Parameter Estimation of MIMO Radar Based on the OMP Algorithm

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Abstract. This paper introduces the concept of compressed sensing (CS) into parameter estimation, and proposes a Multiple-input Multiple-output (MIMO) radar parameter estimation algorithm based on the Orthogonal Matching Pursuit (OMP) algorithm. In this algorithm, MIMO radar received signals are represented by joint sparse representation by establishing a sparse base. Then we use the random gaussian observation matrix and the OMP algorithm to reconstruct the target space to finish the estimation of target parameters. Moreover, the algorithm in this paper considers the radar signal model when array error exists, mainly discusses the influence of array amplitude error and phase error on the parameter estimation of MIMO radar based on OMP algorithm. Then the root mean square error (RMSE) of OMP algorithm and Compressive Sampling Matching Pursuit (COSAMP) algorithm are compared when the array error exists. Simulation shows that when the array amplitude and phase error exists, the estimation accuracy of the target's reflection amplitude and target parameters are reduced, and the OMP algorithm has a lower mean square error than the COSAMP algorithm. In conclusion, the proposed algorithm has high precision in parameter estimation. Even when array error exists, the OMP algorithm still has better performance in parameter estimation of MIMO radar target's reflection amplitude, azimuth Angle and pitch Angle than the COSAMP algorithm.

Keywords: Parameter estimation · The OMP algorithm · Amplitude error and phase error of array · MIMO radar

1 Introduction

The concept of MIMO radar and its related signal processing technologies have attracted more and more attention from scholars all over the world. Many scholars have studied the parameter estimation performance of MIMO radar. In recent years, scholars in the field of signal processing began to use the compressed sensing theory, by using the existing classic method (such as transformation coding, optimization algorithm, etc.) to solve the problem of high rate of analog to digital conversion [1–5]. When the sampling frequency is much less than Nyquist sampling rate, it obtains the discrete samples of the signal through random sampling, and then perfectly reconstructs the signal through the non-linear reconstruction algorithm. Paper [6] is based on the sparsity of the target

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in the angle-doppler-range domain, by using the CS theory, the paper obtains the joint estimation of the target angle-doppler-range information, so as to realize the super resolution of MIMO radar. Paper [7] utilizes the sparsity of the direction of arrival (DOA) in angular space to estimate the super-resolution parameters of MIMO radar with CS at a low sampling rate. Paper [8] establishes a target information perception model, uses compressed sensing to sample the target echo at a rate lower than Nyquist sampling rate, and extractes target scene information under noise background from a small amount of sampled data.

Both the traditional algorithm and the target reconstruction method based on compressed sensing are premised on the exact known array manifold. However, array errors are often unavoidable in the actual situation, so it is of great practical significance to discuss how to achieve robust target Angle estimation when array errors exist. Paper [9] proves the influence of array model error on the resolution of subspace class algorithms. Paper [10] proposed a global correction method, which comprehensively considered the position, phase and gain errors of array elements, and realized the purpose of simultaneous correction of multiple error parameters, which was relatively consistent with the actual application conditions.

This paper studies parameter estimation based on compressed sensing, mainly discusses the influence of amplitude and phase errors of array elements on the accuracy of target reflection amplitude, azimuth Angle and pitch Angle estimation when using the OMP reconstruction algorithm [11] to estimate the direction of arrival of the target, and compares the estimation performance of parameters based on the OMP algorithm and the COSAMP algorithm [12] when array error exists.

2 System Model

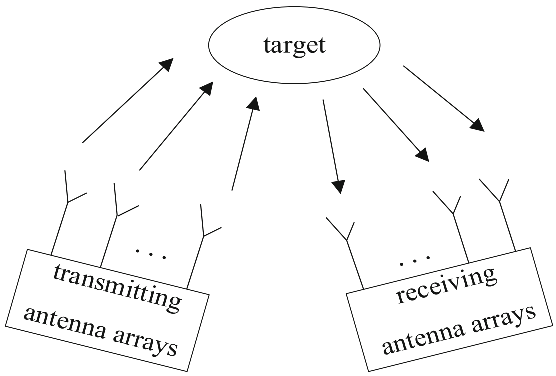

Fig. 1. Structural model of MIMO radar.

