BS distribution Φ; 2) the number of IN requests received by different BSs are mutually independent. With these two assumptions, the dependence of the distributions of the served users and BSs as well as that of Θ_r of different BSs are eliminated, so that the derivation of the expressions for performance metrics can be facilitated. The accuracy of these assumptions will be verified with simulations later. Then, based on these assumptions, we can use the Poisson distribution to approximately model the random variable Θ_r, and the PMF of this variable can be written as

$$\mathbb{P}\left[\Theta_r = \theta_r\right] \approx \frac{\bar{\Theta}(U, R_c)^{\theta_r}}{\theta_r!} \exp\left(-\bar{\Theta}(U, R_c)\right), \tag{4}$$

where $\bar{\Theta}(U, R_c)$ is the average number of IN requests received by a BS, and can be given by

$$\bar{\Theta}(U, R_c) = U\left(\pi\lambda R_c^2 + e^{-\pi\lambda R_c^2} - 1\right). \tag{5}$$

Proof. For an arbitrarily selected BS B_0, there are averagely $\pi R_c^2 U\lambda$ users served within the circle of radius R_c with B_0 as the center. Among these users, only those not served by B_0 will send IN requests to B_0. Consider a randomly selected user with serving distance Z. The probability that Z is smaller than R_c can be given by

$$\mathbb{P}[Z \le R_c] = \int_0^{R_c} f_Z(z)\mathrm{d}z = 1 - e^{-\pi\lambda R_c^2}, \tag{6}$$

where $f_Z(z) = 2\pi\lambda z \exp(-\pi\lambda z^2)$ is the PDF of Z. Therefore, for all the U users served by B_0, there are averagely $U\mathbb{P}[Z \le R_c]$ users which are located within the circle with radius R_c. These users do not send any interference nulling request to B_0. Thus, the average number of interference nulling requests received by B_0 is

$$\bar{\Theta}(U,R_c)=\pi R_c^2 U\lambda - U\mathbb{P}[Z\le R_c]=U\left(\pi\lambda R_c^2+e^{-\pi\lambda R_c^2}-1\right), \tag{7}$$

which finishes the proof.

Due to the limitation of the antenna resource, up to L IN requests can be satisfied by each BS. Denote by $\Theta_s \in \{0,1,\cdots,L\}$ the number of interference nulling requests *satisfied* by a BS. Then, for a BS, if $\Theta < L$, $\Theta_s = \Theta_r$; otherwise, $\Theta_s = L$. Therefore, the PMF of Θ_s can be written by

$$\mathbb{P}\left[\Theta_s=\theta\right]=\begin{cases}\mathbb{P}\left[\Theta_r=\theta\right], & 0\le\theta\le L-1,\\ \sum_{\theta'=L}^{\infty}\mathbb{P}\left[\Theta_r=\theta'\right], & \theta=L.\end{cases} \tag{8}$$

Let $\varepsilon(U,L,R_c)$ represent the probability that a base station has received the IN request from the typical user u_0 but fails to suppress its interference, and is referred to as the *IN missing probability*. Consider a randomly chosen interfering BS from the IN range of u_0. Suppose it receives Θ_r' more IN requests besides the one from u_0. If $\Theta_r'+1\le L$, the request from u_0 will always be satisfied; otherwise, if $\Theta_r'+1>L$, the request from u_0 will not be satisfied with probability $1-\frac{L}{\Theta_r'+1}$. Since the decision of a served user to send an IN request to its interfering BSs is independent of other users, given the BS has received the IN request from u_0, Θ_r' has the same Poisson distribution with PMF in (4). Hence, we have

$$\begin{aligned}\varepsilon(U,L,R_c)&=\sum_{\theta=L}^{\infty}\left.\frac{\Theta_r'+1-L}{\Theta_r'+1}\right|_{\Theta_r'=\theta}\mathbb{P}\left[\Theta_r'=\theta\right]\\&=\sum_{\theta=L}^{\infty}\frac{\theta+1-L}{(\theta+1)!}\bar{\Theta}(U,R_c)^{\theta}\exp\left(-\bar{\Theta}(U,R_c)\right),\end{aligned} \tag{9}$$

with $\bar{\Theta}(U,R_c)$ given by (5).

