Equal Proportional Probability Caching Policy for Heterogeneous Network Based on Stochastic Geometry Theory

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Abstract. In this paper, we compare different caching policies in a cache-enabled heterogeneous network (HetNet). The network is equipped with a tier of macro cell base stations (MBSs) with backhaul link and a tier of small cell base stations (SBSs) with cache memories. The two-dimensional spatial distribution of these two-tier network is modeled by stochastic geometry theory, the probability of the files being requested is modeled by Zipf distribution and the probability of the files being cached on the SBS are modeled by different particular distributions corresponding to different caching policies. We use average outage probability (AOP) to measure the performance of the system. In order to decilne the AOP of a user request, we propose a new caching policy with equal proportional probability. The performance of the new caching policy, the affect caused by SBS density changing and the affect caused by cache skewness changing are verified by Monte-Carlo simulation.

Keywords: Caching policy · Heterogeneous network · Stochastic geometry · Average outage probability · Equal proportional probability

1 Introduction

With the explosive growth of mobile data services, the user's demand for high-speed data streams is rapidly growing. Since requesting contents from the centralized cloud usually generates significant backhaul link pressure, the recent research tends to cache the contents on the edge storage deployed on the SBSs of HetNet [1].

Focusing on the minimization of resource consumption, by combining the caching memory and backhaul link as a unified resource with balance coefficient, the optimal memory size has been obtained in closed-form, and the optimal density of SBSs has been found numerically, which can reduce the required capacity of backhaul but keep the wireless QoS effectively [2].

This work is supported in part by National Natural Science Foundation of China (No. 62071146, 62071147).

Q. Guo et al. (Eds.): WiSATS 2021, LNICST 410, pp. 508–519, 2022.
https://doi.org/10.1007/978-3-030-93398-2_47

Focusing on different performance metrics, to reduce time delay, by jointly modeling the backhaul and wireless transmission phases, a theoretical framework for partially connected edge cache network was derived to characterize the trade-off relations between time delay and deployment parameters, and the optimal deployment strategy is obtained [3]. To maximize cache hit probability, by modeling the network with limited number of BSs, cached files and users, [4] relaxed the problem to a convex optimization problem and obtained the optimal caching policy with randomized rounding algorithm. When considering outage probability of the service, [5] compared the performance of different classical caching policies and analytically derived the outage probability of serving the requested content by jointly considering spectrum allocation and storage constraints. To maximize the success probability and area spectral efficiency (ASE), [6] takes SBS idling into consideration and obtained the optimal caching probability respectively maximizing the success probability and ASE, which concluded that each SBS tends to cache the most popular files to maximize ASE.

Focusing on special network structure, with extra D2D communication inserted, [7] proposed a near optimal MOAC algorithm and a low-complexity MRAC algorithm, which demonstrated that the proposed algorithms solved the problem of user selfishness through the natural social efficiency and individual rationality of the proposed auction model.

This paper characterizes the AOPs of user request corresponding to different caching policies by modeling the spatial distribution of the BSs as several independent Poisson Point Processes (PPPs). Unlike [5], we propose a new caching policy which could avoid the shortcomings of the classical policies and provide lower outage probability than the classical policies. Furthermore, we investigate the relationship between the density of SBS and AOP and the relationship between the sliding parameter and AOP.

2 System Model

Consider the downlink transmission scenario of HetNet with cache-enabled SBSs and backhaul link-deployed MBSs as shown in Fig. 1. The two dimensional spatial distribution of the MBSs and SBSs follows two independent PPPs denoted by Φ_{M} and Φ_{S}, respectively, which represent the spatial location sets of MBSs and SBSs. The density values of MBS, SBS are λ_{M} (nodes per square meter) and λ_{S}, respectively. Since we only consider the performance from the user side, we assume the spectrum allocation of both SBSs and MBSs and the same. The transmit power of MBS, SBS is denoted by P_{M} and P_{S}, respectively. The radium of service area for each SBS is denoted by $R_{\max}$.

