




# Initial Phrase Blind Estimation of Co-channel Time-Frequency Overlapped Signals

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**Abstract.** In order to successfully implement blind demodulation of the received signal, it is necessary to make a comprehensive and accurate estimation of the parameters of the received signal. Therefore, parameter estimation of communication signals is an indispensable key technology in underlay cognitive networks. This paper focuses on the initial phase estimation method of time-frequency overlapped signals and the initial phrase blind estimation method based on four-order cyclic cumulants is proposed. In this paper, the four-order cyclic cumulants is computed firstly, then the test statistics are constructed by the ratio of four-order cyclic cumulants in specific frequency and specific cycle frequency. Finally, and the initial phrase of signal component are estimated based on the test statistics. Simulation results show that the proposed method achieves initial phrase estimation of time-frequency overlapped signals effectively and the performance is better.

**Keywords:** Co-channel signal · Cyclic spectrum · Forth-order cyclic cumulants · Initial phrase estimation · Time-frequency overlapped signal

## 1 Introduction

In communication systems, multiple signals often appear in the same frequency band at the same time, which is the so-called time-frequency overlapped signal, such as co-channel or adjacent channel interference in mobile communication, mutual interference of frequency bands in satellite communication, multiple echoes in radar systems. In the non-cooperative communication mode, in order to successfully implement blind demodulation of the received signal, it is necessary

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to make a comprehensive and accurate estimation of the modulation parameters of the received signal. Therefore, parameter estimation of communication signals is an indispensable key technology in non-cooperative communication.

The parameter estimation of the time-frequency overlap signal refers to the extraction of parameter information about the signal from the intercepted received signal. The parameter estimation of the modulated signal includes the modulation parameters such as the carrier frequency, symbol rate, amplitude, initial phase and time delay of the received signal. Furthermore, the initial phase estimation methods for the signal can be divided into two categories, one is the initial phase estimation based on auxiliary data [1–3], and the other is the blind estimation of the initial phase without auxiliary data. In non-cooperative communication systems, auxiliary data such as pilot frequency and training sequence are all unknown information and cannot be obtained from the received signal. Therefore, the paper mainly studies the blind estimation method of initial phase without auxiliary data.

This paper firstly introduces a initial phase estimation method for time-frequency overlapped signal based on fourth-order cyclic cumulants. Signal method. The method constructs a test statistic based on the ratio of the fourth-order cyclic cumulants, and estimates the initial phase of each signal component based on the phase information of the test statistic. Simulation results show that the method can effectively estimate the initial phase parameters of co-channel time-frequency overlapped signals.

## 2 System Model

The model of time-frequency overlapped signal can be given as [7]

$$x(t) = \sum_{i=1}^N s_i(t) + n(t), \quad (1)$$

where  $s_i(t) = \sum_{m=1}^{M_i} A_i a_i(m) q(t - mT_{bi} - \tau_i) \exp[j2\pi f_{ci}t + \theta_i]$ . These elements  $A_i$ ,  $a_i(m)$ ,  $M_i$  and  $f_{ci}$  represent signal amplitude, symbol sequence, symbol number and carrier frequency of every signal component respectively. Also,  $T_i$  is a symbol cycle, whose reciprocal is symbol rate  $f_{ci}$ ;  $\theta_i$  is initial phase;  $q_i(t)$  is pulse shape function. Note that if  $q_i(t)$  is rectangular shape, it can be expressed as  $q_i(t) = \begin{cases} 1, & |t| \leq \frac{T_i}{2} \\ 0, & \text{others} \end{cases}$ . If cosine raised, it can be expressed as  $q_i(t) = \frac{\sin \pi t/T_i}{\pi t/T_i} \cdot \frac{\cos \alpha \pi t/T_i}{1 - 4\alpha^2 t^2/T_i^2}$ , where  $\alpha$  ( $\alpha = 0.35$  usually) is roll-off factor,  $\tau_i$  is signal delay,  $n(t)$  is the additive Gaussian noise. Signals in the model are independent mutually, and the same to each signal and noise.

### 3 Initial Phrase Estimation of Time-Frequency Overlapped Signals Based on Fourth-Order Cyclic Cumulants

The definition of 4-order cyclic cumulants  $C_{a,40}$  and  $C_{a,42}$  is as follows respectively.

$$C_{40}^{\alpha} = \frac{E^2 C_{a,40} e^{j4\varphi_0}}{T_s} \int_{-\infty}^{\infty} p^4(t) e^{-j2\pi(\alpha-4f_c)t} dt, \quad (2)$$

$$C_{42}^{\alpha} = \frac{E^2 C_{a,42}}{T_s} \int_{-\infty}^{\infty} p^4(t) e^{-j2\pi\alpha t} dt, \quad (3)$$

where  $\varphi_0$  is initial phrase,  $f_c$  is carrier frequency, and  $1/T_s$  is symbol rate.

