A Non-zero Sum Power Control Game with Uncertainty

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Abstract. We consider the communication between a transmitter (user) and a receiver in the presence of a jammer where the jammer, in contrast to the user, has access to local information about jamming fading gain (reflecting distance of the jammer to the receiver) and jamming cost (reflecting its technical characteristics). The problem is modeled as a Bayesian game. Signal-to-interference-plus-noise ratio (SINR) is considered as user's communication utility. Nash equilibrium as well as Stackelberg equilibrium are derived in closed form and compared.

Keywords: Jamming · Bayesian equilibrium · Incomplete information

1 Introduction

Due to the shared and open-access nature of the wireless medium, wireless networks are vulnerable to jamming attacks. Non-cooperative game theory is natural tools to study such jamming problems [11] due to such problems involve multiple agents (say, a user and a jammer) and each of them has own objective. A key characteristic of wireless access networks is that the agents might not have complete information regarding the other agents' identity, channel characteristics, or location [15]. Typically, in literature, uncertainty on one parameter is considered. For example, in [3,5,6,9], uncertainty about rival's type was investigated. In [12], uncertainty on fading channel gains was considered in the context of single carrier communications with non-hostile interference caused by selfish users, while, in [2,10], with hostile interference caused by an adversary. In [16], uncertainty about the jammer's location was examined within a frequency-division multiple access (FDMA) communication scenario.

In [13], it was suggested to consider combined uncertainty about several parameters, namely, fading gains and transmission cost. Due to that, in [13], the agents do not have access to local information it did not lead to an increase in the number of strategies compared with the complete information scenarios.

In this paper, we fill the gap in the literature on the case when one of the agent, namely, the user, does not have access to combined local information about the gain of the jammer's channel (reflecting the distance from the jammer to the receiver) and the jamming cost (reflecting technical characteristics of the jammer). While the jammer has access to such local information. This corresponds the most dangerous scenarios of the user's communication with the

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receiver. We consider SINR as user's communication utility. This work is a complementary work to [10], where throughput was considered as user's communication utility and the user has incomplete information only about one network parameter, namely, the jammer's channel gain.

2 Communication Model

In this paper we consider a single carrier communication between a transmitter (user) and a receiver in the presence of a jammer, who intends to degrade the user's communication by generating interference. The channel is assumed to be flat fading. As examples of studying single carrier communication, see, for example, [1,4,7,8,10,13,14,17–20]. We assume that the user does not have access to local information about the jammer's channel gain (reflecting its distance to the receiver) and jamming power cost (reflecting technical characteristics of the jammer). While the jammer has access to such local information. Namely, the user knows that the jammer's channel gain can be g_i with a priori probability α_i where $i \in \mathcal{N} \triangleq \{1, \dots, n\}$ and $\sum_{i=1}^{n} \alpha_i = 1$, and the (normalized) jamming power cost can be $C_{J,j}$ with a priori probability β_j, for $j \in \mathcal{M} \triangleq \{1, \dots, m\}$, where $\sum_{j=1}^{m} \beta_j = 1$. The strategy for the user is its transmission power P, with $P \geq 0$. We say that the jammer is of type-(i, j) if its channel gain and the jamming cost are g_i and $C_{J,j}$, respectively. The strategy for the (i, j)-type jammer is its jamming power $J_{i,j}$, with $J_{i,j} \geq 0$. Let[1] $\boldsymbol{J} = (J_{1,1}, \dots, J_{1,m}, \dots, J_{n,1}, \dots, J_{n,m})$. Each type jammer as well as the user have complete knowledge about all possible network parameters. But, in contrast to the jammer, the user does not know what type of the jammer occurs. We consider SINR as the user's communication utility. Let the payoff to the user is the difference between the expected SINR and transmission power cost, while the payoff to the (i, j)-type jammer is the negative of sum of the SINR and jamming power cost:

$$v_U(P, \boldsymbol{J}) = \sum_{(i,j)\in\mathcal{N}\times\mathcal{M}} \alpha_i \beta_j hP/(\sigma^2 + g_i J_{i,j}) - C_P P, \tag{1}$$

$$v_{J,i,j}(P, J_{i,j}) = -hP/(\sigma^2 + g_i J_{i,j}) - C_{J,j} J_{i,j},\ (i, j) \in \mathcal{N} \times \mathcal{M}, \tag{2}$$

where C_P is (normalized) transmission power cost and σ^2 is the background noise.