We assume that each user requests the content from a library which contains N files, and the cache space of each SBS is $M\,(M \leq N)$, where each file is assumed to follow unit size. The files are indexed according to their popularity, ranking from the most popular (the 1st file) to the least popular (the Nth file). The probability of each file being requested follows the Zipf distribution,

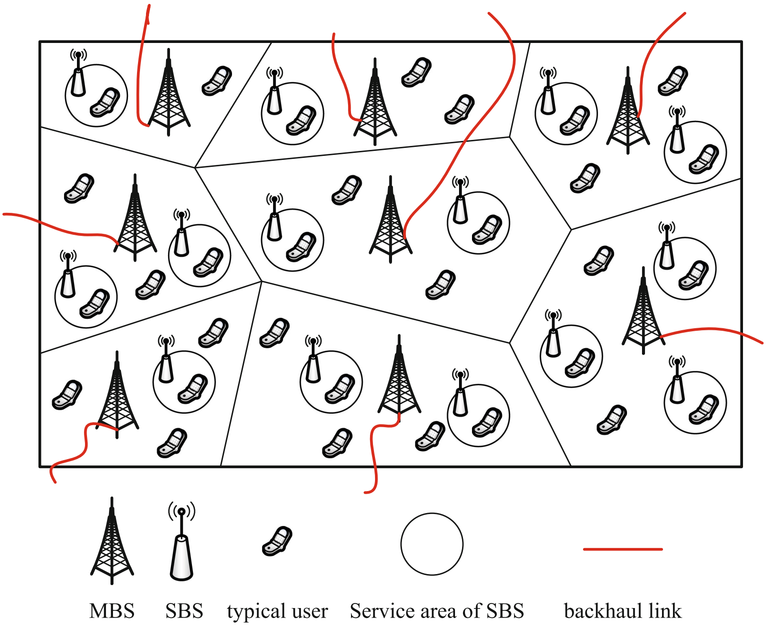

Fig. 1. HetNet with cache-enabled SBSs and backhaul-equipped MBSs.

according to which the requesting probability of the ith popular file is

$$p_i = \frac{i^{-\delta}}{\sum\limits_{j=1}^{N} j^{-\delta}}, \tag{1}$$

where δ ($\delta > 0$) is the skew parameter of the popularity of different files, as δ grows larger, the gap between the high-popularity files and the low-popularity files becomes larger.

We assume each SBS selects the files to be cached independently. The probability of the ith popular file to be cached on SBS is denoted as q_i [8], since each file follows unit size, the average space occupied by the ith popular file is q_i, thus, the summation of q_i is

$$\sum_{i=1}^{N} q_i = M \tag{2}$$

The most commonly used caching policies are uniform caching policy (UCP) and popularity-based caching policy (PCP), where UCP makes SBSs cache each file with equal probability and PCP only cache the most M popular files, denoted as

$$q_{i,\mathrm{UCP}} = \frac{M}{N}(i = 1, 2, ..., N) \tag{3}$$

$$q_{i,\mathrm{PCP}} = \begin{cases} 1 & i = 1, 2, ..., M \\ 0 & i = M+1, M+2, ..., N \end{cases} \tag{4}$$

Since each SBS caches the files independently, the spatial location set of the SBS which caches the ith file is denoted as Φ_{Si}, with the density value thinned by value q_i, i.e., $\lambda_{\mathrm{Si}} = \lambda_{\mathrm{S}} q_i$. On the contrary, $\Phi_{\mathrm{Si}'}$ and $\lambda_{\mathrm{Si}'} = \lambda_{\mathrm{S}}(1 - q_i)$ denote the spatial location set of SBSs which do not cache the ith file and their density value, respectively.

The downlink signal-to-interference-plus-noise ratio (SINR) for a typical user requesting the ith file when associating with SBS (MBS) is

$$\mathrm{SINR}_{\mathrm{S}i} = \frac{P_{\mathrm{S}} h_{\mathrm{S}i} R_{\mathrm{S}i}^{-\alpha}}{I_{\mathrm{S}i} + \sigma^2}, \tag{5}$$

$$\mathrm{SINR}_{\mathrm{M}} = \frac{P_{\mathrm{M}} h_{\mathrm{M}} R_{\mathrm{M}}^{-\alpha}}{I_{\mathrm{M}i} + \sigma^2}, \tag{6}$$

