



Research on Satellite Fault Detection Method Based on MSET and SRPRT

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Abstract. Since the beginning of the 21st century, the increasing requirements of mankind for spacecraft technology have forced spacecraft technology to become more and more complex and increase investment. Most current fault detection methods only use single features and features or single fault detection data, and do not involve satellite fault monitoring under multi-parameter conditions. This paper proposes a satellite fault detection method that can be used in a multi-parameter state. The multivariate state estimation algorithm (MSET) is used to obtain the residual between the multivariate state and historical health data, and then the actual residual of the data to be measured is input into the sequential rank sum probability ratio test method (SRPRT) to test. This paper verifies the effectiveness of the combination of MSET algorithm and SRPRT algorithm through experiments, and finds the optimal parameters and accuracy relative to SPRT method.

Keywords: Fault detection · MSET · SRPRT

1 Background

Since the beginning of the 21st century, the increasing requirements of mankind for spacecraft technology have forced spacecraft technology to become more and more complex and increase investment. In such a system with relatively high complexity and relatively high investment, the reliability, safety and fault diagnosis technology of the spacecraft are forced to be on the agenda, which has become an important link restricting the development of the spacecraft. According to incomplete statistics, of the 764 spacecraft launched in 1990–2001, a total of 121 failed, accounting for 15.8% of the total spacecraft [1].

In the field of spacecraft on-orbit fault detection, the more mature engineering is the threshold detection based on the original telemetry data, and is currently actively exploring and applying detection methods based on feature quantities.. Fuertes et al. [2] analyzed the limitations of the threshold detection method of the CNES spacecraft condition monitoring and the monthly statistical feature monitoring method. In view of the shortcomings of the traditional fixed threshold detection method, D. DeCoste et al. [3] proposed an envelope learning and monitoring method based on error relaxation,

which can continuously update and generate a tight upper and lower bound function envelope to reduce the rate of misdiagnosis and false alarm. Shan Changsheng et al. [4] pointed out that at present, the three main threshold abnormal methods based on telemetry parameter over-limit alarms include telemetry parameter threshold judgment, relative value judgment and associated diagnosis. Wang Weiwei et al. [5] pointed out that the abnormal state mutation can usually be detected in time using the threshold detection method, but there are also a considerable part of the satellite telemetry parameters that do not exceed the limit when the on-orbit abnormality occurs. Yang Tianshe et al. proposed a space system state symptom variable prediction model based on gray system theory [6], and a satellite fault diagnosis and prediction method based on knowledge [7]. Based on data mining and decision tree, Wang Xiaole et al. [8] selected the maximum gain rate information as the segmentation attribute, and obtained the optimal segmentation point by mining the data, and generated the fault diagnosis decision tree after pruning.

In summary, the more mature engineering in the field of rail fault detection is the threshold detection based on the original telemetry data, and is currently actively exploring and applying detection methods based on feature quantities. The current mainstream methods are regression tree, machine learning, Bayesian network, adaptive threshold, and gray system theory. However, most of the above methods only use single features and features or single fault detection data, and do not involve satellite fault monitoring under multi-parameter conditions.

2 MSET Algorithm

2.1 Basic Concepts

Some concepts related to MSET, including data matrix: observation matrix X_{obs} , Training data T , Memory matrix D , Residual training data L , Estimation matrix. X_{est} . For Observation matrix X_{obs} . The observation matrix defined by MSET, as shown in Fig. 1, contains n parameters and each parameter has m values. And n represents the number of monitoring parameters, m represents the number of time states. Each column of the matrix lists all parameter values from parameter X_1 to parameter X_n at the same time state t_i . Since each column contains all monitoring parameters of the system at the same time, it is called the system state.

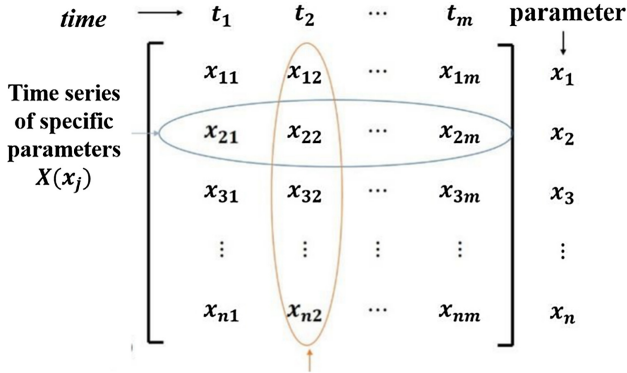
When the fixed time is t_i , the system observation matrix X_{obs} is represented by a vector $X(t_i)$ or $X_{obs}(t_i)$ of length n , where n represents the number of system monitoring parameters.