The user and each type jammer want to maximize their own payoffs. Thus, we look for Nash equilibrium (NE) [11]. Recall that $(P, \boldsymbol{J})$ is an NE if and only if

$$v_U(\tilde{P}, \boldsymbol{J}) \leq v_U(P, \boldsymbol{J}),\ \forall \tilde{P} \geq 0, \tag{3}$$

$$v_{J,i,j}(P, \tilde{J}_{i,j}) \leq v_{J,i,j}(P, J_{i,j}),\ \forall \tilde{J}_{i,j} \geq 0 \text{ and } (i, j) \in \mathcal{N} \times \mathcal{M}. \tag{4}$$

Denote this game by Γ_N. Note that, in game Γ_N there is at least one NE since $v_U(P, \boldsymbol{J})$ is linear in P and $v_{J,i,j}(P, J_{i,j})$ is concave in $J_{i,j}$ [12].

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[1] We use bold face font to denote vectors.

In the following sections we derive the NE in closed form via solving the best response equations. By (3) and (4), $(P, \boldsymbol{J})$ is an NE if and only if P is the best response to $\boldsymbol{J}$, while $J_{i,j}$ is the best response to P for each (i,j), i.e., they are solution of the following best response equations:

$$P = \mathrm{BR}_U(\boldsymbol{J}) \triangleq \operatorname*{argmax}_{P \geq 0} v_U(P, \boldsymbol{J}), \tag{5}$$

$$J_{i,j} = \mathrm{BR}_{J,i,j}(P) \triangleq \operatorname*{argmax}_{J_{i,j} \geq 0} v_{J,i,j}(P, J_{i,j}),\ (i,j) \in \mathcal{N} \times \mathcal{M}. \tag{6}$$

2.1 Auxiliary Notations and Results

Let us introduce auxiliary notations and results employed in the next section to derive in closed form. First let us split the set $\mathcal{N} \times \mathcal{M}$ into subsets $\mathcal{S}_r$, $r = 1, ..., R$ with equal $C_{J,j}/g_i$. Specifically, let

$$\mathcal{N} \times \mathcal{M} = \cup_{r=1}^{R} \mathcal{S}_r \text{ with } C_{J,j}/g_i = C_{J,\tilde{j}}/g_{\tilde{i}},\ \forall (i,j) \in \mathcal{S}_r \text{ and } (\tilde{i},\tilde{j}) \in \mathcal{S}_r. \tag{7}$$

Thus, $C_{J,j}/g_i \neq C_{J,\tilde{j}}/g_{\tilde{i}}$ for any $(i,j) \in \mathcal{S}_r$ and $(\tilde{i},\tilde{j}) \in \mathcal{S}_{\tilde{r}}$ with $r \neq \tilde{r}$. Let

$$\gamma_r \triangleq \sum_{(i,j) \in \mathcal{S}_r} \alpha_i \beta_j. \tag{8}$$

Motivated by (7), we can introduce the following notation:

$$\overline{C}_r \triangleq C_{J,j}/g_i \text{ for any } (i,j) \in \mathcal{S}_r. \tag{9}$$

Without loss of generality we can assume that the sets $\mathcal{S}_r$ are arranged in increasing order by $\overline{C}_r$, i.e.,

$$\overline{C}_1 < \overline{C}_2 < \ldots < \overline{C}_R, \tag{10}$$

and let $\overline{C}_0 \triangleq 0$ and $\overline{C}_{R+1} \triangleq \infty$.

Let

$$P_r \triangleq \sigma^4 \overline{C}_r / h \text{ for } r = 0, \ldots, R+1. \tag{11}$$

Thus, by (10), we have that

$$0 = P_0 < P_1 < P_2 < \ldots < P_R < P_{R+1} = \infty. \tag{12}$$

By (12), for each $P > 0$ there is an unique integer $r(P) \in \{0, ..., R\}$ such that

$$P_{r(P)} \leq P < P_{r(P)+1}. \tag{13}$$

Thus, we can introduce the following notation:

$$F(P) \triangleq F_{r(P)}(P) \text{ with } P_{r(P)} \leq P < P_{r(P)+1}, \tag{14}$$

where

$$F_r(P) \triangleq \sum_{i=1}^{r} \gamma_i \sqrt{h\overline{C}_i / P} + \frac{h}{\sigma^2} \sum_{i=r+1}^{R} \gamma_i. \tag{15}$$

In the following proposition auxiliary properties of $F(P)$ are given.