where $h_{\mathrm{S}i}$ and h_{M} are the equivalent channel power from the associated SBS (MBS) i.e., $\mathrm{S}i0\,(\mathrm{M}0)$ to the typical user, we assume both SBS-to-user and MBS-to-user channels are modeled as Rayleigh fading channel. Thus, both h_{Si} and h_{M} follow the exponential distribution with unit mean, i.e., $h_{\mathrm{Si}} \sim exp(1)\,(i = 1, 2, ..., N)$, $h_{\mathrm{M}} \sim exp(1)$ [9], $R_{\mathrm{S}i}$ is the distance between the typical user and the nearest SBS which caches the ith file, R_{M} is the distance between the typical user and the nearest MBS, α is the path loss exponent, for the ease of the calculation below, the value of α is set to 4. The interference generating to the typical user which is associating with SBS is $I_{\mathrm{S}i} = I_{\mathrm{S}i\mathrm{M}} + I_{\mathrm{S}i\mathrm{S}i} + I_{\mathrm{S}i\mathrm{S}i'}$, where $I_{\mathrm{S}i\mathrm{M}} = \sum_{j \in \Phi_{\mathrm{M}}} P_{\mathrm{M}} h_{\mathrm{M}j} R_{\mathrm{M}j}^{-\alpha}$, $I_{\mathrm{S}i\mathrm{S}i} = \sum_{j \in \Phi_{\mathrm{S}i} \backslash \mathrm{S}i0} P_{\mathrm{S}} h_{\mathrm{S}ij} R_{\mathrm{S}ij}^{-\alpha}$ and $I_{\mathrm{S}i\mathrm{S}i'} = \sum_{j \in \Phi_{\mathrm{S}i'}} P_{\mathrm{S}} h_{\mathrm{S}ij} R_{\mathrm{S}ij}^{-\alpha}$ respectively represent the interference produced by the MBSs, the SBSs which cache the ith file, the SBSs which do not cache the ith file. The interference to the typical user associating with MBS is $I_{\mathrm{M}i} = I_{\mathrm{M}i\mathrm{M}} + I_{\mathrm{M}i\mathrm{S}i} + I_{\mathrm{M}i\mathrm{S}i'}$, where $I_{\mathrm{M}i\mathrm{M}} = \sum_{j \in \Phi_{\mathrm{M}} \backslash \mathrm{M}0} P_{\mathrm{M}} h_{\mathrm{M}j} R_{\mathrm{M}j}^{-\alpha}$, $I_{\mathrm{M}i\mathrm{S}i} = \sum_{j \in \Phi_{\mathrm{S}i}} P_{\mathrm{S}} h_{\mathrm{S}ij} R_{\mathrm{S}ij}^{-\alpha}$ and $I_{\mathrm{M}i\mathrm{S}i'} = \sum_{j \in \Phi_{\mathrm{S}i'}} P_{\mathrm{S}} h_{\mathrm{S}ij} R_{\mathrm{S}ij}^{-\alpha}$, which represent the same as above. Since the HetNets are usually interference-limited [10], we neglect the effect caused by the additive noise, i.e., $\sigma^2 = 0$.

3 Caching Policies and Performance Analysis

In this section, we propose a equal proportional probability caching policy and a connection strategy to increase the usage ratio of SBSs, then we use the outage probability as the performance metric to analyze the validity of the system.

3.1 Caching Policies and Connecting Strategy

When using UCP, if the request probability gap between each file is large (when δ is large), the high-ranking files possess the same cache probability as others. On the contrary, when using PCP, if the request probability gap between each file is little (when δ is little), they low-ranking files still won't be cached although the gap of the requesting probability between the low-ranking files and the high-ranking files is not large, resulting in the low cache hit probability.

The primary reason of this problem is the cache probability of a typical file cannot match its corresponding request probability, to solve these problem, the equal proportional probability caching policy (EPPCP) is proposed. In this policy, the 1st popular file is most likely to be cached, i.e., $\arg\max_i q_i = 1$. Then q_i is equally proportional to q_{i-1} with proportionality factor d $(0 < d < 1)$, i.e., $q_i = q_{i-1}d$. So if we sets q_1 $((M/N) < q_1 < 1)$ as a function of sliding parameter β $(0 < \beta < 1)$ which makes q_1 slide from M/N to 1, i.e., $q_1 = \beta + (1-\beta)(M/N)$, according to (2), the summation of their average space is

$$\begin{aligned}
&\sum_{i=1}^{N} q_i = \sum_{i=1}^{N} q_1 d^{i-1} \\
&\overset{(a)}{=} \frac{q_1(1-d^N)}{1-d} \\
&= \frac{[\beta + (1-\beta)(M/N)](1-d^N)}{1-d} \\
&= M,
\end{aligned} \tag{7}$$

where step (a) follows the summation formula of geometric sequence, by solving this Nth degree polynomial equation with dichotomy, we can obtain a unique solution of d within (0,1), then we can obtain the caching probability of each file q_1, $q_2 = q_1 d$, $q_3 = q_1 d^2$, ..., $q_N = q_1 d^{N-1}$, the above is the achievement method of EPPCP.