The commonly used modulation signals in digital communication are BPSK, QPSK, 16QAM and 64QAM, the value of 4-order cumulants are as follows

$$\begin{aligned} C_{a,40-BPSK} &= C_{a,42-BPSK} = -2, \\ C_{a,40-QPSK} &= C_{a,42-QPSK} = -1, \\ C_{a,40-8PSK} &= 0, C_{a,42-8PSK} = -1, \\ C_{a,40-16QAM} &= C_{a,42-16QAM} = -0.68, \\ C_{a,40-64QAM} &= C_{a,42-64QAM} = -0.62. \end{aligned} \quad (4)$$

Hence, the value of the 4-order cyclic cumulants for these signals are

$$C_{40-BPSK}^{\alpha} = -\frac{2E^2 e^{j4\varphi_0}}{T_s} \int_{-\infty}^{+\infty} p^4(t) e^{j2\pi(\alpha-4f_c)t} dt, \quad (5)$$

$$C_{42-BPSK}^{\alpha} = -\frac{2E^2}{T_s} \int_{-\infty}^{+\infty} p^4(t) e^{-j2\pi\alpha t} dt, \quad (6)$$

$$C_{40-QPSK}^{\alpha} = -\frac{E^2 e^{j4\varphi_0}}{T_s} \int_{-\infty}^{+\infty} p^4(t) e^{j2\pi(\alpha-4f_c)t} dt, \quad (7)$$

$$C_{42-QPSK}^{\alpha} = -\frac{E^2}{T_s} \int_{-\infty}^{+\infty} p^4(t) e^{j2\pi\alpha t} dt, \quad (8)$$

$$C_{40-8PSK}^{\alpha} = 0, C_{42-8PSK}^{\alpha} = -\frac{E^2}{T_s} \int_{-\infty}^{+\infty} p^4(t) e^{j2\pi\alpha t} dt, \quad (9)$$

$$C_{40-16QAM}^{\alpha} = -\frac{0.68E^2 e^{j4\varphi_0}}{T_s} \int_{-\infty}^{+\infty} p^4(t) e^{j2\pi(\alpha-4f_c)t} dt, \quad (10)$$

$$C_{42-16QAM}^{\alpha} = -\frac{0.68E^2}{T_s} \int_{-\infty}^{+\infty} p^4(t) e^{j2\pi\alpha t} dt, \quad (11)$$

$$C_{40-64QAM}^{\alpha} = -\frac{0.62E^2 e^{j4\varphi_0}}{T_s} \int_{-\infty}^{+\infty} p^4(t) e^{j2\pi(\alpha-4f_c)t} dt, \quad (12)$$

$$C_{42-64QAM}^\alpha = -\frac{0.62E^2}{T_s} \int_{-\infty}^{+\infty} p^4(t) e^{j2\pi\alpha t} dt. \tag{13}$$

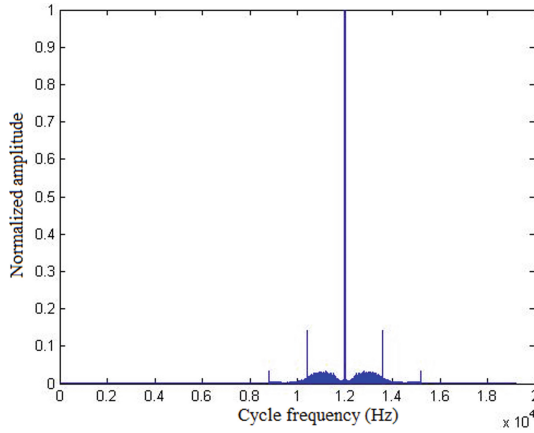
From the above expression of 4-order cyclic cumulants, we can note that  $C_{40}^\alpha$  includes initial phrase information, but  $C_{42}^\alpha$  does not. So that we can use the ratio of  $C_{40}^\alpha$  and  $C_{42}^\alpha$  to estimate the initial phrase.

