Motion Simulation of Rocket Based on Simulink

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Abstract. A system of carrier rockets tracing target points is modeled and simulated in Simulink platform of MATLAB, in order to reduce the workloads of designers in designing, analyzing, calculating and selecting the carrier rockets' motion parameters. Based on the dynamic formula of carrier rockets strictly deduced in this study for tracing targets, a three-degree free tracing system simulation, combined with model simulation techniques, is established, and the proper carrier rockets' design parameters are studied according to the results of the simulation. First, this paper introduces the background and stresses the importance of carrier rockets in national defense, and then analyzes the problems with system modeling and analysis to be solved. After designing some sub-system simulation modules and integrating them into Simulink platform, this research sets up an tracing simulation model. And finally, by executing the trascing system simulation under the condition of different guidance ratio values, this research obtains corresponding simulation results and compares the influences of various guidance ratio values on the tracing effects.

Keywords: Rockets · Simulation · Simulink · Proportion guidance

1 Introduction

Flying by the controlling law is an important task of carrier rockets for targeting to specific target point and fly on special trace. Traditionally, the research and design of carrier rockets' manufacture parameters rely on the relevant professional staff's deduction. The traditional way of completing the task is mainly to program the formula using computer or to calculate the parameters manually by professional staff [1]. There are problems such as large amount of calculation load, low efficiency to get accurate and suitable rocket parameters, tendentiousness of making mistakes and difficulty in programming due to formula constraints in scale and complexity [2]. As a rare method to obtain carrier rockets' actual flight parameters under guidance and control except for actual rocket flight test, the simulation technology is used to simulate the dynamic motion of carrier rockets working in the practical environment with the help of simulation software platforms such as MATLAB/Simulink, KD-HLA, LabVIEW, etc. [3], and modify the model simulation parameters to meet the design expectation, which has become the

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essential method of carrier rockets' design and performance optimization. Between carrier rockets' design, test and finalization, the flight control design optimization, selection of guidance and control law, evaluation of rocket operational effectiveness and so on, almost depend on the trajectory simulation previously. Therefore, basic research on the trajectory simulation of carrier rockets is carried out, in the hope of developing a set of trajectory simulation specifications at some future date.

As one of the core components of MATLAB software, Simulink simulation platform provides users with an integrated environment of dynamic system modeling, simulation and comprehensive analysis [4]. By means of graphical interactive operation, dragging simulation modules, modifying simulation parameters, combining multiple modules as a whole and other humanized operations, complex simulation model system could be constructed effectively and humanly. Simulink provides carrier rockets' tracing system modeling with a wealth of useful library components. Simulink's commonly used blocks contains constant module, delay module in discrete system, integration module in continuous system, input port or output port required when creating tracing system's sub-system, etc., and continuous system simulation block includes modules used in continuous system, such as differential integration, state transfer function, delay, zero pole modules, while discrete system simulation block includes delay, difference, filter, signal control modules and math operation simulation part contains absolute, gain, deviation, division, exponent operation and lookup tables block, which realizes table mapping conveniently by inputting data into required dimensions blanks, includes various query table modules [5]. With the characteristics of wide application fields, clear and concise simulation structure and implementation, it has been widely used in the complex system simulation field of rocket control and guidance.

The existing trajectory simulation research of carrier rockets is almost based on MATLAB/Simulink [6]. For example, the department of automatic control engineering of naval aviation engineering university has established the anti-ship carrier rockets' trajectory simulation model with the help of Simulink, and verified the model; the air force engineering university has built carrier rockets' guidance and control dynamic simulation models in Simulink; based on MATLAB, Zhang Zhenzhong described the UAV's penetration simulation model under the missile interceptor [7]. It could be concluded that the Simulink/MATLAB simulation platform has become a trend in the development and design of carrier rockets. Given the characteristics of carrier rockets, this paper studies basic simulation models of the carrier rockets' trajectory control and guidance system when tracing the target point in Simulink, in order to develop a set of trajectory simulation specifications of the carrier rockets and assist the professional staff with high efficiency and quality.

