Sparse Algorithm for OFDM Underwater Acoustic Channel Estimation

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Abstract. Channel estimation is very important and challenging for underwater acoustic (UWA) systems that use orthogonal frequency division multiplexing (OFDM) technique. The conventional methods are not appropriate to the severe frequency selective fading channel; also current channel estimation algorithms do not use the sparse characteristics of the UWA channel efficiently. In this paper, a novel algorithm about channel estimation is addressed that applies channel sparse features. First, Least Square (LS) algorithm is used to get the pilot channel valuation. Secondly, the sparse degree of channel is estimated through DFT for noise reduction processing. Then the autocorrelation matrix of the channel is obtained approximately. Finally a preliminary calculation of the error threshold is acquired, and the high quality data subcarrier channel impulse can be estimated by the pilot symbols through using the orthogonal matching pursuit (OMP) algorithm. In terms of simulating, we study the performance of the system through bit error rate (BER) and the constellation diagram, and it is indicated that the new method has excellent performance at less computation. So it is very important that the method can be implemented in OFDM systems.

Keywords: Underwater acoustic communications · OFDM · Channel estimation · OMP algorithm

1 Introduction

Due to the large delay, time-varying and multipath, the channel of UWA is one of the most challenging wireless channels [1]. Recently the UWA channels are proved to have sparse feature, namely that the most energy of the channel is focused on a few channels. Considering the sparse nature of the channels, many sparse methods about the channel estimation have been addressed. OFDM technique has low computation cost and can resist frequency selective fading. This promotes the application of OFDM in wireless underwater acoustic communication systems. Over the past decade, the sparse estimation methods based compressed sensing (CS) are widely employed. There are many algorithms about CS. Basic Pursuit (BP) and OMP methods belong to greedy algorithms among them, and the two ways studied as a solution to the UWA channel estimation [2, 3]. In the literature [4], through exploiting the sparse feature of the channel, some estimation algorithms described have been mainly proposed. In the works [5, 6], BP and

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other OMP methods have been verified, and through simulating and experimenting, it was indicated that BP has better performance than OMP for the UWA channel. However, the BP method has much higher computational complexity. In [7], through comparing conventional CS methods, a field test have been done by exploiting joint sparse nature about the continuous OFDM blocks. In the experiment, the authors verified the superiority of the method. In addition, the LS method used is greatly affected by noise for channel estimation in the system [8]. To solve the above problems, this paper proposes an optimization algorithm based on CS about the noise of the LS method. Combined with CS algorithm, pilot data are used to optimize the LS channel estimation at the receiving end. Simulation results and theoretical analysis indicate that the impact of Gaussian white noise can be reduced in the process of the channel estimation.

This thesis will make the following structure arrangement. In the second part of the paper, we will introduce the model of signal and system. Section 3 briefly explores the error threshold values about OMP approach and the details proposed method for estimating UWA channel. The results of simulation will be contained in Sect. 4. Conclusions are summed up in Sect. 5.

2 Signal and System Model

2.1 UWA Sparse Channel Model

We usually use the model of time-varying channel for underwater acoustic communication [9]

$$h(t) = \sum_{k=0}^{L-1} A_k(t) e^{j\beta_k(t)} \delta(t - \tau_k(t)) + g(t) \tag{1}$$

where, let's say there are L paths in the channel. A_k and τ_k denote the gain and delay of the kth path respectively. β is related to the Doppler shift, and g denotes noise. In addition, the gain and delay of the propagation path will vary greatly over a long period of time [10], it is assumed that the path delay remains constant in several adjacent OFDM blocks, and that Doppler scale factor and the path gain are stable in one block, but different from one block to another.