Considering the IN missing probability, the densities of the interfering BSs in Φ_a and Φ_b denoted by λ_a and λ_b, are respectively given by

$$\lambda_a=\begin{cases}\varepsilon(U,L,R_c)\lambda & Z<R_c,\\ 0 & Z\ge R_c,\end{cases}\qquad \lambda_b=\lambda. \tag{10}$$

3.2 Approximation on the SIR Meta Distribution

Before we derive the approximation on the SIR MD, we first present several definitions. For a network where the location distribution of BSs Φ is given, the CSP of the typical user u_0 is defined as follows.

Definition 1 (Conditional Success Probability). *Given the location distribution of the BSs Φ, the conditional success probability of u_0 is defined as*

$$P_s(\tau)\triangleq\mathbb{P}\left[\Upsilon\ge\tau\mid\Phi\right]. \tag{11}$$

where Υ is the received SIR of u_0, and τ is a predefined SIR threshold.

In the network model this paper considers, the SIR model takes the randomness of the point process and channel fading into consideration. However, the CSP only takes expectation over the channel fading with a given Φ. Therefore, the CSP reflects the SP of each link with a given realization of network point process, and can capture the performance of link reliability of individual links in a network. Therefore, in the rest of this paper, we may also refer to the CSP given in (11) as the *link reliability*.

Definition 2 (SIR Meta Distribution). *The SIR meta distribution is the CCDF of the CSP $P_s(\tau)$, i.e.,*

$$\bar{F}_{P_s}(x) \triangleq \mathbb{P}\left[P_s(\tau) > x\right], \quad x \in [0,1], \tag{12}$$

where x denotes some predefined link reliability threshold.

Given that the point processes of BSs and users, Φ and Φ_u, are ergodic, $\bar{F}_{P_s}(x)$represents the proportion of users whose link reliability exceeds the threshold x in each network realization. This metric reveals the distribution of per-link reliability across the network.

From [15], we observe that the exact expression for the SIR MD can be obtained using the Gil-Pelaez theorem:

$$\bar{F}_{P_s}(x) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\Im\left(e^{-jt\log x} M_{jt}\right)}{t}\,\mathrm{d}t, \quad x \in [0,1], \tag{13}$$

where $j \triangleq \sqrt{-1}$; $M_i = \mathbb{E}\left[P_s(\tau)^i\right]$ represents the i-th moment of the CSP $P_s(\tau)$, M_{jt} is the imaginary moment of $P_s(\tau)$; and $\Im(s)$ returns the imaginary part of $s \in \mathbb{C}$.

Due to the extreme complexity of the Gil-Pelaez theorem in (13), it is usually prohibitive to calculate the exact value of the SIR meta distribution. Therefore, we adopt a commonly used beta approximation to obtain an excellent approximation for the SIR MD, as in [9,16]. Specifically, the first moment M_1 and the second moment M_2 of the CSP are calculated to match the first moment and the second moment of the beta distribution. Consequently, the SIR MD can be approximated by

$$\bar{F}_{P_s}(x) \approx 1 - I_x\left(\frac{M_1\kappa}{1-M_1}, \kappa\right), \quad x \in [0,1], \tag{14}$$

where $\kappa = \frac{(M_1-M_2)(1-M_{1,n})}{(M_2-(M_1)^2)}$; $I_x(a,b)$ is the regularized incomplete Beta function, i.e., $I_x(a,b) = \frac{\int_0^x t^{a-1}(1-t)^{b-1}\,\mathrm{d}t}{B(a,b)}$ with parameters $a,b > 0$; and $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\,\mathrm{d}t$ is the Beta function.

From (14), we observe that by calculating the first and second moments on the CSP, i.e., M_1 and M_2, the SIR MD can be approximated by the Beta function. However, for the multi-antenna system considered in this work, the exact expression for M_2 is prohibitive to obtain. Therefore, we would like to consider a tractable upper bound on the CSP, which is denoted by $P_s^u(\tau)$, for the approximation on the SIR MD.

Lemma 1 (An Upper Bound on the CSP $P_s(\tau)$). *An upper bound on the CSP $P_s(\tau)$ can be given by*

$$P_s^u(\tau) = \sum_{\theta=0}^{L} \mathbb{P}\left[\Theta_s = \theta\right] \sum_{i=1}^{D} (-1)^{i+1} \binom{D}{i} \prod_{x \in \Phi_a \cup \Phi_b} \left(1 + i\beta\tau Z^{\alpha} \|x\|^{-\alpha}\right)^{-U}, \quad (15)$$

where $D = M - U + 1 - \theta$, $\beta = (D!)^{-1/D}$ and Z is the serving distance of the typical user under a realization of Φ.