From (2), we know that when  $\alpha = 4f_c + 1/T_s$  the expression of  $C_{40}^\alpha$  is

$$C_{40}^{4f_c+1/T_s} = \frac{E^2 C_{a,40} e^{j4\varphi_0}}{T_s} \int_{-\infty}^{\infty} p^4(t) e^{-j2\pi t/T_s} dt, \tag{14}$$

and when  $\alpha = 1/T_s$  the expression of  $C_{42}^\alpha$  is given as

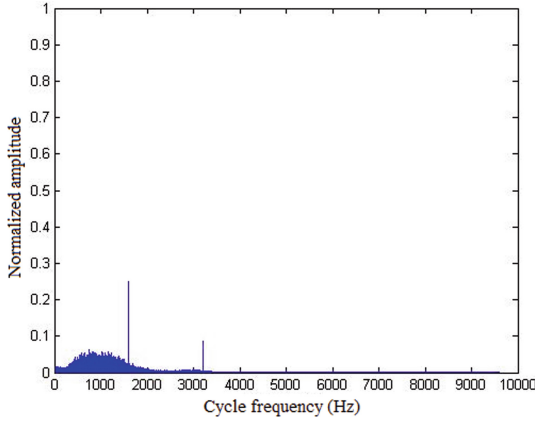
$$C_{42}^{1/T_s} = \frac{E^2 C_{a,42}}{T_s} \int_{-\infty}^{\infty} p^4(t) e^{-j2\pi t/T_s} dt. \tag{15}$$



**Fig. 1.** The 4-order cyclic cumulants  $C_{40}^\alpha$  of QPSK.

In order to make it more intuitive, the 4-order cyclic cumulants  $C_{40}^\alpha$  and  $C_{42}^\alpha$  are shown in Fig. 1 and Fig. 2, respectively. It can see that the value of  $C_{40}^\alpha$  and  $C_{42}^\alpha$  is bigger and not zero at  $\alpha = 4f_c + 1/T_s$  and at  $\alpha = 1/T_s$  respectively. Besides, from Eqs. 14 and 15, it can find it that the integral part of the two expressions are the same, but only Eq. 14 has initial phrase information. Therefore we can estimate the signal components initial phrase by structuring characteristic parameters  $T$ . The expression of  $T$  is show as

$$T = \frac{C_{40}^{4f_c+1/T_s}}{C_{42}^{1/T_s}} = \frac{\frac{E^2 C_{a,40} e^{j4\varphi_0}}{T_s} \int_{-\infty}^{\infty} p^4(t) e^{-j2\pi t/T_s} dt}{\frac{E^2 C_{a,42}}{T_s} \int_{-\infty}^{\infty} p^4(t) e^{-j2\pi t/T_s} dt} = \frac{C_{a,40} e^{j4\varphi_0}}{C_{a,42}}. \tag{16}$$



**Fig. 2.** The 4-order cyclic cumulants  $C_{42}^\alpha$  of QPSK.

As  $C_{\alpha,40}$  of BPSK, QPSK, 16QAM and 64QAM are all real, the initial phase information is included in parameter  $T$ . Thus we can estimate the initial phase by detecting  $T$ , the estimator of initial phase  $\phi_i$  is as follow.

$$\hat{\phi}_i = \frac{1}{4} \arg(T), \tag{17}$$

where  $\arg()$  means compute phrase for plural.

From the above, the specific steps of initial phase estimation method based on 4-order cyclic cumulants are as follows:

**Step 1:** Computing 4-order cyclic cumulants  $C_{x,40}^\alpha$  and  $C_{x,42}^\alpha$  of time-frequency overlapped signals by (3–8) and (3–9), then get the 4-order cyclic cumulants spectrum  $\alpha - C_{x,40}^\alpha$  and  $\alpha - C_{x,42}^\alpha$ ;

**Step 2:** In light of the known signals carrier frequency  $f_{c1}$ ,  $f_{c2}$  and symbol rate  $1/T_{s1}$ ,  $1/T_{s2}$ , get the value  $C_{x,40}^{4f_{c1}+1/T_{s1}}$  of  $\alpha - C_{x,40}^\alpha$  at  $\alpha = 4f_{c1} + 1/T_{s1}$  and  $C_{x,40}^{4f_{c2}+1/T_{s2}}$  at  $\alpha = 4f_{c2} + 1/T_{s2}$ ;

**Step 3:** According to symbol rate of the known signals  $1/T_{s1}$  and  $1/T_{s2}$ , get the value  $C_{x,42}^{1/T_{s1}}$  of  $\alpha - C_{x,42}^\alpha$  at  $\alpha = 1/T_{s1}$  and  $C_{x,42}^{1/T_{s2}}$  at  $\alpha = 1/T_{s2}$ ;

**Step 4:** In line with the value coming from step 2 and step 3, structure characteristic parameters  $T_1 = C_{x,40}^{4f_{c1}+1/T_{s1}} / C_{x,42}^{1/T_{s1}}$  and  $T_2 = C_{x,40}^{4f_{c2}+1/T_{s2}} / C_{x,42}^{1/T_{s2}}$ ;

**Step 5:** Computing phrase angle  $\Phi_1$  of characteristic parameters  $T_1$  and  $\Phi_2$  of  $T_2$ , then get the initial phase  $\varphi_{01} = \Phi_1/4$  and  $\varphi_{02} = \Phi_2/4$ .