We establish a bistatic signal model, in which the transmitting and receiving antenna arrays are equidistant linear arrays, and the transmitting and receiving arrays are remotely separated. The radar system has M transmitting antennas and N receiving antennas. The space between transmitting and receiving antennas is d_t and d_r, respectively. To ensure that the received signal of each receiving signal unit does not have resolution ambiguity, the spacing of receiving array elements should meet the half wavelength condition, that is, $d_r \leq \lambda/2$. Assume that the far field has K objects, the K *th* object's azimuth is (θ_k, φ_k), and $K < MN$. θ_k is the direction of departure, and φ_k is the direction of arrival.

For bistatic MIMO radar, when there are multiple targets in space, and the MIMO radar transceiving array contains amplitude-phase error, the received data can be expressed as follows:

$$X_{n-error} = A_{Mr-error}(\varphi_k)\eta A_{Mt-error}^{T}(\theta_k)s + W \tag{1}$$

$A_{Mt-error}(\theta_k) = \Gamma_{Mt}A_{Mt}(\theta_k)$ and $A_{Mr-error}(\varphi_k) = \Gamma_{Mr}A_{Mr}(\varphi_k)$ are the guidance vector matrix with errors in the transmit and receive arrays respectively. $\Gamma_{Mr} = \mathrm{diag}[\rho_{Mr1}e^{j\phi_{Mr1}}, \rho_{Mr2}e^{j\phi_{Mr2}}, \ldots, \rho_{Mrn}e^{j\phi_{MrN}}]^T$ is diagonal matrix, which represents the amplitude and phase error of N receiving array elements. $\Gamma_{Mt} = \mathrm{diag}[\rho_{Mt1}e^{j\phi_{Mt1}}, \rho_{Mr2}e^{j\phi_{Mt2}}, \ldots, \rho_{Mtn}e^{j\phi_{MtM}}]^T$ represents the amplitude and phase error of N Transmitting array elements. The ρ_{Mti} and ϕ_{Mti} are the i*th* a transmitting array amplitude and phase errors respectively.

$A_{Mr}(\varphi_k)$ and $A_{Mt}(\theta_k)$ are the ideal receiving guidance vector and emission guidance vector matrix of K targets, respectively.

$$A_{Mr}(\varphi_k) = [a_{M1}(\varphi_1), a_{M2}(\varphi_2), \ldots, a_{Mr}(\varphi_k)] \tag{2}$$

$$A_{Mt}(\theta_k) = [a_{M1}(\theta_1), a_{M2}(\theta_2), \ldots, a_{Mt}(\theta_k)] \tag{3}$$

$$a_{Mt}(\theta_k) = [1, e^{j2\pi d_t \sin\frac{\theta_k}{\lambda}}, \ldots, e^{j2\pi d_t \sin\theta_k (M-1)/\lambda}]^T \tag{4}$$

$$a_{Mr}(\varphi_k) = [1, e^{j2\pi d_r \sin\frac{\varphi_k}{\lambda}}, \ldots, e^{j2\pi d_r \sin\varphi_k (N-1)/\lambda}]^T \tag{5}$$

$\eta = [\eta_1, \eta_2, \ldots, \eta_k]^T$ is the amplitude of k target reflected signals. S is M orthogonal independent waveforms transmitted by the system. $W \in C^{N \times L}$ is the complex white gaussian noise.

3 Parameter Estimation Principle Based on the OMP Algorithm When Array Error Exists

We discrete each target angle space into $P_1 \times P_2$ directions. Define α as angle P_1 of the signal's direction of departure space. Define β as angle P_2 of the signal's direction of arrival space. $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_{P_1}]^T$, $\beta = [\beta_1, \beta_2, \ldots, \beta_{P_2}]^T$.

When the k *th* goal exists in $(\alpha_{P_1}, \beta_{P_2})$, then $\sigma_{P_1P_2} = \eta_k$. When $(\alpha_{P_1}, \beta_{P_2})$ have no target, then $(\alpha_{P_1}, \beta_{P_2}) = 0$.

