Finite Time Trajectory Tracking of a Mobile Robot Using Cascaded Terminal Sliding Mode Control Under the Presence of Random Gaussian Disturbance

Adisu Safo Bosera[1(✉)], Ayodeji Olalekan Salau[2], Asrat Gedefa Yadessa[1], and Kaheli Anteneh Jembere[1]

[1] Department of Electrical and Computer Engineering, Mettu University, Mettu, Ethiopia
addisuusaafoo@gmail.com

[2] Department of Electrical/Electronics and Computer Engineering, Afe Babalola University, Ado-Ekiti, Nigeria

Abstract. In this paper, dynamic modeling of a differential drive mobile robot (DDMR) using Langrage formulation and terminal sliding mode trajectory tracking control is presented. The proposed controller is a cascaded controller designed to improve the dynamic response of the system, i.e. kinematic and dynamic problems, asymptotical convergence, and chattering problem using terminal sliding mode control (TSMC). The terminal sliding mode control provides faster convergence and higher-precision control than the conventional linear hyperplane sliding control which guarantees the asymptotic stability. This is due to fact that the terminal sliding mode control system guarantees a finite time convergence to the sliding phase. The entire control design consists of an outer loop kinematics control and inner loop speed control system. Here, outer kinematic control system provides an appropriate velocity control input for the inner loop angular velocity control of each wheel. An angular and linear velocity control input is designed in order to make angular and posture error to converge to zero in a finite time based on global fast terminal sliding mode control (GFTSMC). Then, the inner loop GFTSMC of the robot is designed to ensure that the tracking error between the actual and desired angular velocity of each wheels converges to zero in a finite time. Both the inner and outer closed loop controllers achieve path following in a finite time and avoids high frequency switching in the closed loop such that the overall dynamic response of the system is improved using the cascaded control technique and the stability of each controller was checked using lyapunov criteria. Generally, the proposed control system shows the performance and effectiveness of the proposed method compared to conventional SMC, and the simulation results indicate good convergence and robustness of the system for circular trajectories under both model uncertainty and random Gaussian disturbances using GFTSMC.

Keywords: Differential drive mobile robot · Langrage formulation approach · TSMC · Finite time · Trajectory tracking

B. H. Woldegiorgis et al. (Eds.): ICAST 2022, LNICST 455, pp. 63–78, 2023.
https://doi.org/10.1007/978-3-031-28725-1_5

1 Introduction

Wheeled mobile robotics is one of the most developed fields in robotics, having many uses in different fields like entertainment, military, and exploration. Mobile robots have caught the interest of many researchers in robotic control because of their complications in control schemes and practice applications [1]. Non-holonomic constraint is a feature of a wheeled mobile robot. The problem of tracking control of a mobile robot has been addressed by a number of control approaches [2, 3]. Two models (kinematic and dynamic) are used to categorize research approaches in two axes and the tracking control problem as a kinematic or dynamic challenge [4]. In [5], a navigation system slide mode control was proposed for a mobile robot, but in the paper, the influence of dynamics of the mobile robot was not well thought-out and only steering system (kinematic problem) controller was designed. A back stepping controller for a Wheeled Mobile Robot was presented by the authors in [6]. The kinematic control unit's job was to provide velocity outputs to the robot, which assisted to keep the posture mistakes to a minimum. The response, on the other hand, was slow. Due to the large amount of difficult computations, the control has a reduced reaction time [7]. Authors in [8] designed a cascaded control system that has a master feedback kinematic and slave feedback dynamics based PID with the translational velocity kept constant and only the robot's rotational velocity controlled, i.e., the robot's pose is controlled by varying the rotational velocity alone, resulting in a low response time. In [9], a traditional sliding mode control (SMC) system which used Gao's reaching method was implemented for a mobile robot, while a fuzzy kinematic controller with dynamic proportional integral control was designed for a mobile robot in [10]. The results show that the controller has finite time convergence problems.

In this paper, a controller is proposed to eliminate the chattering occurrence and asymptotical convergence in traditional sliding mode control systems by using cascaded robust finite time global fast terminal SMC for trajectory tracking of a differential drive mobile robot. Furthermore, accuracy and effectiveness of the proposed control system is evaluated using different performance index such as ITSE, IAE and ITAE and compared to conventional SMC.

This remaining sections of the paper are structured as follows. Section 2 describes the kinematic model of the differential robot and proposed Langrage formulation. Section 3 describes both the GFTSMC and SMC controller design. Section 4 presents the analysis and simulation results, and Sect. 5 concludes the paper.

2 Kinematic and Dynamic Model of DDMR

In this section, both kinematic model and dynamic model of differential drive mobile robot was derived.

2.1 Kinematic model of DDMR

We consider the following a differential drive mobile robot which has two wheels and one castor wheel at front. Geometrical parameters of the differential drive mobile robot

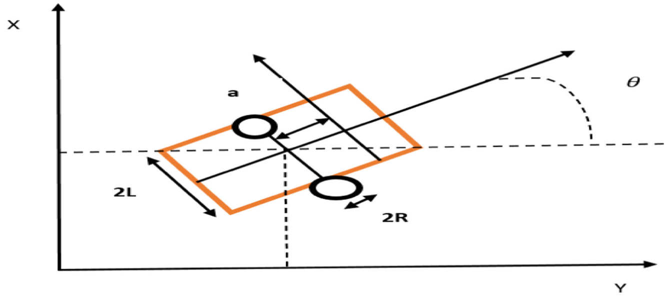

Fig. 1. Posture description of DDMRM

is specified such that the radius of R is placed with a distance L from the center of the mobile robot which illustrated in Fig. 1.