## 4 Simulation Results and Discussion

The validity of the method in the paper is verified via MATLAB simulate experiment. Time-frequency overlapped signals model is used in the paper, and the noise is white Gaussian noise. For evaluating performance from different sides,

the next simulation experiment uses time-frequency overlapped signals whose type are MPSK and MQAM. The signals use raised cosine shaping function whose roll-factor is 0.35. And do Monte Carlo experiments 1000 times, the evaluative criteria of initial phrase is MSE.

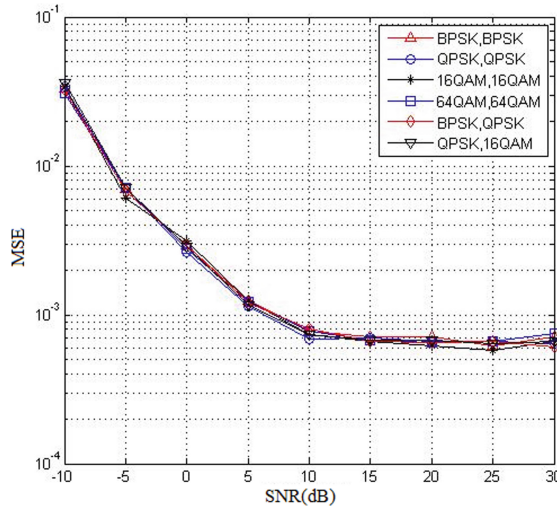
In order to test impact of SNR to time-frequency overlapped signals initial phrase estimation, any two random combination from BPSK, QPSK, 16QAM and 64QAM. Any two signals parameter setting is as follows: carrier frequency  $f_{c1} = 2.7$  KHz and  $f_{c2} = 3.3$  KHz, symbol rate  $1/T_{s1} = 1.2$  KBaud and  $1/T_{s2} = 1.6$  KBaud, sampling frequency  $f_s = 19.2$  KHz, data length is 5000. The simulation result is shown in Fig. 3.

As seen from Fig. 3, in the case of two overlapped signals, when input SNR is bigger than 5 dB, the method in the paper can achieve ideal performance. With the increase of SNR, the estimation performance is also improved.

For the purpose of test the impact of data length on time-frequency overlapped signals initial phrase estimation, we should combine any two random signals from BPSK, QPSK, 16QAM, 64QAM (the SNR is 10 dB). Signals parameter setting is as follows: carrier frequency  $f_{c1} = 2.7$  KHz and  $f_{c2} = 3.3$  KHz; symbol rate  $1/T_{s1} = 1.2$  KBaud and  $1/T_{s2} = 1.6$  KBaud; sampling frequency  $f_s = 19.2$  KHz. The simulation result is shown in Fig. 4.

As can be seen from Fig. 4, the MSE of time-frequency overlapped signals the estimation performance increase with the increase of sampling data length. As the cyclostationarity reflected by cyclic cumulants is an asymptotic behavior, thus we can improve estimated performance by increasing data length.

In order to test the impact of spectrum overlap rate on initial phrase estimation performance for overlap signals, we should use any two random signals from



**Fig. 3.** Initial phrase estimation performance of time-frequency overlapped signals with different SNRs.

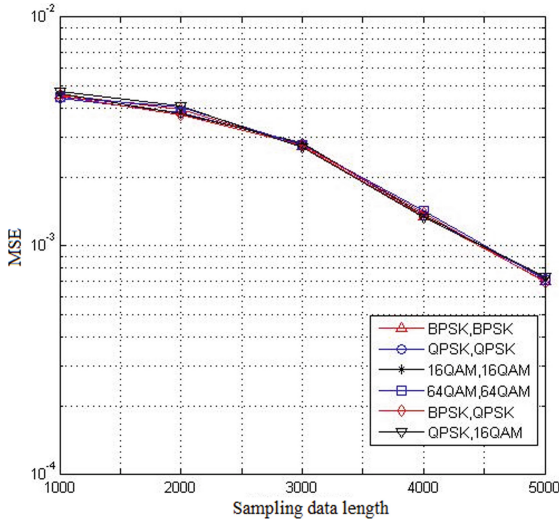


Fig. 4. Initial phase estimation performance of time-frequency overlapped signals with different data length.