Then the received signal of bistatic uniform linear array MIMO radar when array error exists can be expressed as:

$$X_{n-error} = \sum_{P_2=1}^{P_2}\sum_{P_1=1}^{P_1} A_{Mr-error}(\beta_{P_2})\sigma_{P_1P_2}A_{Mt-error}^{T}(\alpha_{P_1})s + W \tag{6}$$

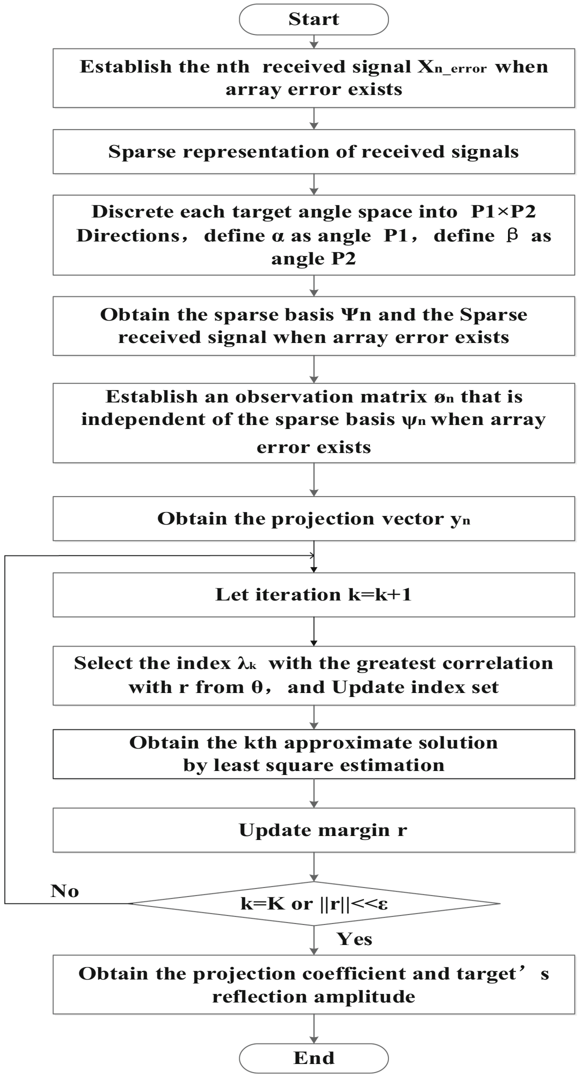

Fig. 2. Flowchart of the algorithm.

Suppose:

$$\psi_n = [A_{Mr-error}(\beta_1)S^T A_{Mt-error}(\alpha_1), \ldots A_{Mr-error}\left(\beta_{P_2}\right)S^T A_{Mt-error}(\alpha_1), \ldots$$
$$A_{Mr-error}(\beta_1)S^T A_{Mt-error}\left(\alpha_{P_1}\right), \ldots, A_{Mr-error}\left(\beta_{P_2}\right)S^T A_{Mt-error} \quad (7)$$

And:

$$\sigma = [\sigma_{11}, \ldots, \sigma_{P_2 1}, \ldots, \sigma_{P_2 P_1}, \ldots, \sigma_{P_2 P_2}]^T \quad (8)$$

Then:

$$X_n^{'} = (\psi_n \sigma)^T \quad (9)$$

The transformation basis matrix ψ_n(n = 1,2,…L) has been determined. It is the sparse basis containing the phase information of the array element, σ is the sparse vector of the signal on the sparse basis. The position of the non-zero element P_2P_1 indicates the target Angle, and its value is the target's reflection amplitude. There are a total of n received signals, and the two-dimensional angle of the target determines the common coefficient structure of these signals. The number of the target is sparsity, and the target's reflection amplitude is the non-zero coefficient of each received signal in the transformation domain. Then take a smooth random gaussian M × L observation matrix ϕ_n (n = 1,2,…M), it is uncorrelated with transformation basis matrix ψ_n, and M < L. Therefore, the projection vector of the received signal $X_n^{'}$ (n = 1,2,…L) of the n*th* receiving antenna on the observation matrix ϕ_n is:

$$y_n = \phi_n X_n^{'T} = \phi_n \psi_n \sigma = \Theta_n \sigma \quad (10)$$

Θ_n(n = 1,2,…M) is Perception matrix.