2.2 OMP Algorithm Model

OMP is one algorithm about CS theory that is given in the form as follows

$$\boldsymbol{v} = \boldsymbol{\Phi u} = \boldsymbol{\Phi \Psi \theta} \tag{2}$$

where, $\boldsymbol{\Phi}$ represents $M \times N$ measurement matrix and $\mathbf{v}$ the $M \times 1$ measurement vector; $\boldsymbol{\Psi} = [\psi_1, \psi_2, \cdots, \psi_N]$ is $N \times N$ orthonormal basis matrix and $\boldsymbol{u} \in R^N$ the $N \times 1$ column vector. Usually in vector $\boldsymbol{\vartheta}$, there are some K non-zero elements that $K \ll N$. In addition, $\mathbf{v}$ can be acquired from a K-sparse $\boldsymbol{u}$ by using $\boldsymbol{\Phi}$. Due to $M \ll N$, Eq. (2) is underdetermined. Therefore, in order to search for the optimal one in the solution space, an appropriate algorithm is needed.

BP algorithm and OMP algorithm are two typical search optimization algorithms in common use [11]. For the second algorithm, in order to achieve sparse recovery, the minimum sparse estimation can be obtained by optimizing the constrained norm problem [12].

$$\hat{\theta} = \arg\min \|\boldsymbol{\vartheta}\|_1 \; s.t. \; \|\boldsymbol{\Phi}\boldsymbol{\Psi}\boldsymbol{\vartheta} - \boldsymbol{v}\|_2 \leq \varepsilon \tag{3}$$

where, ε is the l_2 norm threshold value.

2.3 System Model of OFDM

In one OFDM cycle, $X(k)$ represents the complex information code to be transmitted on the kth subcarrier. Doing N point IDFT, the following time domain expression will be got

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi nk/N}, \quad n = 0, 1, \cdots, N-1 \tag{4}$$

Then the discrete baseband data in the receiver will be obtained in the following form

$$y(n) = h(n) * x(n) + g(n) \tag{5}$$

Next, the frequency domain model matrix of Eq. (1) will be as follows

$$\boldsymbol{H} = \boldsymbol{F}h + \boldsymbol{G} \tag{6}$$

where $\boldsymbol{F}$ is $N \times L$ DFT matrix. Therefore, the frequency domain data of the pilot subcarrier in the following receiver will be obtained

$$\boldsymbol{Y}_p = diag(\boldsymbol{X}_p)\boldsymbol{F}_p h + \boldsymbol{G}_p \tag{7}$$

where $\boldsymbol{F}_p$ is $N_p \times L$ DFT matrix and $\boldsymbol{X}_p$ denotes pilots data transmitted.

When the number N_p is less than L, Eq. (7) is a problem to solve the underdetermined equation. Then h can be calculated through knowing ε.

$$\hat{h} = \arg\min \|h\|_1 \; s.t. \; \left\|diag(\boldsymbol{X}_p)\boldsymbol{F}_p h - \boldsymbol{Y}_p\right\|_2 \leq \varepsilon \tag{8}$$

To the OMP algorithms, the sparse degree is unknown, so we need to know the threshold value ε that determines the time of iterations. On the basis of LS algorithm, this paper proposes an innovative algorithm which uses pilot frequency to set threshold.

3 Estimation of Error Threshold Values

The pilot channel estimation is obtained by LS channel estimation.

$$\hat{H}_{\mathrm{p}}(k) = \frac{Y_{\mathrm{p}}(k)}{X_{\mathrm{p}}(k)}, \; k = 0, 1, 2, \cdots, N_{\mathrm{P}} - 1 \tag{9}$$

These data are then carried out N_P point IDFT to obtain the point time domain sequence as fellows

$$\hat{h}_p(n) = \frac{1}{N_p}\sum_{k=0}^{N_P-1}\hat{H}_p(k)e^{j\frac{2\pi k}{N_P}n} \tag{10}$$

Then denoising is performed for Eq. (10), and L effective paths are reserved for each response value and the rest are set as “0”. If the delay information of the channel is unknown, the circular prefix can be retained or the value of L can be determined by setting noise threshold. The N_p point time domain sequence is obtained after denoising is given by

$$h_p(n) = \begin{cases} \hat{h}_p(n)\, n = 0, 1, \cdots, L-1 \\ 0\, n = L, L+1, \cdots, N_p - 1 \end{cases} \tag{11}$$