Proposition 1. *(a) $F(P)$ is continuous and decreasing from $F(0) = h/\sigma^2$ to zero as $P \uparrow \infty$.*

(b) Let $h/\sigma^2 > C_P$. Then there is an unique r_* such that

$$\chi_{r_*^N+1} < C_P \le \chi_{r_*^N}, \tag{16}$$

where

$$\chi_r \triangleq \frac{h}{\sigma^2}\left(\sum_{i=1}^{r}\gamma_i\sqrt{\overline{C}_i/\overline{C}_r} + \sum_{i=r+1}^{R}\gamma_i\right),\ r = 1,\dots,R \tag{17}$$

and $\chi_{R+1} \triangleq 0$.

(c) The following equation has the unique positive root

$$F(P) = C_P. \tag{18}$$

Moreover, this root P is equal to P_N, where

$$P_N \triangleq h\left(\frac{\sum_{i=1}^{r_*^N}\gamma_i\sqrt{\overline{C}_i}}{C_P - (h/\sigma^2)\sum_{i=r_*^N+1}^{R}\gamma_i}\right)^2. \tag{19}$$

Proof: (a) Note that, by (11), $h/\sigma^2 = \sqrt{h\overline{C}_r/P_r}$ for any r. This and (14) imply that $F(P)$ is continuous. Also, $F(P)$ is decreasing in P as sum of decreasing functions (15). Moreover, by (15), $F(0) = h/\sigma^2$, and $F(P) = \sum_{r=1}^{R}\gamma_r\sqrt{h\overline{C}_r/P}$ for $P_R \le P$, and (a) follows.

By (11) and (15), we have that $\chi_r = F(P_r)$ for $r = 1,\dots,R$. This jointly with (a) imply that χ_r is strictly decreasing from $\chi_1 = h/\sigma^2$ to $\chi_{R+1} = 0$, and (b) follows.

By (a) and (b), Eq. (18) has the unique positive root. Moreover, by (b), this root belongs to $[P_{r_*^N}, P_{r_*^N+1}]$. Then, substituting (15) into (18) via straightforward calculation we obtain that this root is given by (19). ■

2.2 Equilibrium Strategies

In this section we derive equilibrium strategies in closed form.

Theorem 1. *In game Γ_N, the jammer's equilibrium strategy $\boldsymbol{J}$ is unique, while user equilibrium strategy is unique except of a particular case (b), where a continuum of user's equilibrium strategies arise. Specifically,*

(a) if

$$h/\sigma^2 < C_P \tag{20}$$

then

$$P = 0 \text{ and } \boldsymbol{J} = \boldsymbol{0} \triangleq \{J_{i,j} = 0 : (i,j) \in \mathcal{N} \times \mathcal{M}\}; \tag{21}$$

(b) if

$$h/\sigma^2 = C_P \tag{22}$$

then

$$\boldsymbol{J} = \mathbf{0} \text{ and } P \text{ is any such such that } P \le P_1; \tag{23}$$

(c) if

$$h/\sigma^2 > C_P \tag{24}$$

then $P = P_N$ *is uniquely given by (19) and* $J_{i,j} = BR_{i,j}(P)$*, where*

$$J_{i,j} = BR_{J,i,j}(P) = \begin{cases} \frac{1}{g_i}\left(\sqrt{g_i h P/C_{J,j}} - \sigma^2\right), & P > \sigma^4 C_{J,j}/(hg_i), \\ 0, & P \le \sigma^4 C_{J,j}/(hg_i). \end{cases} \tag{25}$$

Proof: By (2), the payoff of (i,j)-type jammer coincides with jammer's payoff in power control game with complete information [19]. This remark and [19, Lemma 1] imply that the best response of (i,j)-type jammer is given by (25).