In Fig. 1, let's assume that the robot can never slide which makes the robots movement along y axis always zeros, i.e. $\dot{Y}_r = 0$. Since, the robot is subjected to Nonholonomic constraints, namely: pure rolling constraints and no side move (forward and backward) motion only. A two different coordinate system (frames) must be defined to characterize the WMR's position in its environment such as world coordinate and robot coordinate system.

The linear velocity of the robot in its robot coordinate system frame is:

$$\dot{X}_r = \frac{R\dot{\Phi}_r}{2} + \frac{R\dot{\Phi}_l}{2} \tag{1}$$

Given the speeds $\dot{\Phi}_r$ and $\dot{\Phi}_l$ of the right and left wheels, respectively, the rotational velocity is expressed as:

$$\dot{\theta} = \frac{R\dot{\Phi}_r}{2} + \frac{R\dot{\Phi}_l}{2} \tag{2}$$

The differential drive mobile robot's velocities can be obtained also in the inertial frame (world frame) with respect to the robot coordinate system as follows using the rotational matrix along z axis:

$$\begin{pmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta}_I \end{pmatrix} = \begin{pmatrix} \frac{R\cos\theta}{2} & \frac{R\cos\theta}{2} \\ \frac{R\sin\theta}{2} & \frac{R\sin\theta}{2} \\ \frac{R}{2L} & -\frac{R}{2L} \end{pmatrix} \begin{pmatrix} \dot{\Phi}_r \\ \dot{\Phi}_l \end{pmatrix} \tag{3}$$

The relationship of linear and angular velocities to angular velocities of the wheels are given by Eqs. (1) and (2).

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = M \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} \tag{4}$$

where, $M = \begin{bmatrix} \frac{R}{2} & \frac{R}{2} \\ \frac{R}{2L} & -\frac{R}{2L} \end{bmatrix}$ which forward kinematics of DDMR.

2.2 Langrange Formulation of Dynamic Model of DDMR

We begin by describing the Lagrangain (L) of the mobile robot as the difference of its kinetic energy (T) and potential energy (U) [11].

The kinetic energy of the robot is expressed as:

$$T_c = \frac{1}{2}m_c v_c^2 + \frac{1}{2}I_c\dot{\theta}^2 \tag{5}$$

While the kinetic energy of the right and left wheel is expressed as:

$$T_{wr} = \frac{1}{2}m_w v_{wr}^2 + \frac{1}{2}I_m\dot{\theta}^2 + \frac{1}{2}I_w\dot{\Phi}_r^2 \tag{6}$$

$$T_{wl} = \frac{1}{2}m_w v_{wl}^2 + \frac{1}{2}I_m\dot{\theta}^2 + \frac{1}{2}I_w\dot{\Phi}_l^2 \tag{7}$$

where, m_c is the mass of the DDMR, m_w is the mass of each driving wheel (with actuator), I_c is the moment of inertia of the DDMR about the vertical axis through the center of mass, I_w is the moment of inertia of each driving wheel with a motor about the wheel axis, and I_m is the moment of inertia of each driving wheel with a motor about the wheel diameter. All velocities are first articulated as a function of the generalized coordinates using the general velocity equation in the inertial/world frame.

$$v_i^2 = x_i^2 + y_i^2 \tag{8}$$

The kinetic energy of the system is then written conveniently by

$$T(q, q') = \frac{1}{2}m\left(x_i^2 + y_i^2\right) + m_c a\dot{\theta}\left(\cos\theta\dot{y}_I - \sin\theta\dot{x}_I\right) + \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}I_w(\dot{\Phi}_r^2 + \dot{\Phi}_l^2) \tag{9}$$

where, the following new parameters are introduced as follows, $m = m_c + 2m_w$, and $I = 2m_w D^2 + m_c a^2 + 2I_m + I_c$ and the potential energy of the DDMR is considered to be zero, because the DDMR is travelling in the x_i- y_i plane.

The dynamic equations are simplified to the following form using state space representation, we get

$$\left(I_w + \frac{R^2\left(mL^2 + I\right)}{4L^2}\right)\dot{\omega}_r + \left(\frac{r^2\left(mL^2 - I\right)}{4L^2}\right)\dot{\omega}_l + \frac{R}{L}\left(\frac{R^2 m_c a}{2L}\right)(\omega_r - \omega_l)\omega_l = \tau_r \tag{10}$$

$$\left(I_w + \frac{R^2\left(mL^2 + I\right)}{4L^2}\right)\dot{\omega}_l + \left(\frac{r^2\left(mL^2 - I\right)}{4L^2}\right)\dot{\omega}_r - \frac{R}{L}\left(\frac{R^2 m_c a}{2L}\right)(\omega_r - \omega_l)\omega_r = \tau_l \tag{11}$$

where, the angular velocities of right and left wheel (ω_r, ω_l) of the DDMR and the driving motor torques ((τ_r, τ_l) are right and left wheel torque respectively.

3 Controller Design

To have improved motion over all control performance speed control system must employed because, due to additional load, surface, dynamics of mobile robot and among others perfect velocity tracking impossible. In this case the controller structure should be split into two stages as shown Fig. 2 [13].

Inner loop used for control of angular velocity of each wheel's, depending on the mobile robot dynamics. Whereas, the outer loop position control system is used to control both the translational and rotational positions of the mobile robot steering system. The outer loop finds suitable or desired velocity control inputs, which stabilize the kinematic closed loop control.