BPSK, QPSK, 16QAM, 64QAM (the SNR is 10 dB). Sampling rate  $f_s = 19.2$  KHz, data length 5000, and any two signals performance setting is as follows: the combination of carrier frequency are  $f_{c1} = 1.9$  KHz and  $f_{c2} = 3.3$  KHz,  $f_{c1} = 2.2$  KHz and  $f_{c2} = 3.3$  KHz,  $f_{c1} = 2.5$  KHz and  $f_{c2} = 3.3$  KHz,  $f_{c1} = 2.8$  KHz and  $f_{c2} = 3.3$  KHz,  $f_{c1} = 3.1$  KHz and  $f_{c2} = 3.3$  KHz. The spectrum overlap rate of each combination is 0%, 25%, 50%, 75% and 100%, respectively. Symbol rate are = 1.2 KBaud and = 1.6 KBaud. The simulation result is shown in Fig. 5.

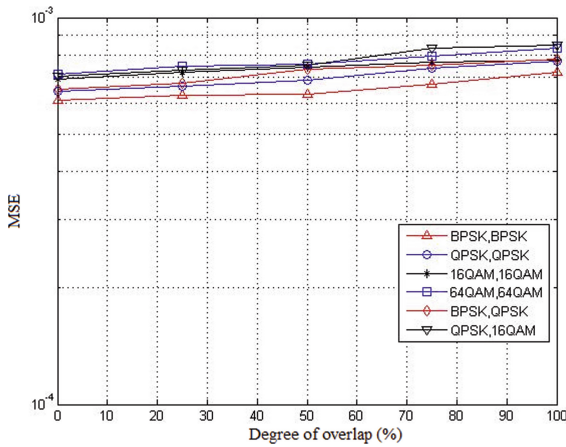
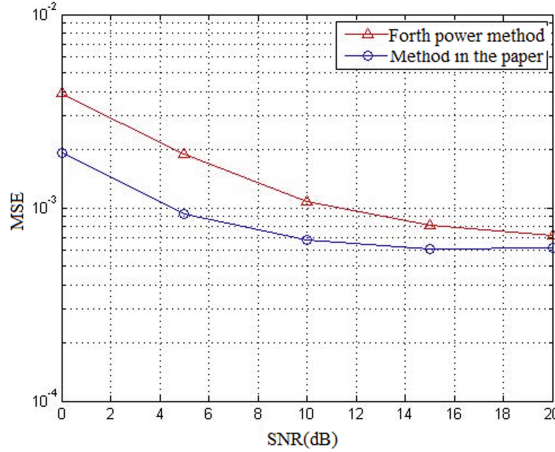


Fig. 5. Initial phase estimation performance of time-frequency overlapped signals with different spectrum overlapped rates.



**Fig. 6.** Initial phrase estimation performance comparison with different methods.

From Fig. 5, we can know that the impact of spectrum overlap rate on initial phrase estimation is small. Because as the increase of spectrum overlap rate, the interval of signal components cyclic cumulants will become smaller. Therefore it can make the signal components be influenced more. As long as the symbol rate of every signal component are different, due the cyclostationarity that we can get the amplitude and phrase information, and decrease the impact of other signal component at the same time.

In order to assess the performance of the method in the paper and the existing method, the overlap signal contains two QPSK. Under the same simulation environment and parameter setting, the method introduced in the paper has to compare with the method in literature [16]. And the result is shown in Fig. 6.

From Fig. 6, it draws that as the increase of SNR, the EMS of initial phrase estimation decrease. Under the same simulation condition, the method introduced in the paper is better than the existing method based on biquadrate. The method of biquadrate needs  $2(N - 1)^2 + 1$  times addition of complex quantities and  $8N(N + 1)$  times complex multiplication. The method in the paper needs addition of complex quantities and complex multiplication  $2N\log_2 N + N$  times and  $N\log_2 N + 5N$  times and the calculation complexity reflected in complex multiplication. In a word, the calculation complexity in the paper is smaller.

## 5 Conclusion

In this paper, an initial phrase blind estimation method based on 4-order cyclic cumulants is introduced. The proposed method employs the ratio of 4-order cyclic cumulants by  $C_{40}^\alpha$  and  $C_{42}^\alpha$  in carrier frequency and symbol rate of every signal component constructing characteristic parameter, then it adopts the phrase

angle information of characteristic parameter to estimate initial phase. Simulation results show that the proposed method has a better estimation performance compared with the existing methods.

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