To release the load of the backhaul link, we use the SBS-priority connecting strategy (SPCS). When a typical user requests the ith file, it firstly searches and associates with the nearest SBS which caches the ith file within $R_{\max}$, if there is no SBS which satisfies the requirement, the nearest MBS would fulfill the request and obtain the requested file from the core network through backhaul link. By this strategy, the backhaul link would be occupied only when the SBSs cannot provide service, resulting in link pressure to be released.

The probability of the user associating with SBS tier when requesting the ith file is denoted as $p_{\mathrm{S}i}$, which is equivalent with the probability that there's at least one SBS within the circle area centered on the typical user of radius $R_{\max}$.

$$p_{\mathrm{S}i} = 1 - e^{-q_i \lambda_{\mathrm{S}} \pi R_{\max}^2}, \tag{8}$$

When a typical user connects with SBS tier, the probability density function of user-SBS distance is

$$\begin{aligned} f_{R_{\mathrm{S}i}|R_{\mathrm{S}i}<R_{\max}}(r) &= \frac{d(F_{R_{\mathrm{S}i}|R_{\mathrm{S}i}<R_{\max}}(r))}{dr} \\ &= \frac{2\pi q_i \lambda_{\mathrm{S}} r e^{-q_i\lambda_{\mathrm{S}}\pi r^2}}{1-e^{-q_i\lambda_{\mathrm{S}}\pi R_{\max}^2}}, \end{aligned} \tag{9}$$

similar to (9), when a typical user connects with MBS tier, the probability density function of user-MBS distance is

$$f_{R_{\mathrm{M}}} = 2\pi\lambda_{\mathrm{M}} r e^{-\lambda_{\mathrm{M}}\pi r^2} \tag{10}$$

3.2 Outage Probability

We use AOP, i.e. $\mathbb{P}_{\mathrm{out}}$ as the performance metric to reflect the quality of service, which is the average probability that the downlink channel SINR be inferior to the given threshold γ, which is

$$\mathbb{P}_{\mathrm{out}} = \sum_{i=1}^{N} p_i[p_{\mathrm{S}i}\mathbb{P}_{out,\mathrm{S}i} + (1-p_{\mathrm{S}i})\mathbb{P}_{out,\mathrm{M}i}] \tag{11}$$

where $\mathbb{P}_{out,\mathrm{S}i}$ represents the outage probability for a SBS-connected user when requesting the ith file, which is

$$\begin{aligned} \mathbb{P}_{out,\mathrm{S}i} &= \mathbb{E}_{R_{\mathrm{S}i}}[p(\mathrm{SINR}_{\mathrm{S}i} < \gamma | R_{\mathrm{S}i} = r)] \\ &= 1 - \int_0^{R_{\max}} p(\mathrm{SINR}_{\mathrm{S}i} > \gamma | R_{\mathrm{S}i} = r) f_{R_{\mathrm{S}i}|R_{\mathrm{S}i}<R_{\max}}(r)dr \\ &= 1 - \int_0^{R_{\max}} \mathcal{L}_{I_{\mathrm{SiM}}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}})\mathcal{L}_{I_{\mathrm{SiSi}}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}})\mathcal{L}_{I_{\mathrm{SiSi}'}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}})\frac{2\pi q_i\lambda_{\mathrm{S}} r e^{-q_i\lambda_{\mathrm{S}}\pi r^2}}{1-e^{-q_i\lambda_{\mathrm{S}}\pi R_{\max}^2}}dr, \end{aligned} \tag{12}$$

where $\mathcal{L}_{I_{\mathrm{SiM}}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}})$, $\mathcal{L}_{I_{\mathrm{SiSi}}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}})$ and $\mathcal{L}_{I_{\mathrm{SiSi}'}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}})$ respectively represent the Laplace transform of the probability density function of I_{SiM}, I_{SiSi} and $I_{\mathrm{SiSi}'}$ with parameter $\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}}$.