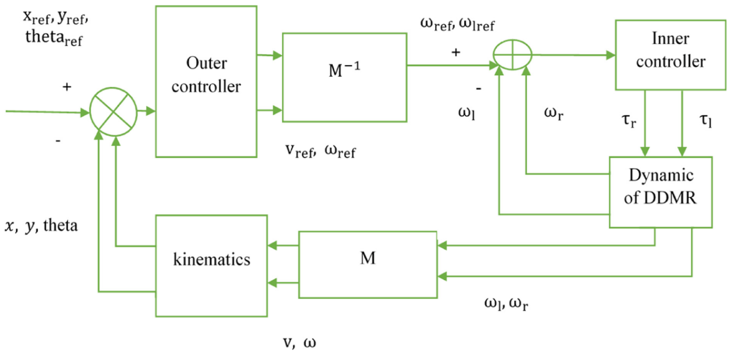

Fig. 2. Entire closed loop control system for DDMR.

3.1 Kinematic Outer Loop Global Fast Terminal Sliding Mode Controller Design

This control method is specially implemented to the kinematic differential drive mobile robot systems which are controlled by linear and angular velocity inputs.

The posture error $p_e = (x_e, y_e, \theta_e)^T$ can be expressed as using from Fig. 3, [12].

$$\begin{pmatrix} x_e \\ y_e \\ \theta_e \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{pmatrix} \tag{12}$$

The sliding surface is chosen as:

$$s = \dot{\theta}_e + \alpha\theta_e + \beta\theta_e^{\frac{q}{p}} \tag{13}$$

rearranging Eq. (13), we get

$$\dot{\theta}_e = \omega_d - \omega = -\alpha\theta_e - \beta\theta_e^{\frac{q}{p}} \tag{14}$$

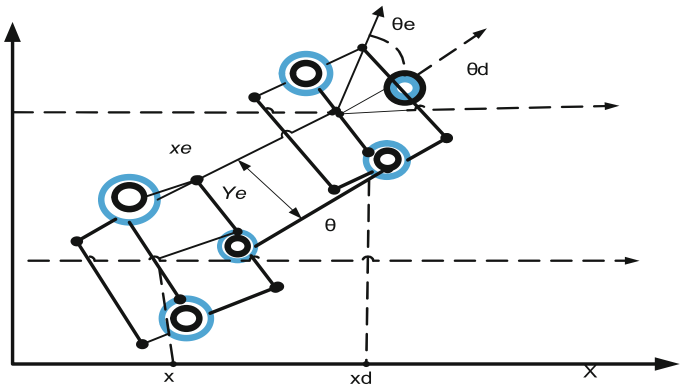

Fig. 3. Posture error description.

Then, the control law can be obtained as [14]:

$$\omega_d - \omega = -\alpha\theta_e - \beta\theta_e^{\frac{q}{p}} = \omega = \omega_d + \alpha\theta_e + \beta\theta_e^{\frac{q}{p}} \tag{15}$$

When, θ_e converges to zero, $\omega_d = \omega$.
The error model becomes to be as follows taking derivative of Eq. (12), we get

$$x_e' = y_e\omega_d - v + v_d \tag{16}$$

$$y_e' = -x_e\omega_d \tag{17}$$

By setting,

$$z = x_e - y_e \tag{18}$$

The control law can be found from the global fast terminal sliding mode as [14]:

$$\dot{z} + \alpha z + \beta z^{p/q} = 0 \tag{19}$$

So, the result can be obtained as:

$$\dot{z} = -\alpha z - \beta z^{p/q} \tag{20}$$

By using the Eqs. (16), (17), and (20), we have

$$\dot{z} = \dot{x}_e - \dot{y}_e = y\omega_d - v + v_d + x\omega_d \tag{21}$$

The control law can be obtained as:

$$\dot{z} = \dot{x}_e - \dot{y}_e = y_e\omega_d - v + v_d + x_e\omega_d \tag{22}$$

$$\dot{z} = -\alpha z - \beta z^{p/q} \tag{23}$$

The control law can be obtained as:

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} y_e\omega_d + v_d + x_e\omega_d + \alpha z + \beta z^{p/q} \\ \omega_d + \alpha\theta_e + \beta\theta_e^{\frac{q}{p}} \end{pmatrix} \tag{24}$$

Analysis of stability of the control system is performed by selecting the lyapunov function as follows

$$v_1 = 0.5\theta_e^2 \tag{25}$$

By differentiating the Eq. (25) is given as:

$$\dot{v}_1 = \theta_e\dot{\theta}_e = -\alpha\theta_e^2 - \beta\theta_e^{p/q+1} < 0 \tag{26}$$

It was observed that $v_1 > 0$ and $\dot{v}_1 < 0$, thus, the error state variables, θ_e and $\dot{\theta}_e$ are stabilized at the equilibrium point.

The posture error control system stability is proved using the lyapunov function as follows

$$v_2 = 0.5z^2 \tag{27}$$

By differentiating the Eq. (27) is given as:

$$\dot{v}_2 z\dot{z} = -\alpha z^2 - \beta z^{p/q+1} < 0 \tag{28}$$

It can be seen that, $v_2 > 0$ and $\dot{v}_2 < 0$, thus, the error state variables, z and $\dot{z}$ are stabilized at the equilibrium point.

3.2 Inner Loop Speed Control System Using Global Terminal Sliding Mode Controller Design

Considering the linear and the terminal sliding surfaces, a new global fast terminal sliding surface is proposed given by Eq. (29).

$$s = \dot{x} + \alpha x + \beta x^{\frac{q}{p}} \tag{29}$$

where, $x \in R$, $\alpha, \beta > 0$ and $p > q$ are positive odd numbers [14].

First by describing the tracking error as follows

$$\dot{e}_i = e_{i+1} \text{ and } e_i = x_{id} - x_i \tag{30}$$

where, e_i is tracking error.