Proof. This proof relies on a lower bound for the incomplete Gamma function, specifically, for a Gamma distributed random variable $X \overset{d}{\sim} \Gamma(D, 1)$, the CCDF of X is $\mathbb{P}[X > x] = 1 - \frac{\gamma(D,x)}{\Gamma(D)})$, and a lower bound on $\frac{\gamma(D,x)}{\Gamma(D)}$ is $\left(1 - e^{-\beta x}\right)^D \leq \frac{\gamma(D,x)}{\Gamma(D)}$, with $\beta = ((D)!)^{-1/(M-\theta)}$. Based on this bound, conditioning on $\Theta_s = \theta$, the upper bound on the CSP can be derived as

$$\begin{aligned}
&\mathbb{P}\left[\Upsilon \geq \tau \mid \Phi, \Theta_s = \theta\right] \\
&= \mathbb{E}_I\left[\mathbb{P}\left[g_{00} \geq \tau Z^{\alpha} I \mid \Phi, \Theta_s = \theta\right]\right] \\
&\overset{(a)}{\leq} 1 - \mathbb{E}_I\left[\left(1 - \exp\left(-\beta\tau Z^{\alpha} I\right)\right)^D\right] \\
&= 1 - \mathbb{E}_I\left[\sum_{i=0}^{D} (-1)^i \binom{D}{i} \exp\left(-i\beta\tau Z^{\alpha} I\right)\right] \\
&= \sum_{i=1}^{D} (-1)^{i+1} \binom{D}{i} \mathbb{E}_{g_x}\left[\exp\left(-i\beta\tau Z^{\alpha} \sum_{x \in \Phi_a \cup \Phi_b} g_x \|x\|^{-\alpha}\right)\right] \\
&= \sum_{i=1}^{D} (-1)^{i+1} \binom{D}{i} \prod_{x \in \Phi_a \cup \Phi_b} \mathbb{E}_{g_x}\left[\exp\left(-i\beta\tau Z^{\alpha} g_x \|x\|^{-\alpha}\right)\right] \\
&\overset{(b)}{=} \sum_{i=1}^{D} (-1)^{i+1} \binom{D}{i} \prod_{x \in \Phi_a \cup \Phi_b} \left(1 + i\beta\tau Z^{\alpha} \|x\|^{-\alpha}\right)^{-U} \\
&\triangleq P_{s,\Theta_s}^u(\tau, \theta),
\end{aligned} \quad (16)$$

where Z is the serving distance of u_0, with a realization of the BS distribution Φ; (a) is from the Gamma distributed wireless channel $g_{00} \overset{d}{\sim} \Gamma(D, 1)$ with $D = M - U + 1 - \theta$; $\beta = (D!)^{-1/D}$; (b) is derived by taking the expectation over the Gamma-distributed variable $g_x \overset{d}{\sim} \Gamma(U, 1)$.

Considering the total probability theorem, we have $P_s^u(\tau) = \sum_{\theta=0}^{L} \mathbb{P}[\Theta_s = \theta] \times P_{s,\Theta_s}^u(\tau, \theta)$, which finishes the proof.

Next, we would like to derive the first moment and second moment of the upper bound $P_s^u(\tau)$.

Theorem 1 (The First and Second Moments on $P_s^u(\tau)$). *The first moment on $P_s^u(\tau)$ can be given by*

$$M_1^u = \sum_{\theta=0}^{L} \mathbb{P}[\Theta_s = \theta] \sum_{i=1}^{D} (-1)^{i+1} \binom{D}{i} \times \int_0^\infty \exp\left(-\pi\lambda\left(az^2 F(i\beta\tau) + bR_c^2 F\left(i\beta\tau\left(\frac{z}{R_c}\right)^\alpha\right)\right)\right) f_Z(z)\mathrm{d}z, \quad (17)$$

where $\mathbb{P}[\Theta_s = \theta]$ is given by (8); $D = M - U + 1 - \theta$, $\beta = (D!)^{-1/D}$; $f_Z(z) = 2\pi\lambda z \exp(-\pi\lambda z^2)$ denotes the PDF of distance Z; a, b, and $F(x)$ are given as

$$a = \begin{cases} \varepsilon(U, L, R_c) & Z \le R_c, \\ 1 & Z > R_c, \end{cases} \quad b = \begin{cases} 1 - \varepsilon(U, L, R_c) & Z \le R_c, \\ 0 & Z > R_c, \end{cases} \quad (18)$$

with $\varepsilon(U, L, R_c)$ given by (9), and

$$F(x) = {}_2F_1\left(-\frac{2}{\alpha}, U; 1 - \frac{2}{\alpha}; -x\right) - 1, \quad (19)$$

where ${}_2F_1(a, b; c; d)$ denotes the Gauss hypergeometric function.