Proof: Since $I_{\mathrm{S}i\mathrm{M}}$, $I_{\mathrm{S}i\mathrm{S}i}$ and $I_{\mathrm{S}i\mathrm{S}i'}$ are independent of each other, so the probability density function of their summation is their convolution, i.e., $f_{I_{\mathrm{S}i}}(u) = f_{I_{\mathrm{S}i\mathrm{M}}}(u) * f_{I_{\mathrm{S}i\mathrm{S}i}}(u) * f_{I_{\mathrm{S}i\mathrm{S}i'}}(u)$, then we get

$$
\begin{aligned}
p(\mathrm{SINR}_{\mathrm{S}i} > \gamma | R_{\mathrm{S}i} = r) &= p(\frac{P_{\mathrm{S}} h_{\mathrm{S}i} r^{-\alpha}}{I_{\mathrm{S}i}} > \gamma) \\
&= \mathbb{E}_{I_{\mathrm{S}i}}[p(h_{\mathrm{S}i} > \frac{\gamma I_{\mathrm{S}i}}{P_{\mathrm{S}} r^{-\alpha}})|I_{\mathrm{S}i}] \\
&\overset{(a)}{=} \mathbb{E}_{I_{\mathrm{S}i}}[e^{-\frac{\gamma I_{\mathrm{S}i}}{P_{\mathrm{S}} r^{-\alpha}}}|I_{\mathrm{S}i}] \\
&= \int_0^{\infty} e^{-\frac{\gamma u}{P_{\mathrm{S}} r^{-\alpha}}} f_{I_{\mathrm{S}i}}(u) du \\
&= \mathcal{L}_{I_{\mathrm{S}i}}(\frac{\gamma}{P_{\mathrm{S}} r^{-\alpha}}) \\
&\overset{(b)}{=} \mathcal{L}_{I_{\mathrm{S}i\mathrm{M}}}(\frac{\gamma}{P_{\mathrm{S}} r^{-\alpha}}) \mathcal{L}_{I_{\mathrm{S}i\mathrm{S}i}}(\frac{\gamma}{P_{\mathrm{S}} r^{-\alpha}}) \mathcal{L}_{I_{\mathrm{S}i\mathrm{S}i'}}(\frac{\gamma}{P_{\mathrm{S}} r^{-\alpha}}),
\end{aligned}
\tag{13}
$$

step (a) is from $h_{\mathrm{M}} \sim exp(1)$, step (b) follows the theorem of convolution, i.e., $\int_0^{\infty} e^{-su}(f_a(u) * f_b(u))du = \int_0^{\infty} e^{-su} f_a(u)du \int_0^{\infty} e^{-su} f_b(u)du$.

The Laplace transform of $\mathcal{L}_{I_{\mathrm{S}i\mathrm{M}}}(\frac{\gamma}{P_{\mathrm{S}} r^{-\alpha}})$ is

$$
\begin{aligned}
\mathcal{L}_{I_{\mathrm{S}i\mathrm{M}}}(\frac{\gamma}{P_{\mathrm{S}} r^{-\alpha}}) &= \mathbb{E}_{I_{\mathrm{S}i\mathrm{M}}}[e^{-\frac{\gamma I_{\mathrm{S}i\mathrm{M}}}{P_{\mathrm{S}} r^{-\alpha}}}|I_{\mathrm{S}i\mathrm{M}}] \\
&= \mathbb{E}_{\Phi_{\mathrm{M}}, \mathrm{h}_{\mathrm{M}}}[\exp(-\frac{\gamma}{P_{\mathrm{S}} r^{-\alpha}}(\sum_{j \in \Phi_{\mathrm{M}}} P_{\mathrm{M}} h_{\mathrm{M}j} R_{\mathrm{M}j}^{-\alpha}))] \\
&\overset{(a)}{=} \mathbb{E}_{\Phi_{\mathrm{M}}}[\prod_{j \in \Phi_{\mathrm{M}}} \frac{P_{\mathrm{S}} r^{-\alpha}}{P_{\mathrm{S}} r^{-\alpha} + \gamma P_{\mathrm{M}} R_{\mathrm{M}j}^{-\alpha}}] \\
&\overset{(b)}{=} \exp(2\pi\lambda_{\mathrm{M}} \int_0^{\infty} (\frac{P_{\mathrm{S}} r^{-\alpha}}{P_{\mathrm{S}} r^{-\alpha} + \gamma P_{\mathrm{M}} x^{-\alpha}} - 1) x dx) \\
&= \exp(\pi\lambda_{\mathrm{M}} \int_0^{\infty} (\frac{P_{\mathrm{S}} r^{-\alpha}}{P_{\mathrm{S}} r^{-\alpha} + \gamma P_{\mathrm{M}} u^{-\frac{\alpha}{2}}} - 1) du) \\
&\overset{(c)}{=} \exp(-\frac{\pi^2}{2} \lambda_{\mathrm{M}} \sqrt{\frac{\gamma P_{\mathrm{M}}}{P_{\mathrm{S}}}} r^2),
\end{aligned}
\tag{14}
$$

step (a) is from $h_{\mathrm{M}} \sim exp(1)$, step (b) is from the probability generating function of PPP [11] and step (c) is from the assumption that the path-loss parameter $\alpha = 4$.