By choosing the sliding surface for angular velocity of right and right wheel according to GFTSMC, we have,

$$\begin{cases} s_1 = \dot{e}_{1r} + \alpha e_{1r} + \beta e_{2r}^{\frac{q}{p}} \\ s_2 = \dot{e}_{1l} + \alpha e_{1l} + \beta e_{2l}^{\frac{q}{p}} \end{cases} \tag{31}$$

where, s_1 and s_2 sliding surface & $e_{1l} = \omega_{ld} - \omega_l$, $e_{1r} = \omega_{rd} - \omega_r$ is error b/n desired and actual angular velocity.

The control law for first order given by,

$$s_i = 0 \tag{32}$$

Therefore, the dynamic equation in Eqs. (10) and (11) using the state space representations are given by Eqs. (33) and (34).

$$\begin{cases} \dot{x}_1 = (m_1\tau_r - m_2\tau_l - m_1V(x_1 - x_2)x_1 - m_2V(x_1 - x_2)x_2)\frac{1}{m_1^2 - m_2^2} \\ \dot{x}_2 = (m_2\tau_r - m_1\tau_l - m_2V(x_1 - x_2)x_1 - m_1V(x_1 - x_2)x_2)\frac{1}{m_2^2 - m_1^2} \end{cases} \tag{33}$$

where, $m_1 = I_w + \frac{R^2(mL^2+I)}{4L^2}$, $m_2 = \frac{R^2(mL^2-I)}{4L^2}$, $V = \frac{R}{L}\left(\frac{R^2m_ca}{2L}\right)$, $x_1 = \omega_r$ and $x_2 = \omega_l$.

The linear and angular velocity speed control system using GFTSMC is given by

$$\begin{cases} u_1 = \left(m_1^2 - m_2^2\right)\left(\alpha e_1 + \beta e_2^{\frac{q}{p}} + \dot{\omega}_{rd}\right) + m_1V(x_1 - x_2)x_1 + m_2V(x_1 - x_2)x_2 \\ u_2 = \left(m_2^2 - m_1^2\right)\left(\alpha e_1 + \beta e_2^{\frac{q}{p}} + \dot{\omega}_d\right) + m_2V(x_1 - x_2)x_1 + m_1V(x_1 - x_2)x_2 \end{cases} \tag{34}$$

where, $u_1 = m_1\tau_r - m_2\tau_l$ and $u_2 = m_2\tau_r - m_1\tau_l$.

In a finite amount of time, we can bring the system state to a condition of equilibrium t_s given by such that the initial state $x(0) \neq 0$ attains at $x = 0$,

$$t_s = \frac{p}{\alpha(p-q)} \ln \frac{\alpha x(0)^{(p-q)/p} + \beta}{\beta} \tag{35}$$

By designing α, β, p, q [15].

3.3 Inner Loop Speed Control System Using Conventional Sliding Mode Control

In conventional sliding mode control, sliding surface is given by

$$s_i = \left(\lambda + \frac{d}{dt}\right)^{n-1} e_i \tag{36}$$

where, s_i is sliding surface, n is order of system and e_i tracking error [15].

Sliding surface is chosen as follow for angular velocity of right and left wheel:

$$\begin{cases} s_1 = e_{1r} \\ s_2 = e_{1l} \end{cases} \tag{37}$$

Taking the derivative of Eq. (37), we obtain

$$\begin{cases} \dot{s}_1 = \dot{e}_{1r} \\ \dot{s}_2 = \dot{e}_{1l} \end{cases} \tag{38}$$

where, $e_{1r} = \omega_{rd} - \omega_r$ and $\omega_{rl} - \omega_l$ is difference b/n actual and desired angular velocity of right and left wheel.

Then, the control law can be obtained as,

$$\begin{cases} \dot{e}_{1r} = \dot{\omega}_{rd} - \dot{\omega}_r = -\eta \text{sign}(s_1) \\ \dot{e}_{1l} = \dot{\omega}_{ld} - \dot{\omega}_l = -\eta \text{sign}(s_2) \end{cases} \tag{39}$$

where, η is positive constant,

Therefore, the control law of speed control systems for inner control using traditional control system is given by Eq. (40).

$$\begin{cases} u_1 = \left(m_1^2 - m_2^2\right)(\eta \text{sign}(s_1) + \dot{\omega}_{rd}) + m_1 V(x_1 - x_2)x_1 + m_2 V(x_1 - x_2)x_2 \\ u_2 = \left(m_2^2 - m_1^2\right)(\eta \text{sign}(s_2) + \dot{\omega}_{ld}) + m_2 V(x_1 - x_2)x_1 + m_1 V(x_1 - x_2)x_2 \end{cases} \tag{40}$$

4 Simulation Results and Analysis

Ntrol parameters for the proposed system assuming $v_d = 2\,\text{m/s}$, and $\omega_d = 2\,\text{rad/s}$ are $\alpha = 10, \beta = 1, q = 3,$ and $p = 5$ for outer GFTSMC, $\beta = 2, \eta = 50{,}000, q = 3,$ and $p = 5$ for inner GFTSMC and for SMC, $\eta = 50{,}000$. The numerical parameter used for demonstration of the proposed control is listed in Table 1 for the DDMR [16].

Table 1. Numerical parameter value of DDMR used for simulation.

Parameter	Value and unit
The mass of the DDMR without the driving wheels and actuators [*mc*]	6 [kg]
The mass of each driving wheel (with actuator) [*mw*]	0.5 [kg]
The moment of inertia of the mobile robot about the vertical axis through the center of mass [*Ic*]	3 [kgm^2]
The moment of inertia of each driving wheel with a motor about the wheel axis [*Iw*]	0.01875 [kgm^2]
The moment of inertia of each driving wheel with a motor about the wheel diameter [*Im*]	0.5 [kgm^2]
Radius of mobile robot wheel [*R*]	0.05 [m]
Half of the distance between the wheels [*L*]	0.5 [m]
Distance between center of mass of robot to the cut axis point of robot [*a*]	0.1 [m]

In order to test the robustness of the inner loop GFTSMC and the SMC speed control system against random gaussian disturbance shown in Fig. 4 is applied after 5 s of simulation time.