$$X(t_i) = [x_{1i}, x_{2i}, \dots, x_{mi}]^T \quad (1)$$

When the fixed parameter is x_j , the system observation matrix X_{obs} is represented by a vector $X(x_j)$ or $X_{obs}(x_j)$ of length m , where m represents the number of system time states.

$$X_j = [x_{j1}, x_{j2}, \dots, x_{jm}] \quad (2)$$

Where X_{ij} is the observation value of parameter $j(j = 1, 2, \dots, n)$ at time t_i .



System parameter sequence at specific time $X(t_j)$

Fig. 1. The observation matrix defined by MSET

The training data T is a matrix consisting of a healthy historical state. The matrix format is defined as:

$$T = [x(t_i), \dots, x(t_i), \dots, x(t_{i+k-1})] \quad (3)$$

Where k is the number of normal states selected for the training data, and this parameter is set by the user in advance.

The memory matrix D is a matrix selected from the training data T according to the corresponding rules. The number of states m is the number of states selected from the training data by the memory matrix. After the number of states m is determined, the memory matrix can be defined as:

$$D = [x_1, x_2, \dots, x_m] \quad (4)$$

The remaining training data L is the data state in the training data that is not selected by the memorized data. The relationship between T , D , and L is given by the following formula:

$$T = D \cup L \quad (5)$$

The estimation matrix X_{est} of the observation matrix X_{obs} and the estimation matrix of the remaining training data L are both estimates of the similarity measure calculated from the health data. The estimation matrix format is the same as the observation matrix format.

2.2 MSET Detailed Steps

Step 1: Acquiring new observation matrix based on actual telemetry data X_{obs} :

Step 2: Selects training data:

The training data T needs to include all health states of all monitoring parameters under the normal operating state of the system. The normal operating state of the system here

includes not only the steady change state of the monitoring parameters but also the normal degradation state of the monitoring parameters. The selection of training data T should meet the following requirements:

1. The state in the training data T can only be established on the basis of the normal operation of the system. If the system is in an abnormal operation state, all monitoring parameter states cannot be selected into the training data T .
2. The training data T must contain dynamic changes in the state of the monitoring parameters.

In addition to the representation mentioned in the previous section, when the number k of normal states selected by the training data is determined, the training data T can also be expressed as:

$$T = [X(t_{1+i}), X(t_{2+i}), \dots, X(t_{k+i})] \quad (6)$$

Step 3: Constructs memory matrix D and generate residual training data. L :

When the number of states l is determined, a $n \times l$ memory matrix can be generated according to certain rules, where n represents the number of monitoring parameters and l represents the number of states selected in the memory matrix.

Referring to the existing research results, the memory matrix can be generated according to the following steps:

3. Number of states determined manually l .
4. Select the extreme state of each monitoring parameter included in the training data.
5. If the number of selected extreme state is less than the number of memory matrix states, the extreme state is added to the memory matrix, then calculate the vector of Euclidean norm of other data in the training data. After sorting, equidistant sampling is added to the memory matrix to complete the construction of the memory matrix.

Delete the state contained in the memory matrix from the training data to generate the remaining training data.

Step 4: Calculates healthy residuals;

When calculating the health residual, MSET is performed in two steps: calculating the estimated value L_{est} of all remaining training data L ; calculating the health residual between the estimated value and the remaining training data. The calculation formula of L_{est} is as follows:

$$L_{est} = D \cdot W \quad (7)$$

Where W is a weight vector, which represents a measure of similarity between the memory matrix and the current state. The weight vector W can be obtained by the following formula:

$$W = (D^T \cdot D)^{-1} (D^T \cdot L) \quad (8)$$

The calculation formula of L_{est} can be obtained by combining the above formulas:

$$L_{est} = D \cdot (D^T \cdot D)^{-1} (D^T \cdot L) \quad (9)$$

The difference between the estimated value and the actual value is the healthy residual. The formula is:

$$R_l = L_{est} - L \tag{10}$$

Step 5: Calculates actual residuals;

When calculating the actual residuals, MSET is also performed in two steps: calculating the estimation matrix X_{est} of the observation matrix X_{obs} ; and calculate the actual residual between the estimated value and the observation matrix.