The basic steps of the parameters of the OMP algorithm are as follows:

The input of the OMP algorithm are perception matrix Θ, measurement vector y_n, signal sparsity k and error threshold ε。

The output of the OMP algorithm are residual component r = $y_n(n = 1, 2 \ldots N)$, index set Ω = ϕ, number of iterations k = 1, estimation of signal sparse coefficient $\hat{x}$ and support domain $\widehat{\Omega}$。

(1) Select the index λ_k corresponding to the column with the greatest correlation with r from Θ, $\lambda_k = arg\max(\Theta_n{}^H r)(1 \ll n \ll N)$, Θ_n represents the n*th* column of the Θ;
(2) Update index set $\Omega = \Omega \bigcup \lambda_k$;
(3) Obtain the approximate solution by least square estimation: $\hat{x}_k = argmin\left|\left|y - \Theta_\Omega X_n^{'}\right|\right|_2$, where $\hat{x}_k$ is the least squares approximate solution of the K*th* iteration, Θ_Ω is a matrix consisting of columns indicated by Ω in Θ;
(4) Update margin $r = y_n - \Theta_\Omega - \hat{x}_k$;
(5) Judge whether the iteration satisfies the stopping condition: k = K or $||r||_2 \ll \varepsilon$. If it is satisfied, then stop the iteration, output $\hat{x} = \hat{x}_k$ and $\widehat{\Omega} = \Omega$. Otherwise, let k = K + 1, and then turn back to the first step.

The index set Ω is determined by the iteration of the OMP algorithm, and Ω represents the position of K non-zero elements, and the values of these elements correspond to the projection coefficient $\hat{x}_k$ (k = 1, 2, …, K) at this time. The projection coefficients of subspaces formed by observation signal $y_n(n = 1, 2 \ldots N)$ for K atoms correspond to non-zero elements in $X_n^{'}(n = 1, 2 \ldots L)$. Therefore, the sparse signal $X_n^{'}$ to be reconstructed can be determined.

4 Simulation

Bistatic MIMO radar, $d_t = d_r \leq \lambda/2$, M = 6, N = 6. Transmitted signal's coding length is 128 L and the received signal's SNR is 5 dB. DOA and DOD's observation intervals are all [−90, 90]. The discrete interval is 5° when constructing sparse base. We assume that there are three targets in radar observation space, DOD and DOA are (−60, −30), (0, 0), (20, 50), the reflection amplitudes are 5, 2 and 3, respectively. Random values of the amplitude error of the transmitting element is 2 (1, M), the phase error is −1 + 2 (1, M), the amplitude error of the receiving element is 2 (1, N), and the phase error is −1 + 2 (1, N).

According to Table 1, on the basis of joint sparse representation of received signals, the weighted OMP algorithm can overcome array noise and achieve accurate estimation of the target's reflection amplitude. When array amplitude and phase errors exist, the accuracy of the algorithm is reduced.

Table 1. Estimation of target reflection amplitude

	Reflection amplitude estimation without Array error			Reflection amplitude estimation with Array error		
(DOD,DOA)	(−60,−30)	(0, 0)	(50, 30)	(−60, 30)	(0, 0)	(50, 30)
r = 1	4.8970	1.8171	2.5170	4.1655	1.7932	3.3294
r = 2	4.8620	1.7851	2.9863	4.2377	1.8326	3.0418
r = 3	4.9603	1.9087	3.2583	3.2743	1.6751	2.6201
r = 4	5.1219	1.9012	2.9426	5.5902	1.4689	2.7151
r = 5	4.8165	1.7412	2.8379	4.2156	1.2983	2.8145
r = 6	4.9714	1.8892	3.2550	4.2846	1.9976	2.6601
Average	4.9381	1.8404	2.9751	4.2945	1.6776	2.6968
Real	5	2	3	5	2	3

As shown in Fig. 1, when SNR = 5 dB, the real position of the target and the estimated position are highly coincident. When the array error exists, the estimation accuracy of the target angle is reduced, and the estimation position of the target is biased.

Figure 2 shows the relationship between the target angle estimation and the signal-to-noise ratio by using the OMP reconstruction algorithm when amplitude and phase error exists or not. The root mean squard error of the target angle estimate used here for comparison takes the average of 3 targets.

It can be more intuitive to see from the Fig. 3, the angle estimation performance curve of the target with the amplitude and phase error is obviously inferior to the angle estimation performance curve when the array has no phase and amplitude error, and the error of the target estimation is large, the estimation accuracy is significantly reduced. When the SNR is increased from 0 to 25 dB at 5 dB intervals, as the signal-to-noise ratio increases, the estimation accuracy becomes higher and higher. The estimated gain performance of the algorithm is obvious between 0 and 10 dB. Continued increase of the signal-to-noise ratio has a lower and lower gain effect on the MIMO radar target parameter estimation. The root mean squard error of the OMP algorithm with array error exists is lower than the root mean squard error of the COSAMP algorithm with array error exists, which indicates that the OMP algorithm used in this paper is better than the COSAMP algorithm in estimating the parameters when array error exists (Fig. 4).