The accuracy and effectiveness of both inner and outer loop controllers against model uncertainty and random external disturbance was tested using different performance

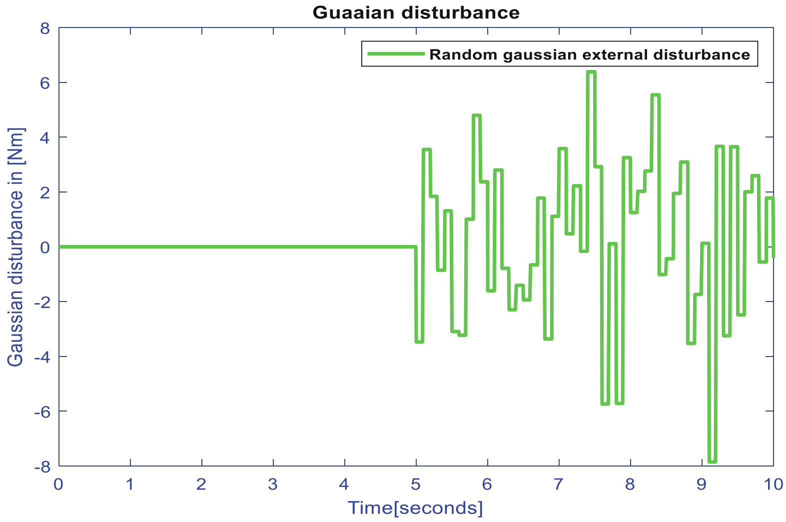

Fig. 4. Random Gaussian external disturbance.

Table 2. Performance index of outer loop controller.

Error along X, Y, and theta	GFTSMC-GTFSMC			GFTSMC-SMC		
	ITSE	IAE	ITAE	ITSE	IAE	ITAE
Theta	0.000686	0.05354	0.01489	0.007078	0.05745	0.02279
X axis	0.001383	0.07725	0.01114	0.1022	0.6755	0.3016
Y axis	0.01594	0.2469	0.03027	0.1242	0.7078	0.3148

Table 3. Performance index of inner loop controller.

Error along X, Y, and theta	GTFSMC			SMC		
	ITSE	IAE	ITAE	ITSE	IAE	ITAE
Right	0.1146	0.2323	1.712	4.081	2.512	12.53
X axis	0.1146	0.2296	1.712	4.06	2.639	12.53

index measurements such as integral absolute error (IAE), integral time square error (ITSE), and integral time absolute error (ITAE) as presented in Tables 2 and 3.

The results reveal that the along θ with external and uncertainty disturbance achieved the least error 0.000686 using GFTSMC (ITSE), while the largest error was achieved along θ with (0.05745) using SMC (IAE). Similarly, least error along X and Y axis 0.001383 and 0.01594 respectively using GFTSMC (ITSE). The largest error 0.6755 and 0.7078 achieved along X and Y axis respectively using SMC (IAE) as indicated

Table 2. The results show that the right and left wheel angular velocity achieved the least error of 0.1146 using GFTSMC (ITSE), while the Left and right wheel angular velocity achieved the largest error (12.53) with SMC (ITAE) as presented in Table 3. The performance index versus time of GFTSMC and conventional SMC illustrated in Fig. 5 and Fig. 6 respectively. It was observed from Fig. 5 that GFTSMC gives good convergence under model uncertainty and Gaussian external disturbances.

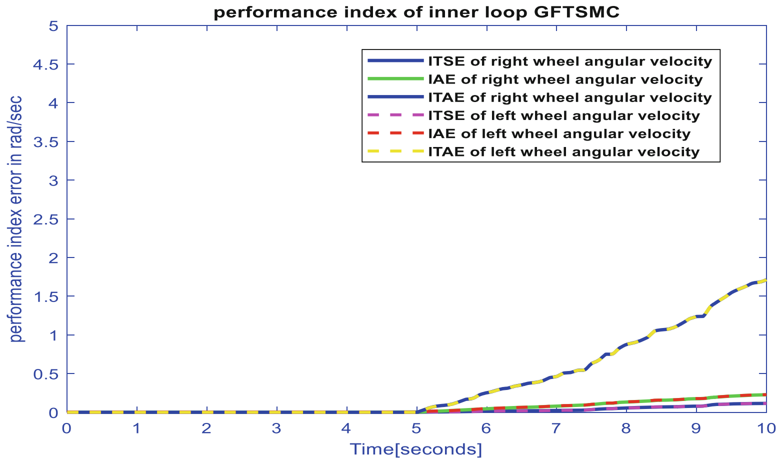

Fig. 5. Performance index of inner loop GFTSMC.

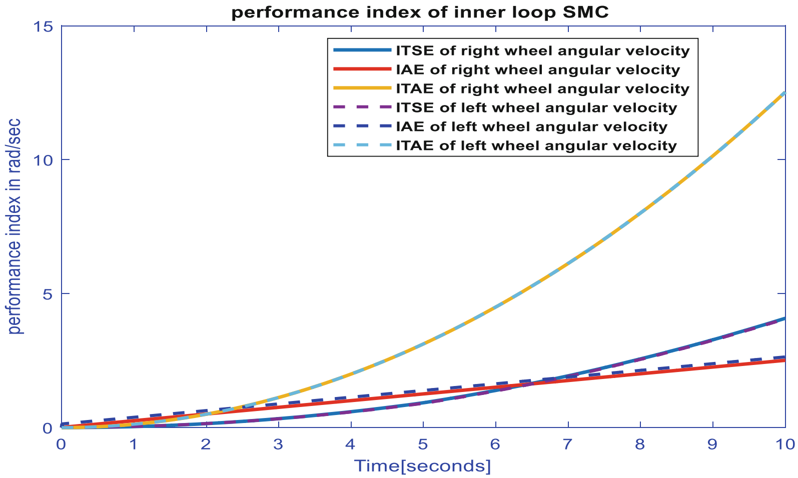

Fig. 6. Performance index of inner loop SMC.