The second moment on $P_s^u(\tau)$ can be given by

$$M_2^u = \sum_{\theta_1=0}^{L} \sum_{\theta_2=0}^{L} \sum_{i=1}^{D_1} \sum_{j=1}^{D_2} \mathbb{P}[\Theta_s = \theta_1] \mathbb{P}[\Theta_s = \theta_2] (-1)^{i+j} \binom{D_1}{i} \binom{D_2}{j} \times \int_0^\infty f_Z(z) \exp\left(-2\pi \sum_{k=a}^{b} \lambda_k \mathcal{I}_{ij}(\Omega_k, \beta_1\tau z^\alpha, \beta_2\tau z^\alpha)\right) \mathrm{d}z, \quad (20)$$

where λ_k and Ω_k, $k \in \{a, b\}$, are given by (10) and (3), respectively; and $\mathcal{I}_{ij}(\Omega, x, y)$ is given by

$$\mathcal{I}_{ij}(\Omega, x, y) \triangleq \int_\Omega \left(1 - \left(1 + ixv^{-\alpha}\right)^{-U} \left(1 + jyv^{-\alpha}\right)^{-U}\right) v\mathrm{d}v. \quad (21)$$

From the expression above, we can observe that $\mathcal{I}_{ij}(\Omega, x, y) = \mathcal{I}_{ji}(\Omega, y, x)$.

Proof. With the total probability theorem, we have

$$
\begin{aligned}
M_1^u &= \mathbb{E}\left[\sum_{\theta=0}^{L} \mathbb{P}\left[\Theta_s = \theta\right] P^u_{s,\Theta_s}(\tau,\theta)\right] \\
&= \sum_{\theta=0}^{L} \mathbb{P}\left[\Theta_s = \theta\right] \sum_{i=1}^{D} (-1)^{i+1} \binom{D}{i} \mathbb{E}_{Z,\Phi_a,\Phi_b}\left[\prod_{x\in\Phi_a\cup\Phi_b} \left(1 + i\beta\tau Z^{\alpha}\|x\|^{-\alpha}\right)^{-U}\right] \\
&= \sum_{\theta=0}^{L} \mathbb{P}\left[\Theta_s = \theta\right] \sum_{i=1}^{D} (-1)^{i+1} \binom{D}{i} \\
&\quad \times \int_0^{\infty} f_Z(z) \exp\left(- 2\pi \sum_{k=a}^{b} \lambda_k \int_{\Omega_k} \left(1 - \left(1 + i\beta\tau z^{\alpha} v^{-\alpha}\right)^{-U}\right) v \mathrm{d}v\right) \mathrm{d}z \\
&= \sum_{\theta=0}^{L} \mathbb{P}\left[\Theta_s = \theta\right] \sum_{i=1}^{D} (-1)^{i+1} \binom{D}{i} \int_0^{\infty} f_Z(z) \exp\left(\chi(s)\right)\Big|_{s=i\beta\tau z^{\alpha}} \mathrm{d}z,
\end{aligned} \tag{22}
$$

where $P^u_{s,\Theta_s}(\tau,\theta)$ is defined by (16). Following some algebraic manipulation, we can derive the final expression as shown in (17).

For the second moment, we have

$$
\begin{aligned}
M_2^u &= \mathbb{E}\left[\left(\sum_{\theta=0}^{L} \mathbb{P}\left[\Theta_s = \theta\right] P^u_{s,\Theta_s}(\tau,\theta)\right)^2\right] \\
&= \int_0^{\infty} \mathbb{E}_{\Phi_a,\Phi_b}\left[\left(\sum_{\theta=0}^{L} \mathbb{P}\left[\Theta_s = \theta\right] P^u_{s,\Theta_s,Z}(\tau,\theta,z)\right)^2\right] f_Z(z)\mathrm{d}z,
\end{aligned} \tag{23}
$$