There are two main practical application significances in this study. In the first place, repeatability and scalability. In actual rocket flight test, due to the random factors' dynamic influence of environment, it is difficult to ensure that the experiment results are obtained under the same condition [8]; however, the simulation method could control the random factors' impact by simulation environment parameters adjustment, which means strong repeatability. At the same time, each practical flight test needs to carry out a complete experiment process, while the trajectory simulation could start from any state among simulation, without needing to restart from the system's zero point, which is flexible and scalable.

The second is high test efficiency with low cost. Among the overall design process of the guidance and control of carrier rockets, it is necessary to carry out the trajectory calculation and optimization analysis for different guidance and control schemes to select the proper one to meet the trajectory index; generally, multiple trajectory flight tests are carried out to determine the rocket performance with high cost and long test time, while trajectory simulation of the carrier rockets as accurate as possible improves the test efficiency of different schemes and reduce the cost.

In this research, Simulink is used to execute the designed simulation model so as to improve the efficiency and quality of carrier rocket development. Section 2 gives a simple problem analysis of carrier rocket system, including the case study description and a collection of parameters. In Sect. 3, the modeling and simulation methodology is detailed. Simulation results are presented in Sect. 4 which follows conclusions in Sect. 5.

2 Problem Analysis

Case applied for this research's simulation is required to model and simulate the tracing system of the carrier rocket guided by the proportional guidance method and target point [9], as shown in Fig. 1. Assuming that an target point is flying horizontally at a constant speed along axis x in negative direction in the vertical plane, while at the same time, the defense side discovers the target and launches an carrier rocket in the same vertical plane in time to tracing the target point. The carrier rocket's flight stage is preset to be divided into two main working stages, of which the first stage is pushed by a high thrust rocket engine so as to be far away from the launching position as fast as possible and reach a high speed value in a short period of time to be comfortable for tracing in the next stage. The second stage engine's thrust is lower than that of the first stage, while the engine's working time of the second stage engine is longer than that of the first stage engine. In the second guidance stage, the rocket is controlled and guided by the proportional guidance method, which means that the rocket's elevation angle change will be corrected according to the change in the line of sight angle of between the rocket and target [10], so that the tracing rocket continuously approaches the target point to implement the task of tracing.

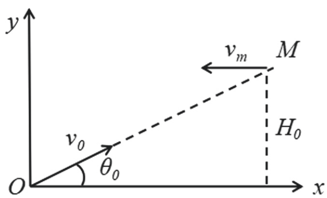

Fig. 1. The proportional guidance methodology of target tracing

The thrust and mass changes curves corresponding to two working stage of the primary and secondary engines in the rocket's flight phase are shown in Fig. 2. Time period 0 ~ t_1 corresponds to the working phase of the primary engine in the first stage, and time period t_1 ~ t_2 corresponds to the working phase of the secondary engine in the second stage.

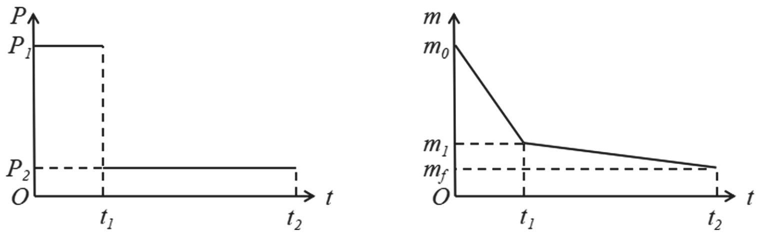

Fig. 2. Thrust and mass changes curves

The parameter symbol representation, initial values and physical definition of rocket design parameters, including dynamic characteristic parameters, motion parameters and target motion parameters [11] are shown in Table 1. The relationship between resistance coefficient caused by atmospheric acting force, Mach number of rocket's velocity and attack angle of rocket is shown in Table 2. The relationship between curve slope of lift coefficient and Mach number of its velocity is shown in Table 3.