From Fig. 7, an initial tracking error along X axis, Y axis, and rotation position starting from point $\left(1, 2, \frac{\mathrm{pi}}{6}\right)$ approached to zero in finite time.

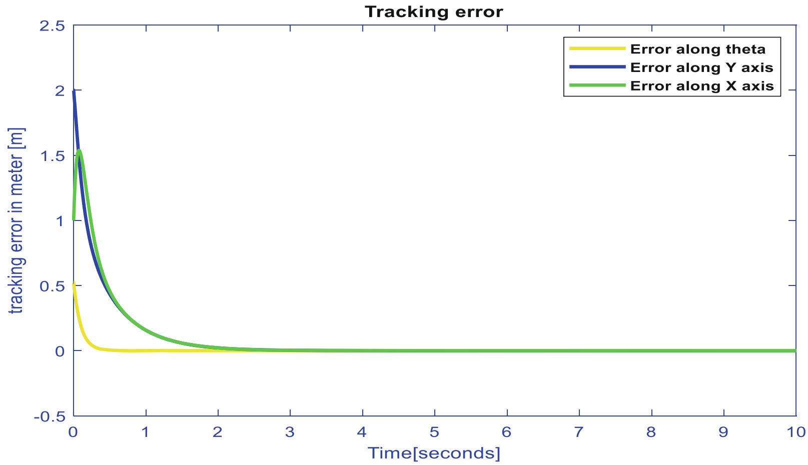

Fig. 7. Tracking error along x, y and orientation of mobile robot using GFTSMC.

From Figs. 8 and 9, it was observed that control signal input angular and linear velocity generated using outer loop GFTSMC changed in to angular velocity of each wheels using transformational matrix in Eq. (4).

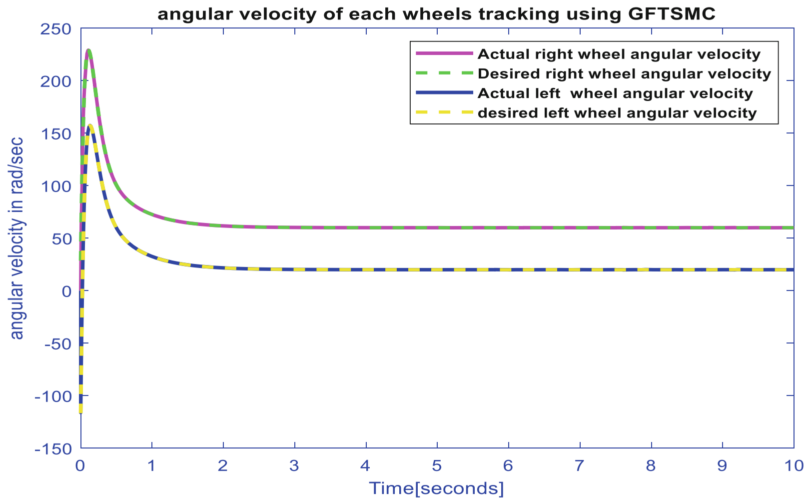

Fig. 8. Right and left wheel angular velocity speed tracking

The inner loop GFTSMC speed control system makes the actual linear and angular velocity to track the desired linear and angular velocity of 2 m/s and 2 rad/s respectively.

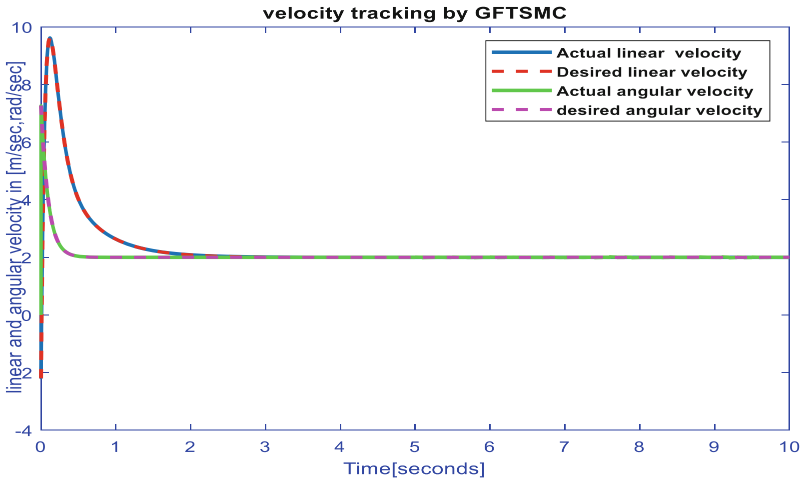

Fig. 9. Angular and linear velocity control signals

XY 2D movement of the mobile robot shown in Fig. 10. The robot starts from the initial point (1 m, −1 m) and moves to the desired circular trajectory radius of 1 m.