Then Let $\hat{h}_p(n)$ be the sequence acquired by using the N_p-IDFT, the channel response estimation at the pilot frequency after denoising is finally obtained as

$$H_P(k) = \sum_{n=0}^{N_P-1} h_p(n)e^{j\frac{2\pi k}{N_P}n} \tag{12}$$

$\boldsymbol{M}$ denotes the sum of channel estimation error and Gaussian white noise, and it can be acquired

$$\boldsymbol{M} = diag(\boldsymbol{X}_P)\left(\boldsymbol{H}_P - \hat{\boldsymbol{H}}_P\right) + \boldsymbol{G}_P \tag{13}$$

where, $\boldsymbol{H}_P$ and $\hat{\boldsymbol{H}}_P$ denote true value and estimated value of LS at the pilot frequency response respectively.

Then let $\boldsymbol{C} = \boldsymbol{H}_P - \hat{\boldsymbol{H}}_P$, Eq. (13) becomes

$$\boldsymbol{M} = diag(\mathbf{X}_P)\mathbf{C} + \mathbf{G}_P \tag{14}$$

The noise $\boldsymbol{M}$ can be assumed a multivariable normal distribution with zero mean value, and its covariance matrix can be expressed by the following equation

$$\boldsymbol{R}_{MM} = diag(\boldsymbol{X}_P)\boldsymbol{R}_{CC}diag(\boldsymbol{X}_P)^{H} + \sigma^2\boldsymbol{I}_{N_P} \tag{15}$$

where, σ^2 denotes the variance of Gaussian white noise and it can be obtained based on LS channel estimation; $\boldsymbol{R}_{CC}$ denotes the covariance matrix of $\boldsymbol{C}$, which is calculated by the following formula

$$\boldsymbol{R}_{CC} = \boldsymbol{R}_{\boldsymbol{H}_P\boldsymbol{H}_P} - \boldsymbol{R}_{\boldsymbol{H}_P\boldsymbol{H}_P}\left(\boldsymbol{R}_{\boldsymbol{H}_P\boldsymbol{H}_P} + \frac{\beta}{SNR}\boldsymbol{I}_{N_P}\right)^{-1}\boldsymbol{R}_{\boldsymbol{H}_P\boldsymbol{H}_P} \tag{16}$$

where, β is a constant, and its value is related to the specific modulation mode; SNR represents the signal-to-noise ratio, and $\boldsymbol{I}_{N_P}$ is $N_P \times N_P$ unit matrix; $\boldsymbol{R}_{\boldsymbol{H}_P\boldsymbol{H}_P}$ denotes the channel autocorrelation $N_P \times N_P$ matrix at pilot frequency. Since it is difficult to obtain the channel autocorrelation matrix, this paper adopts DFT transform domain denoising processing on the basis of LS algorithm to obtain the approximate autocorrelation matrix. According to Eq. (15), the following types can be calculated [13]

$$E\left(\|\boldsymbol{M}\|_2^2\right) = trace(\boldsymbol{R}_{MM}) \tag{17}$$

$$E\left(\|\boldsymbol{M}\|_2^4\right) = \sum_{i=1}^{N_P}\sum_{j=1}^{N_P}\left[R_{MM}(i,i)R_{MM}(j,j) + \|R_{MM}(i,j)\|^2\right] \tag{18}$$

where, $trace(\cdot)$ indicates tracing of a matrix; $R_{MM}(i,j)$ represent the element at the position of row i and column j of the matrix. Then the standard deviation of $\|\boldsymbol{M}\|_2^2$ is calculated by Eqs. (17) and (18)

$$\mathrm{std}\left(\|\boldsymbol{M}\|_2^2\right) = \sqrt{E\left(\|\boldsymbol{M}\|_2^4\right) - \left(E\left(\|\boldsymbol{M}\|_2^2\right)\right)^2} \tag{19}$$

According to the knowledge of probability theory, $\|\boldsymbol{M}\|_2^2$ generally satisfies the following formula

$$\|\boldsymbol{M}\|_2^2 \le E\left(\|\boldsymbol{M}\|_2^2\right) + 2\cdot\mathrm{std}\left(\|\boldsymbol{M}\|_2^2\right) \tag{20}$$