Table 1. Simulation parameters

Parameter	Physical meaning
P_1	First stage thrust
P_2	Second stage thrust
t_1	First stage shutdown time
t_2	Second stage shutdown time
m_0	Initial mass
m_1	First stage mass when shut down
m_f	Second stage mass when shut down
S	Equivalent Area of rocket
v_0	Initial velocity of rocket
θ_0	Initial elevation angle
α_0	Initial attack angle
$H0$	Target height
v_m	Target velocity
r_0	Initial line of sight distance

Table 2. Resistance coefficient map

$M \backslash C_x \backslash \alpha$	0	2	4	6	8
1.5	C_{x11}	C_{x12}	C_{x13}	C_{x14}	C_{x15}
2	C_{x21}	C_{x22}	C_{x23}	C_{x24}	C_{x25}
2.5	C_{x31}	C_{x32}	C_{x33}	C_{x34}	C_{x35}
3	C_{x41}	C_{x42}	C_{x43}	C_{x44}	C_{x45}
3.5	C_{x51}	C_{x52}	C_{x53}	C_{x54}	C_{x55}
4	C_{x61}	C_{x62}	C_{x63}	C_{x64}	C_{x65}

Table 3. Lift coefficient curve slope map

M	c_y^{α}
1.5	$c_y^{\alpha 1}$
2	$c_y^{\alpha 2}$
2.5	$c_y^{\alpha 3}$
3	$c_y^{\alpha 4}$
3.5	$c_y^{\alpha 5}$
4	$c_y^{\alpha 6}$
4.5	$c_y^{\alpha 7}$

3 Modeling and Simulation Methodology

3.1 Tracing System Modeling

Given the requirements of simulation case, it is necessary to establish dynamic system models for the carrier rocket and target point respectively [12]. Considering that the guidance model of the carrier rocket depends on the target point's dynamic models, hierarchical modeling method is applied to tracing system simulation at different model levels of granularity, as shown in Fig. 3.

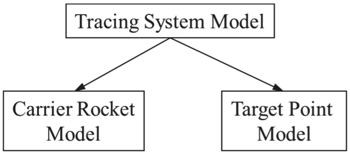

Fig. 3. Thrust and mass changes curves

The carrier rocket model and the target point model are at the same level in total system, and aggregated into the whole tracing system model. Conversely, the tracing system model could be divided into two sub-models: the carrier rocket model and the target point model. Through the multi-level modeling method of combination modeling, different granularity is refined based on the system model's structure [13], which is convenient for modeling and simulation.

The general form of FSM is described as Eq. 1 [13], where Q is the finite and non empty state set of simulation model system, and parameter I is the finite and non empty input set to change the system state, and parameter O is the finite and non empty output set to reflect the influences of input set I, and parameter δ is the transfer function of the system from original state Q to a new state Q with the input set I, representing the mapping relationship between states in a state transfer, and finally parameter λ is the function to describe relation between the input set I and output set O, which represents the mapping relationship between them.

$$FSM = < Q, I, O, \delta, \lambda > \tag{1}$$

The specific process of tracing target point is modeled and simulated by state modeling method of description modeling. The state transfer flow-process diagram is shown in Fig. 4 and Fig. 5, which respectively represents the internal state transfer of the whole tracing system and the inner state transfer of the carrier rocket. Figure 4 shows that the rocket starts from the initial launch state, and enters the first stage after the ignition of the primary engine, and transfers to the second state after the ignition of the secondary engine with the primary engine having shutdown, and finally ends the state transfer when successfully tracing the target point [14]. Relatively the target point starts from initial flight state after initialization, and continuously keeps the normal flight state as long as it's not traced by carrier rocket, or else it transfers to the end state.

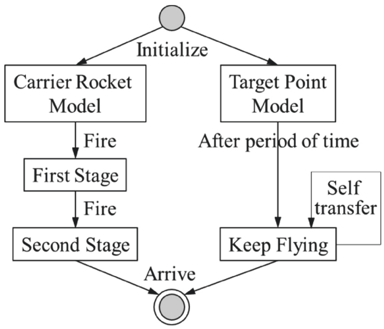

Fig. 4. State transfer of simulation system

The states of the rocket's state machine mainly include the no-guidance state and the guidance state guided by proportional guidance, as shown in Fig. 5. After the primary engine ignites, it enters the no-guidance state to accumulate speed as fast as possible, and then enters the guidance state when igniting the secondary engine. The proportional guidance state is further refined to two sub-states, of which the mutual conversion is realized when the change rate of the rocket's elevation angle or the line of sight angle varies following the proportional guidance law [15].