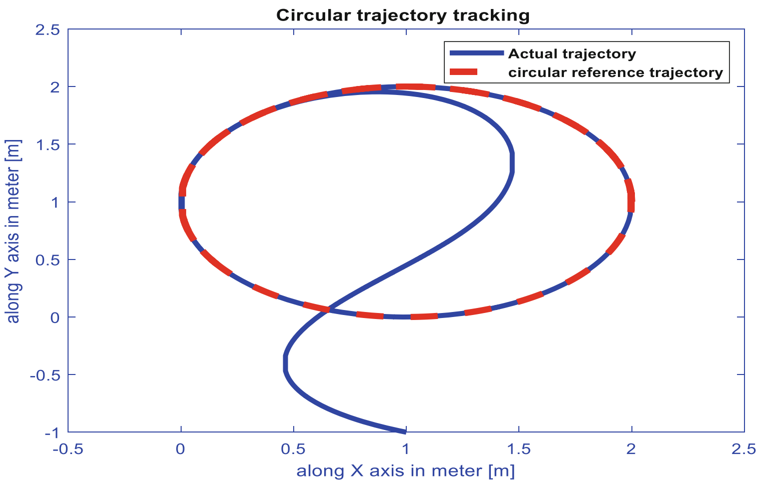

Fig. 10. Circular trajectory tracking starting from point (1, −1)

Similarly, the Fig. 11 shows the inner loop GFTSMC speed control system performance which shows the error between the angular velocity of each wheel to zero at a finite time. In this case, random Gaussian disturbance was applied after 5 s for both wheels and a model uncertainty of $\rho = 0.1 * \sin(20 * t)$ a was taken to test the robustness of the controller, but the inner loop controller is suitable for both disturbances inputs.

In traditional SMC, sliding mode parameters can be adjusted to get faster error convergence, however, this will in turn increase the control gain, which may cause severe

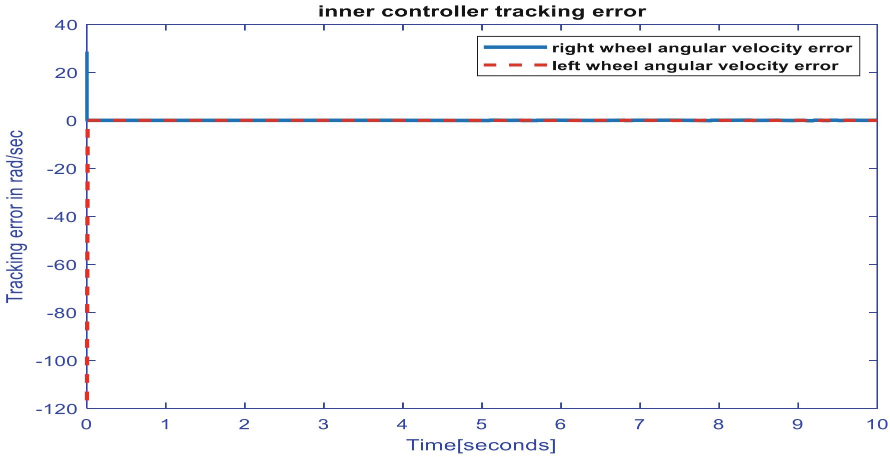

Fig. 11. Inner loop GFTSMC angular velocity each wheels tracking error

chattering of the sliding surface and, therefore, deteriorate the system performance as shown in Fig. 13. But, sliding surface GFTSMC smooth which means that it is free from chattering problems as shown in Fig. 12.

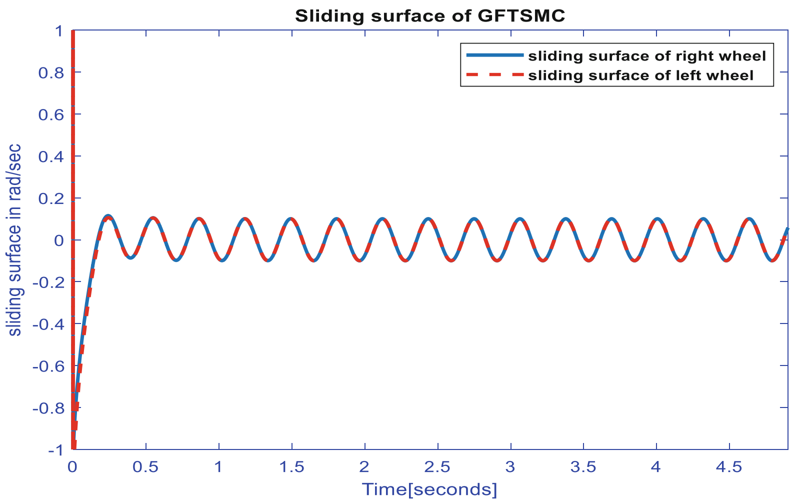

Fig. 12. Sliding surface of inner loop GFTSMC

As observed in Fig. 14, the mobile robot approaches the desired trajectory starting from (0, 0). Here, starting error along X, Y and theta will be (2, 1, pi/6).