Similar to (14), $\mathcal{L}_{I_{\mathrm{S}i\mathrm{S}i}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}})$ and $\mathcal{L}_{I_{\mathrm{S}i\mathrm{S}i'}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}})$ are

$$\begin{aligned}\mathcal{L}_{I_{\mathrm{S}i\mathrm{S}i}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}}) &= \mathbb{E}_{\Phi_{\mathrm{S}i}}[\prod_{j\in\Phi_{\mathrm{S}i}}\frac{r^{-\alpha}}{r^{-\alpha}+\gamma R_{\mathrm{S}ij}^{-\alpha}}] \\ &= \exp(2\pi q_i\lambda_{\mathrm{S}}\int_r^{\infty}(\frac{r^{-\alpha}}{r^{-\alpha}+\gamma x^{-\alpha}}-1)xdx) \\ &= \exp(-\pi q_i\lambda_{\mathrm{S}}\sqrt{\gamma}r^2(\frac{\pi}{2}-\arctan(\frac{1}{\sqrt{\gamma r^2}})))\end{aligned} \tag{15}$$

$$\begin{aligned}\mathcal{L}_{I_{\mathrm{S}i\mathrm{S}i'}}(\frac{\gamma}{P_{\mathrm{S}}r^{-\alpha}}) &= \mathbb{E}_{\Phi_{\mathrm{S}i'}}[\prod_{j\in\Phi_{\mathrm{S}i'}}\frac{P_{\mathrm{S}}r^{-\alpha}}{P_{\mathrm{S}}r^{-\alpha}+\gamma P_{\mathrm{S}}R_{\mathrm{S}ij}^{-\alpha}}] \\ &= \exp(2\pi(1-q_i)\lambda_{\mathrm{S}}\int_0^{\infty}(\frac{r^{-\alpha}}{r^{-\alpha}+\gamma x^{-\alpha}}-1)xdx) \\ &= \exp(-\frac{\pi^2}{2}(1-q_i)\lambda_{\mathrm{S}}\sqrt{\gamma}r^2)\end{aligned} \tag{16}$$

Similar to (12), the outage probability of the typical user request when associating with MBS (i.e., $\mathbb{P}_{out,\mathrm{M}i}$) is

$$\begin{aligned}\mathbb{P}_{out,\mathrm{M}i} &= \mathbb{E}_{R_{\mathrm{M}i}}[p(\mathrm{SINR}_{\mathrm{M}i}<\gamma|R_{\mathrm{M}i}=r)] \\ &= \int_0^{\infty}(1-\mathcal{L}_{I_{\mathrm{M}i\mathrm{M}}}(\frac{\gamma}{P_{\mathrm{M}}r^{-\alpha}})\mathcal{L}_{I_{\mathrm{M}i\mathrm{S}i}}(\frac{\gamma}{P_{\mathrm{M}}r^{-\alpha}})\mathcal{L}_{I_{\mathrm{M}i\mathrm{S}i'}}(\frac{\gamma}{P_{\mathrm{M}}r^{-\alpha}}))2\pi\lambda_{\mathrm{M}}re^{-\lambda_{\mathrm{M}}\pi r^2}dr,\end{aligned} \tag{17}$$

where $\mathcal{L}_{I_{\mathrm{M}i\mathrm{S}i}}(\frac{\gamma}{P_{\mathrm{M}}r^{-\alpha}})$, $\mathcal{L}_{I_{\mathrm{M}i\mathrm{S}i}}(\frac{\gamma}{P_{\mathrm{M}}r^{-\alpha}})$, $\mathcal{L}_{I_{\mathrm{M}i\mathrm{S}i'}}(\frac{\gamma}{P_{\mathrm{M}}r^{-\alpha}}))$ are

$$\begin{aligned}\mathcal{L}_{I_{\mathrm{M}i\mathrm{M}}}(\frac{\gamma}{P_{\mathrm{M}}r^{-\alpha}}) &= \mathbb{E}_{\Phi_{\mathrm{M}}\backslash\mathrm{M}0}[\prod_{j\in\Phi_{\mathrm{M}}\backslash\mathrm{M}0}\frac{r^{-\alpha}}{r^{-\alpha}+\gamma R_{\mathrm{M}j}^{-\alpha}}] \\ &= \exp(\frac{1}{\sqrt{\gamma}r^2}(\frac{\pi}{2}-\arctan(\frac{1}{\sqrt{\gamma r^2}})))\end{aligned} \tag{18}$$