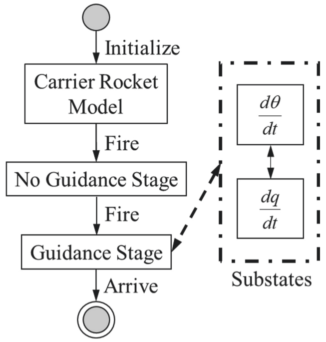

Fig. 5. State transfer of rocket model

The function modeling method could refine the system model for the composition of multiple distinguishable objects and describe the material signal flow with clear direction between different objects. The simulation objects and the material signal flow associated between the objects are designed depending on the carrier rocket model and the target point model.

The dynamic formulas of the carrier rocket model and the target point model are established with the constraint modeling of the function-based functional modeling [16]. The parameters' symbol representation involved and the corresponding physical meanings are listed in Table 4.

According to motion decomposition [17], the thrust of the rocket engine, gravity, lift and resistance are orthogonal decomposed along the speed direction and vertical direction. The resultant force along the speed direction provides the change of the speed value during the rocket flying and the resultant force along the vertical direction provides the change of the speed direction during the rocket flying, as shown in formula 2 and 3.

$$\frac{P(t)\cos\alpha(t) - m(t)g\sin\theta(t) - X(t)}{m(t)} = \frac{dv(t)}{dt} \tag{2}$$

$$\frac{P(t)\sin\alpha(t) - m(t)g\cos\theta(t) + Y(t)}{m(t)v(t)} = \frac{d\theta(t)}{dt} \tag{3}$$

The change rate equations of the rocket's velocity value and elevation angle value indicate the direction of signal flow during simulation modeling [18], that is, the output signal on the right side of the equation is calculated from the input on the left side, and the left side of the equation could be transformed into a corresponding sub-system in Simulink, in which the input ports represent the parameters on the equation's left side, and the output ports represent the right side of the equation.

According to the commonly calculating formula 4 of atmospheric density [19] and the calculation method of lift or resistance of rocket flying in the atmosphere [20] shown in formula 5 and 6.

$$\rho(t) = 1.293e^{-0.00015\,y(t)} \tag{4}$$

$$q_s(t) = \frac{1}{2}\rho(t)v(t)^2 \tag{5}$$

Table 4. Parameters and meanings

Parameter	Physical meaning
$P(t)$	Time-variation thrust
$\alpha(t)$	Time-variation attack angle
$\theta(t)$	Time-variation elevation angle of rocket
$\theta_m(t)$	Time-variation elevation angle of target
$q(t)$	Time-variation line of sight angle
$X(t)$	Time-variation resistance
$Y(t)$	Time-variation lift
$x(t)$	Time-variation location in x of rocket
$y(t)$	Time-variation location in y of rocket
$x_m(t)$	Time-variation location in x of target
$y_m(t)$	Time-variation location in y of target
$m(t)$	Time-variation mass
$v(t)$	Time-variation velocity
$r(t)$	Time-variation line of sight distance
$\rho(t)$	Time-variation atmospheric density
C_x	Resistance coefficient
C_y	Lift coefficient

$$\begin{cases} X(t) = C_x q_s(t) S \\ Y(t) = C_y q_s(t) S \end{cases} \tag{6}$$

On the basis of the proportional guidance method [21], the rocket's elevation angle change rate to time is adjusted depending on the line of sight angle change rate of the rocket and the target point. The parameter, the guidance ratio k, represents the ratio of the rocket's speed elevation angle's rotation angle speed to the target's line of sight angle rotation speed, as shown in the formula 7. Similarly, the constraint relation of the attack angle is shown as formula 8. When the angle between the line of sight and the horizontal plane increases, the rocket's flight elevation increases correspondingly in accordance with the guidance ratio set, in order that the rocket's flight direction is continuously corrected to aim until finally the tracing task is completed.

$$\frac{d\theta(t)}{dt} = k\frac{dq(t)}{dt} \tag{7}$$

$$\frac{\frac{d\theta(t)}{dt} m(t)v(t) + m(t)g\cos\theta(t) - Y(t)}{P(t)} = \alpha(t) \tag{8}$$

Decompose the two objects' velocity parameter along the line of sight between the rocket and the target point and the perpendicular direction to the line of sight. The velocity along the line of sight results in shortening the length of it, that is, the proximity of two objects, while the velocity vertical to the direction causes the effect of rotation centered on the other object.