where $P^u_{s,\Theta_s,Z}(\tau,\theta,z)$ is the upper bound of the CSP conditioning on Φ, $\Theta_s = \theta$, and $Z = z$. Then, we have

$$
\begin{aligned}
&\mathbb{E}_{\Phi_a,\Phi_b}\left[\left(\sum_{\theta=0}^{L} \mathbb{P}\left[\Theta_s = \theta\right] P^u_{s,\Theta_s,Z}(\tau,\theta,z)\right)^2\right] \\
&= \mathbb{E}_{\Phi_a,\Phi_b}\left[\sum_{\theta_1=0}^{L} \sum_{\theta_2=0}^{L} \mathbb{P}\left[\Theta_s = \theta_1\right] \mathbb{P}\left[\Theta_s = \theta_2\right] P^u_{s,\Theta_s,Z}(\tau,\theta_1,z) P^u_{s,\Theta_s,Z}(\tau,\theta_2,z)\right] \\
&= \sum_{\theta_1=0}^{L} \sum_{\theta_2=0}^{L} \mathbb{P}\left[\Theta_s = \theta_1\right] \mathbb{P}\left[\Theta_s = \theta_2\right] \mathbb{E}_{\Phi_a,\Phi_b}\left[P^u_{s,\Theta_s,Z}(\tau,\theta_1,z) P^u_{s,\Theta_s,Z}(\tau,\theta_2,z)\right],
\end{aligned} \tag{24}
$$

where

$$
\begin{aligned}
&\mathbb{E}_{\Phi_a,\Phi_b}\left[P^u_{s,\Theta_I,Z}(\tau,\theta_1,z)P^u_{s,\Theta_I,Z}(\tau,\theta_2,z)\right]\\
&=\mathbb{E}_{\Phi_a,\Phi_b}\Bigg[\Bigg(\sum_{i=1}^{D_1}(-1)^{i+1}\binom{D_1}{i}\prod_{x\in\Phi_a\cup\Phi_b}\frac{1}{(1+i\beta_1\tau z^\alpha\|x\|^{-\alpha})^U}\Bigg)\\
&\qquad\times\Bigg(\sum_{j=1}^{D_2}(-1)^{j+1}\binom{D_2}{j}\prod_{x\in\Phi_a\cup\Phi_b}\frac{1}{(1+i\beta_2\tau z^\alpha\|x\|^{-\alpha})^U}\Bigg)\Bigg]\\
&=\sum_{i=1}^{D_1}\sum_{j=1}^{D_2}(-1)^{i+j}\binom{D_1}{i}\binom{D_2}{j}\\
&\qquad\times\mathbb{E}_{\Phi_a,\Phi_b}\Bigg[\prod_{x\in\Phi_a\cup\Phi_b}\frac{1}{(1+i\beta_1\tau z^\alpha\|x\|^{-\alpha})^U(1+j\beta_2\tau z^\alpha\|x\|^{-\alpha})^U}\Bigg]\\
&=\sum_{i=1}^{D_1}\sum_{j=1}^{D_2}(-1)^{i+j}\binom{D_1}{i}\binom{D_2}{j}\exp\Bigg(-2\pi\sum_{k=a}^{b}\lambda_k\\
&\qquad\times\int_{\Omega_k}\Bigg(1-\frac{1}{(1+i\beta_1\tau z^\alpha\|x\|^{-\alpha})^U(1+j\beta_2\tau z^\alpha\|x\|^{-\alpha})^U}\Bigg)v\mathrm{d}v\Bigg)\\
&=\sum_{i=1}^{D_1}\sum_{j=1}^{D_2}(-1)^{i+j}\binom{D_1}{i}\binom{D_2}{j}\exp\Bigg(-2\pi\sum_{k=a}^{b}\lambda_k\mathcal{I}_{ij}(\Omega_k,\beta_1\tau z^\alpha,\beta_2\tau z^\alpha)\Bigg). \qquad (25)
\end{aligned}
$$

Thus, the proof is finished.

By replacing M_1 and M_2 in (14) with M_1^u and M_2^u given in Theorem 1, we can obtain an approximation of the SIR meta distribution of the network. In the sequel, we will use this approximation to anaylze the distribution of the link reliability of the network.