Since $v_U(P.\boldsymbol{J})$ is linear in P, P is the best response to a fixed $\boldsymbol{J}$ if and only if the following relation holds:

$$P = \begin{cases} 0, & \sum_{(i,j)\in\mathcal{N}\times\mathcal{M}} \alpha_i\beta_j h/(\sigma^2 + g_i J_{i,j}) < C_P, \\ \text{is any}, & \sum_{(i,j)\in\mathcal{N}\times\mathcal{M}} \alpha_i\beta_j h/(\sigma^2 + g_i J_{i,j}) = C_P, \\ \infty, & \sum_{(i,j)\in\mathcal{N}\times\mathcal{M}} \alpha_i\beta_j h/(\sigma^2 + g_i J_{i,j}) > C_P. \end{cases} \tag{26}$$

Thus, to find NE we have to solve the best response equations (25) and (26). Two cases arise to consider separately: (I) $P = 0$ and (II) $P > 0$.

(I) Let $P = 0$. Then, by (25), $\boldsymbol{J} = 0$. Substituting $P = 0$ and $\boldsymbol{J} = 0$ into (26) implies $h/\sigma^2 \le C_P$. Thus, if $h/\sigma^2 < C_P$, then, (a) follows.

(II) Let $P > 0$. Two cases arise to consider: (II-a) $\boldsymbol{J} = \mathbf{0}$ and (II-b) $\boldsymbol{J} \ne \mathbf{0}$.

(II-a) Let $\boldsymbol{J} = \mathbf{0}$. Then, by (26), $h/\sigma^2 = C_P$. Substituting $\boldsymbol{J} = \mathbf{0}$ and $P > 0$ into (25) imply that $P \le \sigma^4 C_{J,j}/(hg_i)$ for any i and j. Thus, by (9) and (12), $P \le \sigma^4 \overline{C}_1/h = P_1$, and (b) follows.

(II-b) Let $\boldsymbol{J} \ne \mathbf{0}$ and $P > 0$. Then, by (26), $h/\sigma^2 > C_P$. Thus, by (26), we have that $\sum_{(i,j)\in\mathcal{N}\times\mathcal{M}} \alpha_i\beta_j h/(\sigma^2 + g_i J_{i,j}) = C_P$. Substituting (25) into this equation implies (19), and (c) follows from Proposition 1. ■

3 Stackelberg Game

In this section we consider Stackelberg game (SG) [11] scenario, where the user is leader while each type jammer is follower. Thus, each type jammer implements the best response strategy, and the user, knowing such jammer's behaviour, maximizes its payoff given as follows:

$$\Psi(P) \triangleq v_U\left(P, \mathbf{BR}_J(P)\right). \tag{27}$$

Then $(P, \mathbf{BR}_J(P))$ is Stackelberg equilibrium (SE), where $P = \text{argmax}_P \varPsi(P)$. Denote this SG by $\varGamma_S$.

By (11), (12) and (25), the payoff to the user $\varPsi(P)$ can be present as follows:

$$\varPsi(P) = \varPsi_{r(P)}(P) - C_P P \text{ for } P_{r(P)} \le P < P_{r(P)+1}, \tag{28}$$

where

$$\varPsi_r(P) \triangleq \sum_{i=1}^{r} \gamma_i \sqrt{h\overline{C}_i P} + \frac{hP}{\sigma^2} \sum_{i=r+1}^{R} \gamma_i. \tag{29}$$

3.1 Auxiliary Notations and Results

In this section we introduce auxiliary notations and present auxiliary properties of user's payoff (27).

Proposition 2. *(a) $\varPsi(P)$ is continuous for $P \ge 0$ such that $\varPsi(0) = 0$ and $\lim_{P\uparrow\infty} \varPsi(P) = -\infty$.*
(b) Derivative of $\varPsi(P)$ with respect to P is given as follows:

$$\frac{d\varPsi}{dP}(P) = \xi(P) - C_P \textit{ for } P \in \mathbb{R}_+ \setminus \mathcal{P}, \tag{30}$$

where $\mathcal{P} \triangleq \{P_1, \dots, P_R\}$ and

$$\xi(P) \triangleq \xi_{r(P)}(P) \textit{ for } P_{r(P)} \le P < P_{r(P)+1} \tag{31}$$

with

$$\xi_r(P) \triangleq \sum_{i=1}^{r} \frac{\gamma_i}{2} \sqrt{\frac{\overline{C}_i h}{P}} + \frac{h}{\sigma^2} \sum_{i=r+1}^{R} \gamma_i \tag{32}$$

and $\xi_{-1}(P) \triangleq \infty$.
(c) $\xi_r(P_r) < \xi_{r-1}(P_r)$.
(d) $\xi(P)$ is strictly decreasing in $\mathbb{R}_+$ and continuous everywhere except on the finite set $\mathcal{P}$.
(e) $\varPsi(P) = (h/\sigma^2 - C_P)P$ for $P < P_1$.