$$\frac{-v(t)\sin(\theta(t) - q(t)) - v_m \sin(q(t) - \theta_m)}{r(t)} = \frac{dq(t)}{dt} \tag{9}$$

$$-v(t)\cos(\theta(t) - q(t)) + v_m \cos(q(t) - \theta_m) = \frac{dr(t)}{dt} \tag{10}$$

The line of sight angle's change rate between the rocket and target point and the length of the line's change rate to the time are separated into output signals in Simulink, and are integrated to get the solution results of the original time-variance angle [22]. The geometric relationship between rocket and target point is expressed as the following formula 11 and 12, which respectively represents the constraint relationship between horizontal position and vertical position of two objects.

$$x_m(t) - r(t)\cos q(t) = x(t) \tag{11}$$

$$y_m(t) - r(t)\sin q(t) = y(t) \tag{12}$$

The normal flight state of the target point is modeled based on the Eqs. 13 and 14, respectively according to the time-variance constraints of the horizontal position and the vertical position.

$$x_m(t) = r_0 \cos\theta_0 - v_m t \tag{13}$$

$$y_m(t) = r_0 \sin\theta_0 \tag{14}$$

3.2 Simulation Methodology

In the Simulink environment, the constraint relations that function modeling depends on are represented using relative simulation modules. According to the parameter being left side or right side of the equation, set equation's left side the module's input ports, and right side the output ports of the simulation module. Overlapping ports are related to solve an equation in Simulink simulation process, and output ports' outputs are written to MATLAB variable workspace as results of the tracing simulation [23]. Each separate equation corresponds to sub-module at sub-system level and united sub-modules corresponds to the total system at system level in Simulink.

Tracing system simulation model are separated into 10 sub-modules of Simulink, which correspond 10 output ports, respectively representing the velocity, elevation angle and attack angle, x position and y position, cornering overload of rocket, x position and y position of target, line of sight length and angle between line and horizontal plane. According to the input signal from input ports and the output signal from the output ports in each sub-system, the related signal is connected through the signal flow direction line, and the total system simulation in Simulink is shown in the Fig. 6.