The calculation process of observation matrix X_{obs} is as follows:

$$X_{est} = D \cdot W^1 \tag{11}$$

Where W^1 is a weight vector, which can be obtained by the following formula:

$$W^1 = (D^T \cdot D)^{-1} (D^T \cdot X_{obs}) \tag{12}$$

Formula of X_{est} :

$$X_{est} = D \cdot (D^T \cdot D)^{-1} (D^T \cdot X_{obs}) \tag{13}$$

The actual residual is the difference between the estimated value and the actual value. The formula is:

$$R_x = X_{est} - X_{obs} \tag{14}$$

After completing the above steps, the fault detection process will compare the actual residual with the healthy residual to determine whether the current system is healthy.

3 SRPRT Algorithm

In the above chapters, a series of calculations are carried out to get the health residuals and the actual residuals. This section mainly introduces the basic principles and operation steps of the sequential rank sum probability ratio test (SRPRT) method.

3.1 SRPRT Nonparametric Test Principle

The core idea of non-parametric testing is to obtain as much necessary information as possible from actual data without assuming the overall distribution in advance. Among many non-parametric test methods, SRPRT only requires that the sample signal is continuous and symmetric about the median or mean, and the sample signal has lower requirements and is easy to implement. The input of the SRPRT method is the average and actual residual of the health residual calculated by MSET, and the output is directly set as a flag bit to judge whether the system is abnormal.

The SRPRT method is based on Wilcoxon symbol rank statistics, and the likelihood function becomes:

$$\Lambda(R_n) = \frac{L(R_n; \theta_1)}{L(R_n; \theta_0)} = \frac{P(W_1, m)}{P(W_0, m)} = \frac{P_1}{P_0} \tag{15}$$

Special attention is needed when the actual residual is expressed as $R = [r_1, r_2, \dots, r_n]$, where n does not indicate the length of the actual residual, but the length of the test result.

The specific steps of the SRPRT method are as follows:

Step 1: builds statistical assumptions;

$$H_0 : \mu_0 = \mu, H_1 : \mu_1 = \delta \cdot \mu \tag{16}$$

Where μ is the average of the health residuals in the MSET result, and δ is the detection ratio, that is, when the actual residual of the parameter is greater than $1 - \delta$ times the health residual, it is determined to be a fault.

Step 2: Under the assumption of H_i , the hypothetical mean μ_i is subtracted from each original residual sample r_j , namely:

$$r_{ij} = r_j - \mu_i, j = 1, 2, \dots, n \tag{17}$$

Where i is the hypothetical serial number and j is the sample serial number; after the calculation, the new residual sample sequence $r_{i1}, r_{i2}, \dots, r_{in}$.

Step 3: sets the sequence of new residual samples from the absolute value from small to large.

$$|r_i^{(1)}| \leq |r_i^{(2)}| \leq \dots \leq |r_i^{(n)}| \tag{18}$$

The updated sample residual sequence is $r_1^{(n)}, r_2^{(n)}, \dots, r_i^{(n)}$.

Step 4: Remove the samples with the absolute value of 0 among the above samples to generate rearranged samples. Let the number of remaining samples in the rearranged sample be m . Then, rank R is assigned according to the position of the remaining samples in the entire sequence. The rank assignment process can be expressed as:

$$R(r_i^{(k)}) = k \tag{19}$$

The function value of the sign indicating function Ψ is determined by the sign of the original values of the rearranged samples. The sign indicating function Ψ is constructed as follows:

$$\Psi(r_i^{(k)}) = \begin{cases} 1, & \text{if } r_i^{(k)} > 0 \\ 0, & \text{if } r_i^{(k)} < 0 \end{cases} \tag{20}$$

Among $k = 1, 2, \dots, m$.