To sum up, the error tolerance value ε can be approximated as fellows

$$\varepsilon \approx \sqrt{E\left(\|\mathbf{M}\|_2^2\right) + 2\cdot\mathrm{std}\left(\|\mathbf{M}\|_2^2\right)} \tag{21}$$

4 Simulation Results

The detailed system specifications of the simulation are shown in Table 1.In the simulation process, the performance of the new method is tested mainly by detecting the BER curve and constellation diagram. Assume that time synchronization and frequency synchronization is ideal. In addition, in order to highlight the performance of the new algorithm, the additive white Gaussian noise is the only factor affecting the performance of the algorithm in UWA channel.

Table 1. Specifications of OFDM system

Parameter	Value	Parameter	Value
FFT size	2048	Cyclic prefix duration	10.7 ms
Carrier frequency	20 kHz	Modulation order	16-QAM

The OMP algorithm proposed in this paper can use the sparse characteristics of UWA channel to overcome the noise. In order to verify the performance of the new algorithm in channel estimation, Fig. 1 shows the bit error rate curve comparison of the results of the LS algorithm and the OMP algorithm. Because the traditional LS method cannot eliminate the Gaussian noise of the receiver, the bit error rate of the system will be reduced seriously. In addition, the above conclusion can be verified according to the constellation diagram in Fig. 3 and Fig. 4.

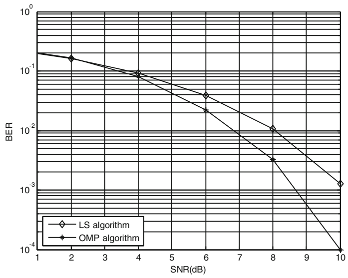

Fig. 1. BER performance of different algorithms

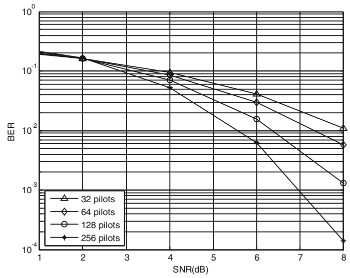

Fig. 2. BER performance of different pilots

Pilot subcarriers are applied to roughly estimate the channel based on OMP algorithm, so the number of pilot subcarriers has some influence on the new algorithm. The influence of bit error rate curve on the number of different pilot subcarriers will be simulated and analyzed. As can be seen from Fig. 2, the system performance will decrease with the reduction of pilot frequency. This is because the frequency interval of the pilot frequency has a great influence on the LS algorithm, that is, the LS algorithm will affect the calculation of the threshold ε.

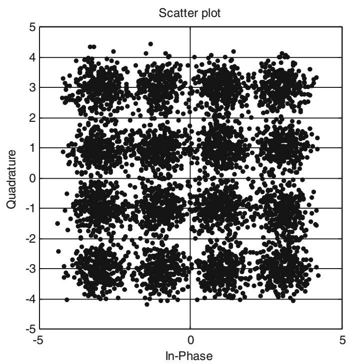

Fig. 3. Constellation diagram of LS algorithms

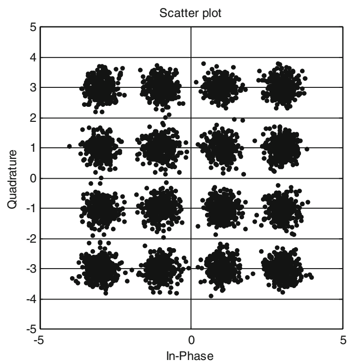

Fig. 4. Constellation diagram of OMP algorithms

5 Conclusions

In this paper, a new UWA channel estimation method based on OMP algorithm is proposed by using the sparsity of UWA channel and CS processing method, combined with LS method. The algorithm can make the receiver estimate the channel pulse accurately and greatly reduce the Gaussian noise. Both theoretical analysis and simulation results indicate that the algorithm is effective in reducing Gaussian noise, and the complexity is low. In conclusion, compared with the traditional LS method, the improved method is more practical for high-speed real-time OFDM communication on the underwater acoustic system.

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