Proof: Note that, by (11), we have that $\sqrt{h\overline{C}_r P_r} = \overline{C}_r \sigma^2 = hP_r/\sigma^2$. This and (29) imply that $\varPsi(P)$ is continuous. By (29), $\varPsi(P) = \sum_{i=1}^{R} (\gamma_i/2)\sqrt{\overline{C}_i h/P} - C_P P$ for $P > P_R$, and (a) follows. (b) follow from (28) and (29).

By (11) and (32), we have that

$$\xi_{r-1}(P_r) - \xi_r(P_r) = \gamma_r \left(\frac{h}{\sigma^2} - \frac{1}{2}\sqrt{\frac{\overline{C}_r h}{P_r}} \right) = \frac{h\gamma_r}{2\sigma^2} > 0,$$

and (c) follows. (d) follows from (30) and (31). (e) follows from (28) and (29). ■

Let us introduce the following auxiliary notations:

$$\overline{\chi}_r \triangleq \xi_r(P_r) = \frac{h}{\sigma^2}\left(\sum_{i=1}^{r} \sqrt{\frac{\overline{C}_i}{\overline{C}_r}} \frac{\gamma_i}{2} + \sum_{i=r+1}^{R} \gamma_i \right), \tag{33}$$

$$\underline{\chi}_r \triangleq \xi_r(P_{r+1}) = \frac{h}{\sigma^2}\left(\sum_{i=1}^{r} \sqrt{\frac{\overline{C}_i}{\overline{C}_{r+1}}} \frac{\gamma_i}{2} + \sum_{i=r+1}^{R} \gamma_i \right), \tag{34}$$

where $r = 0, \dots, R$. Also, let $\underline{\chi}_{-1} \triangleq \infty$ and $\underline{\chi}_R \triangleq 0$.

Proposition 3. *There is an unique $r_*^S \in \{0, \dots, R\}$ such that one of the following two relations holds:*

$$\overline{\chi}_{r_*^S} \le C_P \le \underline{\chi}_{r_*^S - 1}, \tag{35}$$

$$\underline{\chi}_{r_*^S} < C_P < \overline{\chi}_{r_*^S}. \tag{36}$$

Proof: The result follows from (12), (33) and (34) and Proposition 2(c) and (d). ■

3.2 Stackelberg Equilibrium Strategies

In the following theorem we prove the uniqueness of SE and also present SE in closed form.

Theorem 2. *In game Γ_S there is SE $(P_S, BR_{\boldsymbol{J}}(P_S))$. Moreover, jammer equilibrium strategy is unique, while user's equilibrium strategy is unique except of a particular case (a), where a continuum of user's equilibrium strategies arise. Specifically,*

(a) if

$$C_P = h/\sigma^2 \tag{37}$$

then there is a continuum of user's equilibrium strategies. Specifically, any strategy P_S such that $P_S \le P_1$ is equilibrium strategy, while the jammer's equilibrium strategy is unique and it is $\boldsymbol{J} = \boldsymbol{0}$;

(b) if (37) does not hold then user's equilibrium strategy is unique and it is given in closed form as follows:

$$P_S = \begin{cases} P_{r_*^S}, & \text{if (35) holds,} \\ \dfrac{h}{4}\left(\dfrac{\sum\limits_{i=1}^{r_*^S} \gamma_i \sqrt{C_i}}{C_P - (h/\sigma^2) \sum\limits_{i=r_*^S+1}^{R} \gamma_i}\right)^2, & \text{if (36) holds,} \end{cases} \tag{38}$$

where r_*^S is given by Proposition 3.

Proof: First note that, by (27) and Proposition 2, there is at least one SE.

To derive SE two cases arise to consider: (a) $h/\sigma^2 \le C_P$ and (b) $h/\sigma^2 > C_P$.