Step 5: Under the assumption of H_i , Wilcoxon symbolic rank sum statistics W_i^+ and W_i^- are discrete random variables subject to a certain distribution function, which can be calculated by the following formula:

$$W_i^+ = \sum_{k=1}^m R(r_i^{(k)}) \Psi(r_i^{(k)}) \tag{21}$$

When the test condition is small sample size ($m \leq 20$) constructing two sided test statistics:

$$\Lambda(R_n) = W_1^+ \tag{22}$$

Setting confidence is set to α , and the result is compared with the test time. $\Lambda(R_n)$ Critical value $t_{\alpha/2}$ and $\frac{n(n+1)}{2} - t_{\alpha/2}$:

When $\Lambda(R_n) \geq t_{\alpha/2}$ or $\Lambda(R_n) \leq \frac{n(n+1)}{2} - t_{\alpha/2}$ refuse H_0 and receive H_1 , system failure, output fault flag bit 1;

When $\frac{n(n+1)}{2} - t_{\alpha/2} \leq \Lambda(R_n) \leq t_{\alpha/2}$ refuse H_1 and receive H_0 , the system is normal, output fault flag bit 0.

When the test condition is large sample capacity ($m > 20$), calculated in two cases. $P(W_i, m)$:

When the absolute values of all samples in the sample sequence are different, The distribution of the statistic W_i is similar to the normal distribution of mean $T = \frac{m(m+1)}{4}$, variance $V_T = \frac{m(m+1)(2m+1)}{24}$, that is:

$$\frac{1}{\sqrt{2\pi V_T}} \exp\left[-\frac{(W_i - T)^2}{2V_T}\right] \approx N(0, 1) \tag{23}$$

Similarly, $P(W_i, m)$ can be calculated by the following formula:

$$P(W_i, m) = \frac{1}{\sqrt{2\pi\sqrt{V_T}}} \exp\left(-\frac{(W_i - T)^2}{2V_T}\right) \tag{24}$$

When there are samples (called sample groups) with the same absolute value and non-zero in the sample sequence, the variance V_T needs to be corrected. Let g be the number of sample groups composed of samples with the same absolute value, t_i is the number of samples contained in the i -th sample group. Then the variance V_T can be corrected according to the following formula:

$$V'_T = \frac{m(m+1)(2m+1)}{24} - \frac{1}{48} \sum_{i=1}^g t_i(t_i^2 - 1) \tag{25}$$

To sum up, the likelihood ratio function formula is:

$$\Lambda(R_n) = \frac{P(W_1, m)}{P(W_0, m)} = \frac{\frac{1}{\sqrt{2\pi\sqrt{V'_T}}} e^{\left(-\frac{(W_1 - T)^2}{2V'_T}\right)}}{\frac{1}{\sqrt{2\pi\sqrt{V'_T}}} e^{\left(-\frac{(W_0 - T)^2}{2V'_T}\right)}} \tag{26}$$

Step 6: Introduce the constants A and B (usually given by experts in the field), let A be the lower limit, B be the upper limit, and $0 < A < 1 < B$. Then it is determined by comparison that the hypothesis H_0 or hypothesis H_1 is accepted, and the sequential test scheme is constructed as follows:

1. If $\Lambda(R_n) \leq A$, accept assumptions H_0 , the system is normal, and the output abnormal flag is 0;
2. If $\Lambda(R_n) \geq B$, acceptance assumptions H_1 , the system is abnormal and the output abnormal flag is 1.
3. If $A \leq \Lambda(R_n) \leq B$, it is impossible to make a decision. We need to continue sampling, add a subsequent set of data to recalculate and repeat the comparison process.

$$A \cong \frac{\beta}{1 - \alpha}, B \cong \frac{1 - \alpha}{\beta}$$

Among them are the preset first type error probability and second type error probability, namely α is the false alarm rate (FAR) and β is the missing alarm rate (MAR), which can be 0.0001, 0.01, 0.015, 0.02, 0.05, and 0.1 in practical application.

3.2 SRPRT Experiment

In order to verify the effectiveness and accuracy of the SRPRT algorithm, a control experiment was designed according to the following method. The parameter M corresponds to the mean deviation of the fault data in the hypothesis in the algorithm from the normal data. The data used in the experiment are random data that follow the normal distribution with a mean value of 0 and a variance of 1. Therefore, the result of all algorithms determining a failure is an algorithm false alarm. The parameter FAR observation value is the number of algorithm false alarms divided by the total number of samples. The algorithm test method is to observe the false alarm rate as small as possible.