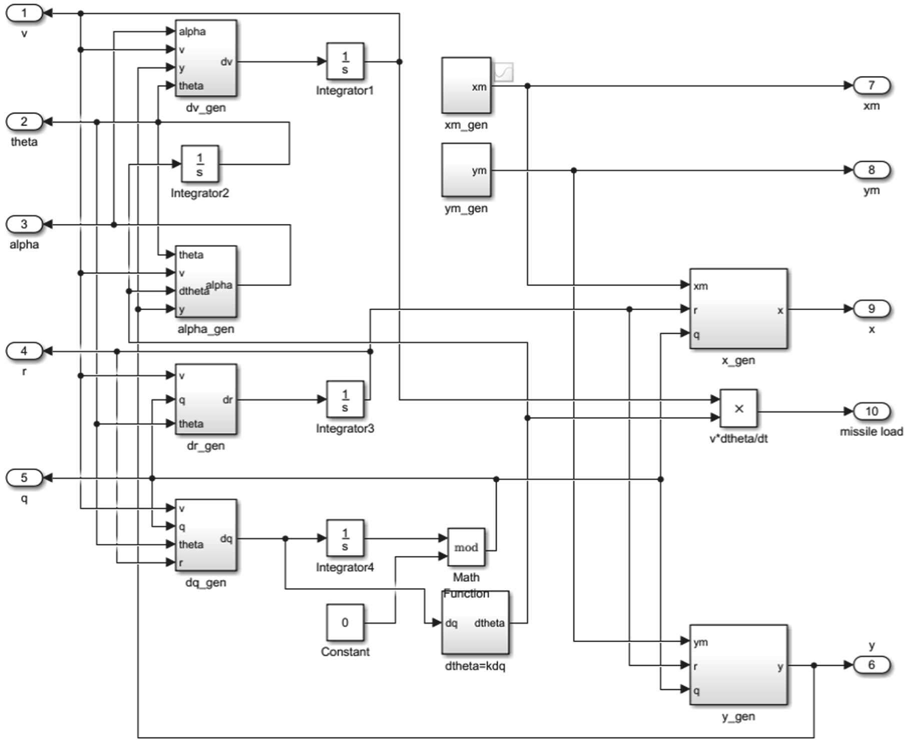

Fig. 6. Simulation system in Simulink

After the tracing system simulation model being built in Simulink, simulation execution options in the solver page and the data import or export page are configured. The total tracing phase continues nearly 45 unit time in this case, and thus set simulation time as the same time as the duration of total tracing phase with fixed time-step being set 0.025 with ode4 Runge-Kutta solver, meaning that 40 simulation points are sampled in 1 s and output the simulation execution results, time matrix *tout* and outputs matrix *yout*, into MATLAB variable workspace for further analysis. The specific simulation execution algorithm of Simulink is shown as following 4 steps:

1) Variable definition and initialization;
2) Loop each iteration in every minimum time step;
3) Each iteration, update the values of simulation time, *rtY.v, rtY.theta, rtY.alpha, rtY.r, rtY.q, rtY.y, rtY.xm, rtY.x* in turn, where *rtY* is the structural variable representing the output port of the simulation system;
4) Judge whether the simulation time is reached. If yes, the simulation ends, otherwise skip to step 2) for execution.

Simulink outputs the simulation time matrix *tout* and outputs matrix *yout* after performing the tracing system model simulation. In order to analyze the influences of different parameters on the carrier rocket's motion state and tracing effects, miss distance is selected as the main evaluation index, which describes the minimum distance deviation between the rocket and the target point [24] under the discrete-time sampling condition, and get the rocket's corresponding motion state under the circumstances of different guidance ratio values.

4 Simulation Results

Different tracing effects are listed in Table 5 under the circumstances of different guidance ratio *k* values ranging from 2.1 to 2.5. From the simulation results table, when the guidance ratio value equals 2.3, the corresponding miss distance reach the minimum distance as 8.20 unit distances. According to the common size of the target aircrafts, 20 unit distances is select as the maximum miss distance threshold of mission success and it means that the rocket's radius of explosion could reach to 20 unit distances at most. Therefore, to obtain the proper range of the guidance ratio value meeting the condition of tracing mission success, the curve of relationship between guidance ratio and miss distance is calculated and shown in Fig. 7.

Table 5. Simulation results

Guidance ratio	Miss distance
2.1	39.13
2.2	31.83
2.3	8.20
2.4	31.21
2.5	36.78

From the curve in Fig. 7, it is concluded that the guidance ratio *k*'s proper value ranges from 2.25 to 2.35, and *k* greater than or less than proper range may cause the addition of miss distance, meaning the failure of tracing mission.

Under the circumstances of guarding tracing mission successfully with different *k*'s value, the corresponding rocket's motion state changes are shown in Fig. 8. The relationship description curve between the guidance ratio *k* and carrier rocket's final velocity is shown in Fig. 8(a) when it just reaches the miss distance. The final velocity reaches higher than 700 unit velocity. In the meanwhile, the relationship description curve between the guidance ratio *k* and carrier rocket's elevation angle is shown in Fig. 8(b). With the increase of *k*, the final elevation angle decreases to less than 1 unit angle. Similarly, the relationship description curve between the guidance ratio *k* and carrier rocket's attack angle is shown in Fig. 8(c). With the increase of *k*, the final attack angle decreases and levels off to zero. The space position change curve of the rocket and target is shown in Fig. 8(d) in tracing process. Motion curve of the rocket is divided into two phase, the straight line phase corresponding to the first stage engine working and the