$$\begin{aligned}\mathcal{L}_{I_{\mathrm{M}i\mathrm{S}i}}(\frac{\gamma}{P_{\mathrm{M}}r^{-\alpha}}) &= \mathbb{E}_{\Phi_{\mathrm{S}i}}[\prod_{j\in\Phi_{\mathrm{S}i}}\frac{P_{\mathrm{M}}r^{-\alpha}}{P_{\mathrm{M}}r^{-\alpha}+\gamma P_{\mathrm{S}}R_{\mathrm{S}ij}^{-\alpha}}] \\ &= \exp(-\pi q_i\lambda_{\mathrm{S}}\sqrt{\frac{\gamma P_{\mathrm{S}}}{P_{\mathrm{M}}}}r^2(\frac{\pi}{2}-\arctan(\frac{R_{\max}}{\sqrt{\frac{\gamma P_{\mathrm{S}}r^4}{P_{\mathrm{M}}}}})))\end{aligned} \tag{19}$$

$$\begin{aligned}\mathcal{L}_{I_{\mathrm{M}i\mathrm{S}i'}}(\frac{\gamma}{P_{\mathrm{M}}r^{-\alpha}}) &= \mathbb{E}_{\Phi_{\mathrm{S}i'}}[\prod_{j\in\Phi_{\mathrm{S}i'}}\frac{P_{\mathrm{M}}r^{-\alpha}}{P_{\mathrm{M}}r^{-\alpha}+\gamma P_{\mathrm{S}}R_{\mathrm{S}ij}^{-\alpha}}] \\ &= \exp(-\pi\lambda_{\mathrm{M}}\sqrt{\gamma}r^2(\frac{\pi}{2}-\arctan(\frac{1}{\sqrt{\gamma}r})))\end{aligned} \tag{20}$$

Thus, we can obtain the AOP of different caching policies by (11).

4 Numerical Results

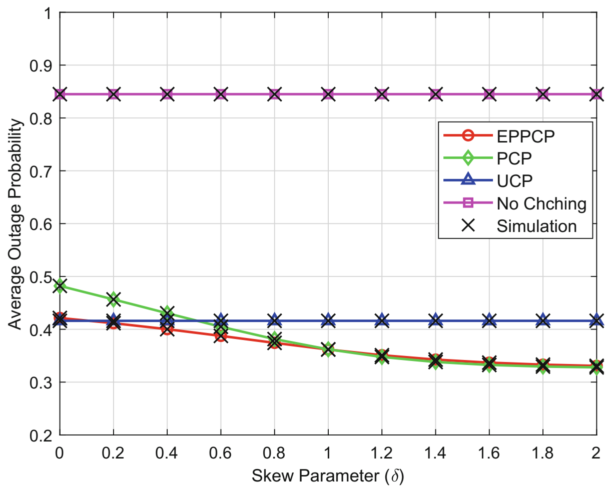

Fig. 2. Outage probability v.s. skew parameter. $\lambda_S = 1000\,\text{km}^{-1}$, $\beta = 1$.

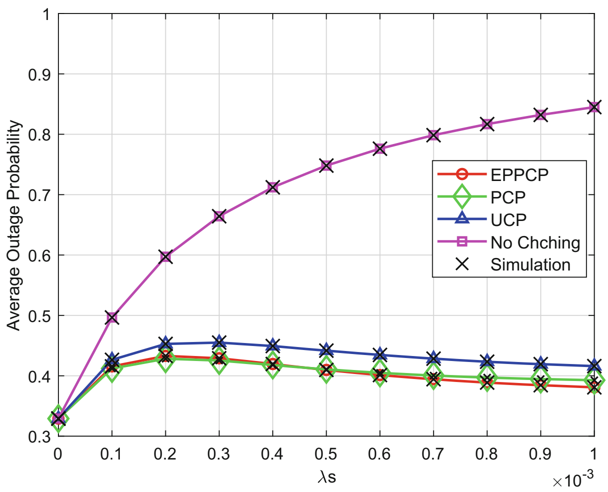

Fig. 3. Outage probability v.s. λ_S. $\delta = 1$, $\beta = 1$.