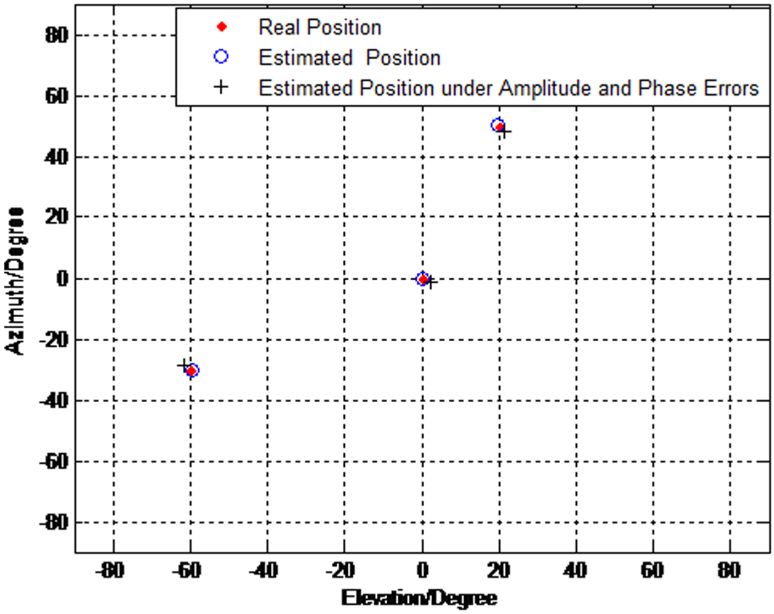

Fig. 3. Comparison of real angle, ideal estimated angle and estimated angle when array error exists

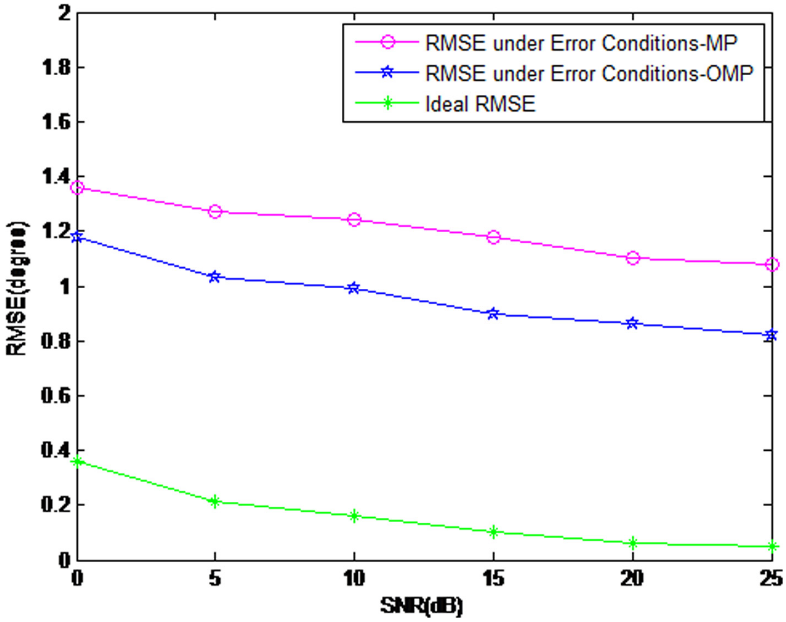

Fig. 4. Relationships between the mean square error of the angle and the signal to noise ratio

5 Conclusion

In this paper, a parameter estimation algorithm for MIMO radar based on the OMP algorithm is proposed. The effects of the array error on the parameter estimation based on the OMP algorithm are studied. Through the above simulations, it is found that the amplitude and phase error of the array error affects the MIMO radar received signal model. At this time, when the parameters are estimated by the OMP algorithm, the accuracy of the estimation is reduced both in the reflection amplitude of the target and in the parameters of the target. But in this case, the OMP algorithm is still better than the COSAMP algorithm in parameter estimation. In summary, the proposed algorithm has high accuracy in parameter estimation. Even when array error exists, the performance of the OMP algorithm in estimating the parameters of MIMO radar target's reflection amplitude, azimuth and elevation angle is better than that of the COSAMP algorithm.

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