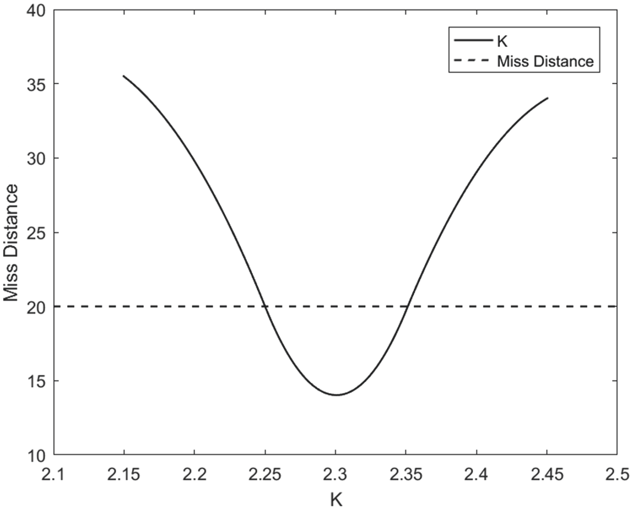

Fig. 7. *k*-Miss distance curve

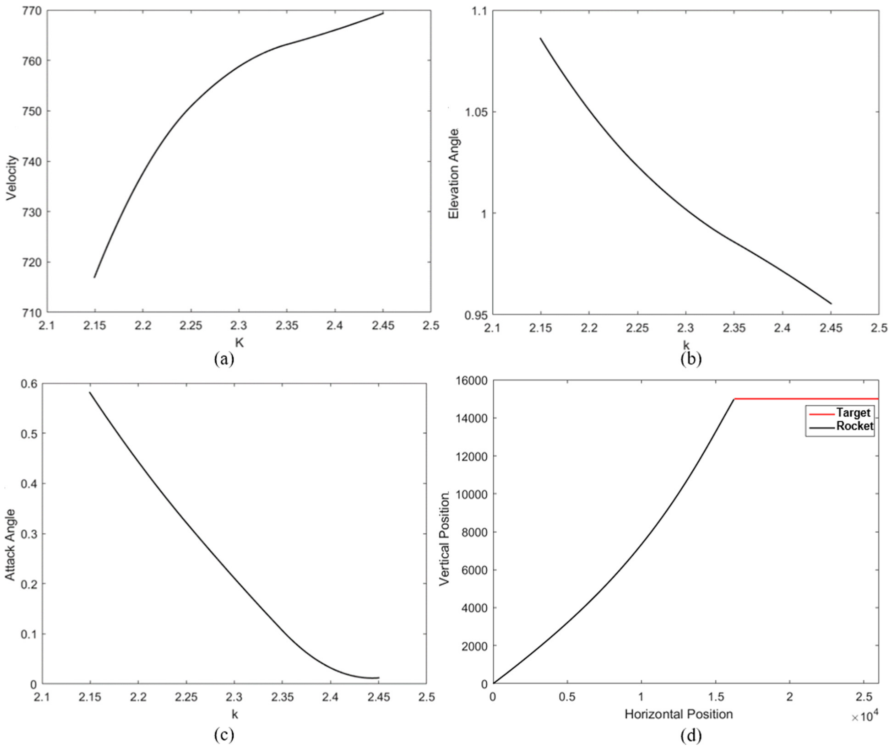

Fig. 8. Motion state curves

winding curve phase corresponding to the second stage engine working with guidance. Ultimately, the rocket arrives the target point in cross area of two motion curves.

Different guidance rate k represents the rotation speed of rocket velocity vector relative to line of sight, which could adapt to a variety of targets with different maneuvering characteristics. From the analysis of tracing effect, the appropriate k value is between 2.25 and 2.35, which meets the requirements of tracing mission. The higher the proportional guidance coefficient k is, the greater the final tracing speed of the rocket is, the smaller the final elevation angle and attack angle are. Within the tracing range, the interception time decreases with the increase of k, and the trajectory curvature also decreases, that is, the trajectory tends to be straight. Due to the limitation of missile structure and technology, the available overload could not be increased unlimited. Therefore, the selection of k value should not be too large, which means to select the appropriate value by the design indices.

5 Conclusions

Based on the Simulink platform and the trajectory characteristics, the tracing system model of the case that the carrier rocket aims to tracing the target point is developed on the dynamic differential equations' constraint conditions. And the simulation is carried out by Simulink simulation execution algorithm on condition of different guidance ratio k values to obtain proper guidance ratio and its corresponding influences on the rocket's motion state.

Finally, the proper range of guidance ratio meeting the tracing requirement and its influences on the carrier rockets' final motion state parameters, such as velocity, elevation angle, attack angle and motion position, are analyzed in detail, which aims at combining the method of simulation modeling with guidance and control laws of carrier rockets. With trajectory simulation models continuously developing, the simulation specifications of carrier rockets may be finally issued to promote design efficiency and quality.

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