4 Simulation Results

In this section, we present simulation results to exam the obtained SIR MD expression and further provide some system design insights. Unless otherwise stated, we take the following settings as the default: $M=8$, $U=3$, $L=3$, $R_c=82.37\,\mathrm{m}$, $P=46\,\mathrm{dBm}$, $\alpha=4$, $\lambda=1\times10^{-4}\,\mathrm{m}^{-2}$, $\lambda_u=8\times10^{-4}\,\mathrm{m}^{-2}$, $\tau=0\,\mathrm{dB}$.

To implement the Monte Carlo simulation, first, we generate 10^4 realizations of the distribution of the BSs Φ and those of the users Φ_u. In each realization, the interference nulling requests for each BS and the user association are determined based on the IN scheme described in Sect. 2. Then, we further generate 10^4 realizations of the small-scale fading channel for each realization of Φ and Φ_u, so that the randomness of the performance of the individual link can be simulated with the given locations of BSs and users. By averaging the results of these 10^4

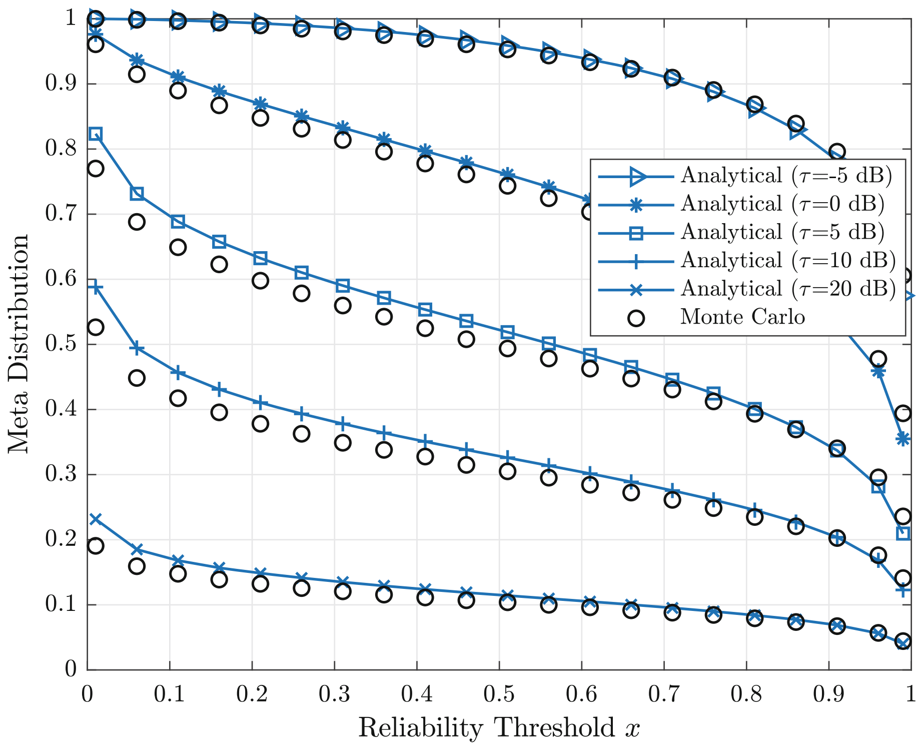

Fig. 1. Verification of the theoretical results.

simulations, the CSP can be obtained for one realization of Φ and Φ_u. Overall, we can obtain 10^4 different results of CSP; and the SIR meta distribution of the network can be obtained by calculating the CCDF of these results.

Figure 1 verify the analytical results of the SIR MD. In Fig. 1, the "Analytical" results are from the approximation of the SIR MD, whereas the "Monte Carlo" are from the real CSP. From the figure, we observe that the "Analytical" results match the "Monte Carlo" results well, indicating that the approximations of the SIR MD are feasible for performance evaluation under the considered network scenario. Furthermore, from the figure, we can also observe that given a value of x, the percentage of users with link reliability of at least x decreases with the increase of the SIR threshold τ. For instance, when $x = 0.5$, the percentages of users with link reliability of at least 0.5 when $\tau = 0\,\text{dB}$ and $\tau = 20\,\text{dB}$ are about 75% and 10%, respectively.

Figure 2 presents the results of SIR MD with different values of L when $U = 3$. From the figure, we observe that with the decrease of L, the average performance of the network (which is reflected by the area of the graph under each curve) increases, which indicates that with current parameter settings, the IN is not beneficial to the average network performance.