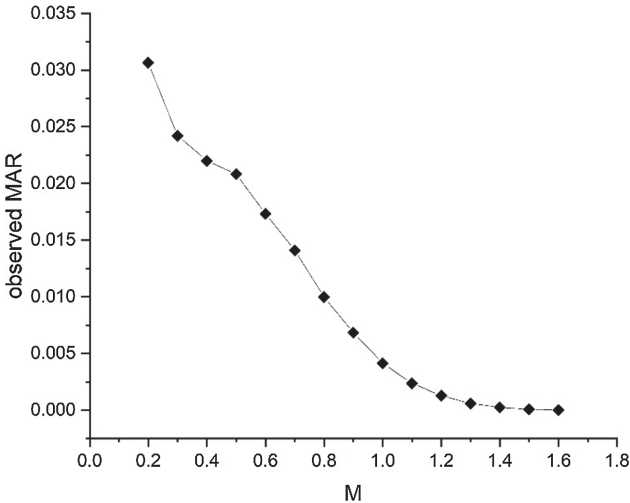


Fig. 2. Schematic diagram of FAR observation value changes with M

Experiment 1: verify the effect of the parameter M indicating the degree of mean shift on the experimental results. Set $MAR = 0.1$, $FAR = 0.003$, the number of samples is 100000, and observe the change law of FAR observation value with M (Fig. 2).

As shown in the above figure, the FAR observation value decreases with the increase of the mean value M of the abnormal data offset. The larger M is, the higher the data is judged as failure criterion, and the more difficult it is for the sample data to be judged as failure, which conforms to the original design assumption of the algorithm. M can be used as a confidence index to join the algorithm using process, which is convenient for adjusting the accuracy of the algorithm.

4 Example

In order to verify the effectiveness and accuracy of MSET and SRPRT, a set of control experiments were designed according to the following methods. The parameter M corresponds to the mean deviation of the fault data in the hypothesis in the algorithm from the normal data. The normal data used in the experiment are random data that follow the normal distribution with a mean of 0 and a variance of 1. The abnormal data use random data that follow a normal distribution with a mean of 0.8 and a variance of 0.01.

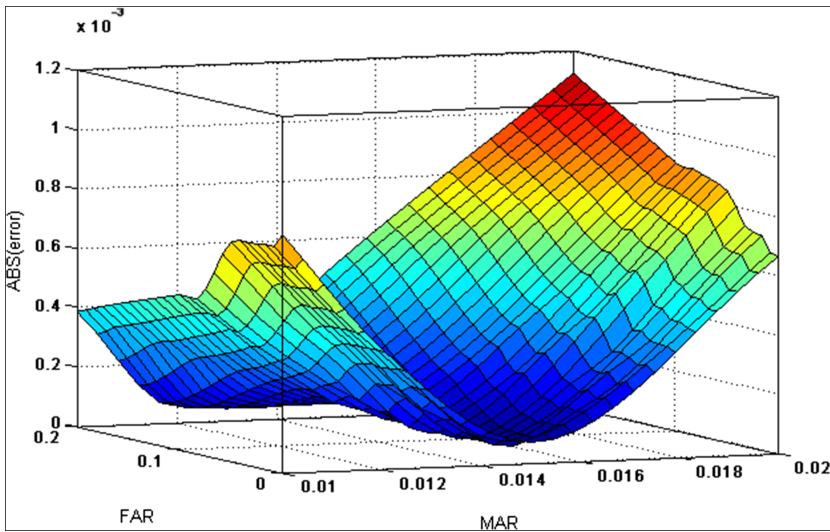


Fig. 3. Schematic diagram of failure rate error along with MAR and FAR

Experiment 1: Adjust the parameters and find the optimal MAR and FAR parameters. Change the MAR and FAR values and conduct multiple experiments to find the MAR and FAR corresponding to the smallest difference between the actual failure rate and the theoretical failure rate. Set the failure rate and add abnormal data to the normal data in proportion to the failure rate. Normal data and abnormal data run SRPRT algorithm separately. After the algorithm runs, the observed FAR is the number of abnormalities

in normal data divided by the total number of normal data, and the observed MAR is the number of non-abnormalities in abnormal data divided by the total number of abnormal data.

As shown in Fig. 3, the X-axis and Y-axis respectively represent the theoretical values of MAR and FAR, and the Z-axis represents the absolute value of the difference between the actual measured value of the failure rate and the theoretical value of the failure rate. Therefore, the curved surface is similar to the parabolic surface, and the lower middle zone is the zone with the smallest failure rate error. When the measured failure rate is closest to the failure rate (the lowest point in the figure), the corresponding MAR and FAR are selected as the optimal parameters.