In this section, we present the numerical analysis of the outage probability, and verify the analysis results with 10,000-time Monte-Carlo simulations of a $5\,\text{km} \times 5\,\text{km}$ size network. γ is $20\,\text{dBm}$, P_M is $50\,\text{dBm}$, P_S is $30\,\text{dBm}$, $M = 70$, $N = 100$, $R_{\max} = 30\,\text{m}$, $\lambda_M = 10\,\text{km}^{-1}$.

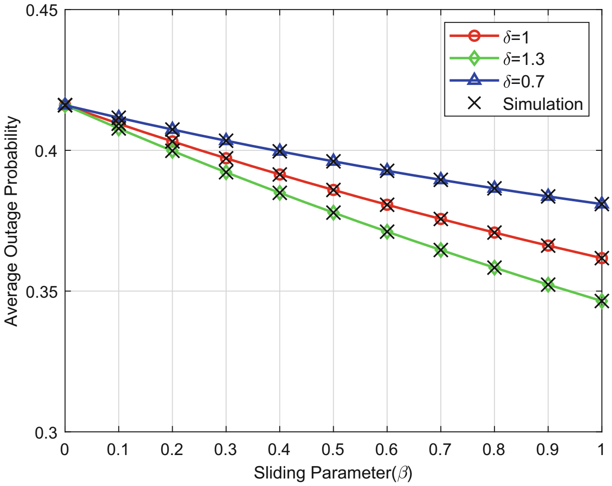

Fig. 4. Outage probability v.s. sliding parameter. $\lambda_{\mathrm{S}} = 1000\,\mathrm{km}^{-1}$.

Figure 2 shows the AOP for different caching policies as the function of skew parameter δ. We can see the results of the performance analysis match the simulation results, the red line, i.e., the AOP of EPPCP is lower than others at the range of (0.2, 1.0). When there's no cache deployed on SBSs (i.e. $q_i = 0$, $i = 1, 2, ..., N$), from (12) we have $\mathbb{P}_{out,\mathrm{S}i} = 1$, $i = 1, 2, ..., N$, since the typical user associates with the nearest MBS regardless of the file it requests and the interference produced by SBSs keeps unchanged with i, the SINR does not change with δ, resulting in the AOP keeps stable. When using UCP, the outage probability when requesting the ith file keeps unchanged, resulting in the AOP stabilizes with δ. Since the requesting probability gap between the high-ranking files and the low-ranking files is not large between (0,1), the EPPCP has lower $\mathbb{P}_{out,\mathrm{S}i}$ when requesting low-ranking files than PCP, resulting in lower AOP. On the contrary, the AOP of EPPCP is higher than UCP between (0,0.2) for the same reason. Since δ has the typical value of 0.5$\sim$1.0 [12], EPPCP would be a better option than other caching policies.

Figure 3 shows the AOP as the function of λ_{S}. We find that for different caching policies, the AOP firstly grows larger, that's because when $\lambda_{\mathrm{S}} = 0$ (i.e. there is no SBS), there's no interference produced by SBS, which is the main part of the total interference, resulting in large SINR. As λ_{S} grows larger, the AOP turns to fall since the average distance to the associating SBS gets shorter, resulting in larger signal power received and larger SINR. Thus, when we build ultra-dense HetNet with cache-enabled SBS, we would sacrifice a little bit of AOP in exchange for the release of backhaul link pressure and lower time delay. Unlike the above, when there's no cache deployed, the AOP could only rise with λ_{S} since there's only interference increasing caused by higher densed SBSs.

Figure 4 shows the AOP as the function of β when using EPPCP. We find that the AOP becomes smaller with β, which means the performance becomes better with a more skewed caching policy.

5 Conclusion

In this paper, we applied an equal proportional probability caching policy with SBS-priority connecting strategy on cache-enabled heterogeneous celluar network. By modeling the spatial distribution of the BSs as two independent Poisson Point Processes, we obtained the average outage probability for a typical user's request. The simulation result showed that the proposed policy resulted in lower AOP than the classical policies when δ is in the interval (0.2, 1.0). In addition, we investigated the relationship between the density of SBS and AOP, which concluded the high-densed SBSs would sacrifice a little bit of AOP in exchange for the release of backhaul link and lower time delay. Lastly, we investigated the relationship between the sliding parameter and AOP, which concluded the performance becomes better with a more skewed caching policy. This paper highlights that the new caching policy performs better than the conventional policies under cache-enabled HetNets, underscoring the fact that using high-densed network in conjunction with a more skewed caching policy would obtain a better system performance.

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