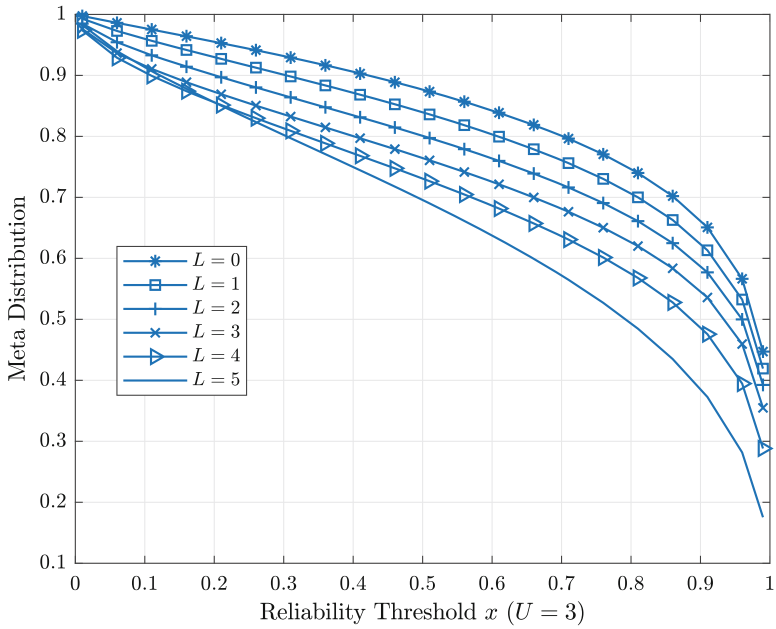

Fig. 2. The SIR MD of the IN scheme with various choices of L.

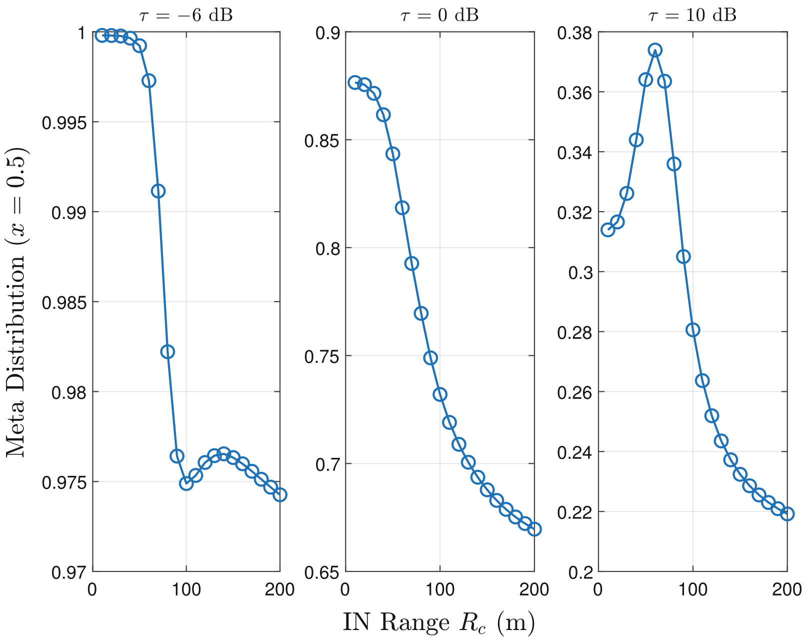

Fig. 3. The SIR MD of the IN scheme versus the IN range R_c with various choices of τ when $x = 0.5$.

Figure 3 shows the effects of the IN range R_c on the SIR MD with different SIR threshold τ when the reliability thresholds is $x = 0.5$. From the figure, we can observe that: 1) when $\tau = -6$ dB, the percentage of users with link reliability of that is greater than 0.5 first decreases, then increases, and then decreases with R_c; 2) when $\tau = 0$ dB, the percentage of users with link reliability that is greater than 0.5 decreases with R_c; 3) when $\tau = 10$ dB, the percentage of users with link reliability that is greater than 0.5 first increases and then decreases with R_c. These phenomena indicate that the value of R_c needs to be carefully selected to obtain desirable link reliability performance.

5 Conclusion

In this work, we investigated the SIR MD for a multi-antenna multi-user network where interference nulling is conducted. Since it is difficult to calculate the exact expression of the SIR MD, we focused on an upper bound on the CSP and derived the first and second moments on it. With further using the beta approximation, we obtained an approximated expression for the SIR MD. We then validated the accuracy of the expression using numerical simulations, and analyzed the impact of system design parameters on the performance from the perspectives of individual link reliability distribution.

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