To clearly select the lowest point in the figure, project the three-dimensional map on the X-Z plane to find the lowest point of Z as shown in Fig. 4. In order to visualize the relationship between the Z value and size, make a straight line parallel to the Z-axis plane. It can be seen that the point in the red box on the right is lower than the lowest point in the valley on the left, so the area in the box is enlarged.

As shown in Fig. 5, there are two points in this area that have very similar values, so they are parallel to the X axis, and the point in the red circle on the right is slightly lower than the lower point on the left. Therefore, the lowest point takes MAR = 0.015 and FAR = 0.02, and the value of Z at this time is 4.59E-05, which is the lowest point, so the algorithm selects the optimal parameters 0.015, 0.02, in line with the experimental results.

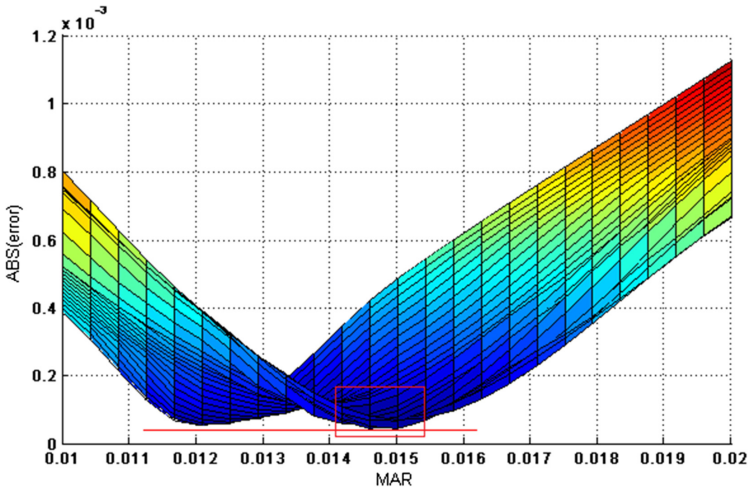


Fig. 4. Failure rate error schematic projection on X axis Z axis

Experiment Two

Experiment 2: The experiment case selects satellite attitude control system for fault detection. Compare the advantages and disadvantages of the existing MSET algorithm combined with SPRT algorithm and MSET algorithm combined with SRPRT algorithm.

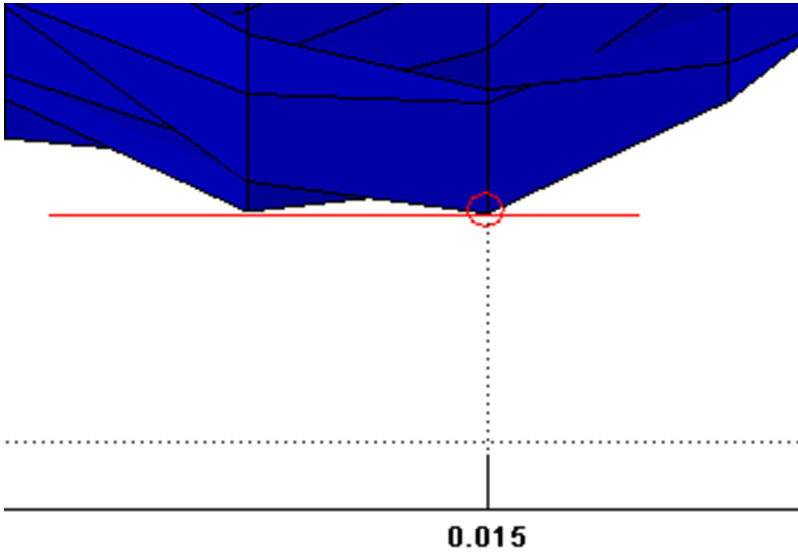


Fig. 5. Local sketch of failure rate error

All data are normal data, and the total sample data is 100,000, with a total of 19 parameters. According to the alphabetical order of English names, the parameters follow the normal distribution with mean values of 0, 1...18 and variance of 1. Since the mean value of the residuals after MSET calculation of 19 parameters is not 0, the mean value of variance needs to be shifted. For the SRPRT algorithm, the parameters $M = 0.5, 0.8, 1.5$ and $MAR = 0.015$ are set in the experiment. The parameters of the SPRT algorithm are the same as the SRPRT algorithm.