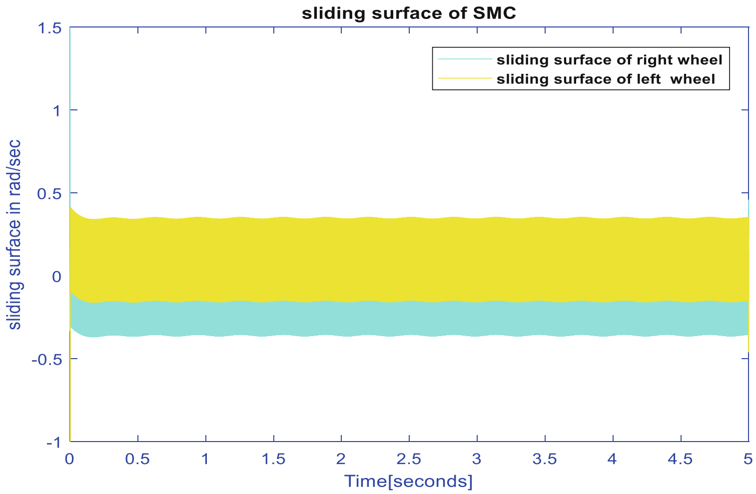

Fig. 13. Sliding surface of inner loop SMC

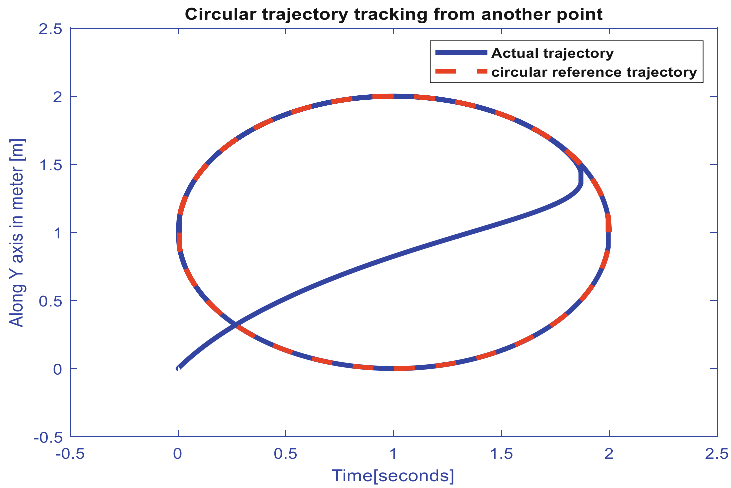

Fig. 14. Circular trajectory tracking starting from another point (0, 0)

5 Conclusion

In this paper, the dynamics of DDMR is obtained using Langrage formulation. The trajectory tracking control for the DDMR is designed to achieve the desired reference trajectory such that the angle error and posture error comes to zero with a finite time global fast terminal sliding mode control (GFTSMC). The GTSMC without chattering effect for kinematic outer closed loop and dynamic inner loop velocity controller was designed to achieve velocity tracking so that errors between the actual and desired velocity control input are reduced to zero at a finite time. The robustness of both controllers was tested using different performance indexes. The results show that both controllers are insensitive to any random external and model uncertainty disturbance. Moreover, the overall dynamic response of the system was improved by integrating the dynamics using cascaded motion control approach for the DDMR.

References

1. Do, K.: Bounded controller for global path tracking of unicycle type mobile robot. Rob.t Auton. Syst. **61**, 775–784 (2013)
2. Ashagrie, A., Salau, A.O., Weldcherkos, T.: Modeling and control of a 3-DOF articulated robotic manipulator using self-tuning fuzzy sliding mode controller. Cogent Eng. **8**(1), 1950105 (2021). https://doi.org/10.1080/23311916.2021.1950105
3. Martins, N.A., Bertol, D.W., De Pieri, E.R., Castelan, E.B., Dias, M.M.; Neural dynamics control of a Nonholonomic mobile robot incorporating the actuator dynamics. In: CIMCA, IAWTIC and ISE, pp. 563–568 (2008)
4. Taganamathar, T.d.a.S.: Control of Nonholonomic mobile robot formations back stepping kinematics in to dynamics. In IEEE, Singapore (2007)
5. Benaziza, W., Slimane, N., Mallem, A.: Mobile robot trajectory tracking using terminal sliding mode control. In: 6th International Conference on Systems and Control (ICSC), Banta, pp. 538–542 (2017)
6. Zidani, G., Drid, S., Chrifi-Alaoui, L., Benmakhlouf, A., Chaouch, S.: Back stepping controller for a wheeled mobile robot. In: IEEE, Sousse, Tunisia April 28–30 (2015)
7. Butt, C., Rahman, M.A.: Limitations of simplified fuzzy logic controller for IPM motor drive. In: Industry Applications Conference, **3**, 1891–1898 (2004)
8. Vinod Raj, N., Mathew, A.T.: Design, simulation and implementation of cascaded path tracking controller for a differential drive mobile robot. In: 2015 International Conference on Advances in Computing, Communications and Informatics (ICACCI), Calicut, India (2015)
9. Salichs, M.A., Puente, E.A., Gachet, D., Moreno, L.: Robust controller for trajectory tracking of a mobile robot. In: 1st IEEE International Conference on Power Electronics. Intelligent Control and Energy Systems (ICPEICES-2016), Calicut, India (2016)
10. Ushikoshi, T., Peixoto, K.P., Souto, F.H.S., das Chagas, T.P., Schnitman, L.: Fuzzy maneuvering controller applied to a dynamic model of a differential drive mobile robot. IEEE (2018)
11. Dhaouadi, R., Hatab, A.A.: Dynamic modelling of differential-drive mobile robots using Lagrange and Newton-Euler methodologies: a unified Framework. Adv. Robot. Autom. **2**(2) (2013)
12. Kandalama, Y., Kimura, Y., Miyazaki, F., Noguchi, T.: A stable tracking control method for an autonomous mobile robot. In: Robotics and Automation, 1990. Proceedings, IEEE (1990)
13. Kolmanovsky, I., McClamroch, N.H.: Development in nonholonomic control problems. IEEE Control Systems 20–36 (1995)
14. Park, K.B., Tsuiji, T.: Terminal sliding mode control of second-order nonlinear uncertain systems. Int. J. Robust Nonlin. Control **9**(11), 769–780 (1999)
15. Liu, J., Wang, X.: Advanced Sliding Mode Control for Mechanical Systems. Springer, Beijing, China (2011). https://doi.org/10.1007/978-3-642-20907-9
16. Sharma, A., Panwar, V.: Control of mobile robot for trajectory tracking by sliding mode control technique. In: International Conference on Electrical, Electronics, and Optimization Techniques (ICEEOT) - 2016, Greater Noida, India (2016)