Since the FAR and FAR observation values are both small, in order to better display the relationship between the horizontal and vertical coordinates, the horizontal coordinate takes the logarithmic function of FAR to lengthen the distance between the horizontal coordinates. As shown in Fig. 6, MSET combined with SPRT algorithm under the conditions of FAR, M, are not as good as SRPRT algorithm. When M is unchanged, the FAR increases, and the FAR observations of the SPRT and SRPRT algorithms increase. However, when the FAR is greater than 0.001, the FAR observations of the SPRT algorithm increase significantly and are much larger than the FAR observations of the SRPRT algorithm. Generally speaking, the SPRT algorithm false alarm rate is greater than the SRPRT algorithm when M is unchanged, and the growth rate is extremely fast, which is obviously inferior to the SRPRT algorithm. When FAR is unchanged, M increases, and FAR fault detection values increase, which conforms to the algorithm design idea, and the SPRT algorithm performs well when $M = 1.5$; although the observed false alarm rate of both algorithms is smaller than the theoretical false alarm rate, the SPRT algorithm false alarm rate is still much higher than the SRPRT algorithm false alarm rate. Therefore, it can be proved that the SPRT algorithm is effective but not as accurate as the

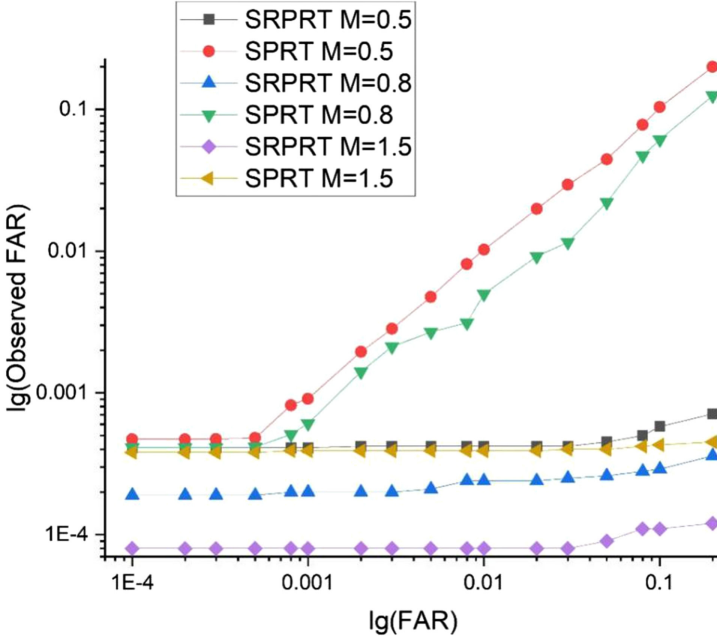


Fig. 6. Schematic diagram of comparison between SPRT and SRPRT algorithm

SRPRT algorithm. It also proves the effectiveness and accuracy of the SPRT algorithm from the side.

Moreover, because the SPRT method requires samples to follow a priori distribution with unknown parameters, satellite measured data may not necessarily meet this requirement. However, the sample signal required by the SRPRT method is continuous and symmetrical about the median or mean value, and the satellite measured data can be satisfied after adjustment. And the SRPRT algorithm has higher precision than the SPRT algorithm, so in this paper, the non-parametric method SRPRT is selected as the method of fault monitoring with the MSET method.

5 Summary

This paper proposes a satellite fault detection method that combines a multivariate state estimation algorithm with a sequential rank and probability ratio test method. This method can be used for satellite fault detection in multi-parameter states. First, the multivariate state estimation algorithm (MSET) is used to obtain the residual between the multivariate state and the historical health data, and then the actual residual of the data to be tested is input into the sequential rank and probability ratio test method (SRPRT) for testing. In terms of experiments, this paper firstly validates the effectiveness of SRPRT through experiments, and finally verifies the effectiveness and accuracy of the combination of MSET and SRPRT through the actual calculation examples of satellite attitude control system, and the superiority of combining MSET with SPRT.

The advantage of the algorithm in this paper is that it can be used for multi-parameter fault detection and can be used for satellite measured data. It is not necessary to be bound by theoretical research. The accuracy of the algorithm is significantly higher than the accuracy of the combination of the MSET algorithm and the SPRT algorithm. The disadvantage is that the running time of the algorithm is slightly longer than the SPRT algorithm, and it still needs to be improved in the future. Follow-up work will focus on comparing this algorithm with other fault detection algorithms such as regression trees, gray system theory, Bayes network in the performance of accuracy and algorithm operation efficiency, and to improve the weakness of the algorithm.

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