(a) Let $h/\sigma^2 \le C_P$. By (33) and (34), this condition is equivalent to $\overline{\chi}_0 \le C_P < \infty = \underline{\chi}_{-1}$, Thus, (35) holds for $r_*^S = 0$. Also, by Proposition 2(f) and (32), $\xi(P) < C_P$ for $P = 0$. Thus, by Proposition 2(d), $\Psi(P)$ gets the maximum the the unique point $P = 0 = P_0$, and (38) holds.

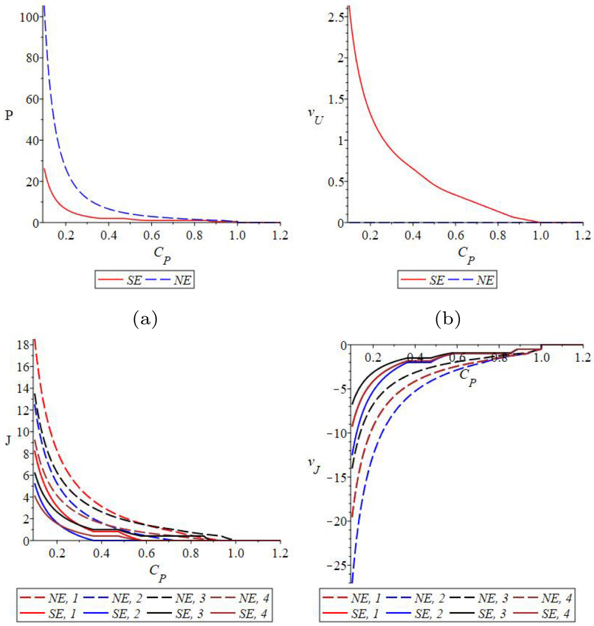
(a) Let $h/\sigma^2 > C_P$. Then, by Proposition 2, we have that there is the unique P_S such that $\Psi(P)$ is increasing for $P < P_S$ and it is decreasing for $P > P_S$. Thus, such $P = P_S$, is the unique equilibrium strategy. Then, two cases arise to consider: (b-i) (35) holds and (b-ii) (36) holds.

(b-i) Let (35) hold. Then $P_S = P_{r_*^S}$, and (38) follows.

(b-ii) Let (36) hold. Then $\xi_{r_*^S}(P)$ is continuous and strictly decreasing in $[P_{r_*^S}, P_{r_*^S+1}]$ from $\xi_{r_*^S}(P_{r_*^S}) > C_P$ to $\xi_{r_*^S}(P_{r_*^S+1}) < C_P$ Thus, equation $\xi(P) = C_P$ has the unique root in $[P_{r_*^S}, P_{r_*^S+1}]$, and straightforward calculation implies that this root is given by (38). ■

4 Discussion of the Results

Let us illustrate the obtained results by an example of a specific network where $n = m = 2$, $\sigma^2 = 1$, $h = 1$, $(g_1, g_2) = (0.5, 1)$, $C_J = (0.5, 1)$, and $\alpha = \beta = (1/3, 2/3)$. Then $\mathcal{N} \times \mathcal{M} = \cup_{r=1}^3 \mathcal{S}_r$, where $S_1 = \{(2,1)\}, S_2 = \{(1,1),(2,2)\}$ and $S_3 = \{(1,2)\}$. Figure 1 illustrates that an increase in user's transmission cost leads to a decrease in user's NE and SE strategies, and a decrease in its payoff in SG. In NG, due to linear structure of user's payoff on the user's strategy, its equilibrium payoff is equal to zero. Moreover, user's SE strategy is smaller as compared to its NE strategy, and this also follows from (19) and (38). This leads to that jammer's SE strategy is smaller as compared to its NE strategy (see, (25)). Flat segments in agents SE strategies reflect higher sensitiveness of the the NE strategies to varying the network parameters, as compared to the SE strategies (see, also (19) and (38)).



(a)

(b)

(c)

(d)

Fig. 1. (a) User's strategy, (b) user's payoff, (c) jammer's strategies and (d) jammer's payoffs.

5 Conclusions

The communication between an user and a receiver in the presence of a jammer, where the jammer, in contrast to the user, has access to local information about jamming fading gain and jamming cost, has been modeled by a Bayesian game in Nash game and Stackelberg game frameworks. The equilibrium are derived in closed form and compared. Higher sensitiveness of the the Nash equilibrium strategies to varying the network parameters, as compared to the Stackelberg equilibrium strategies has been proven and illustrated.

References

1. Al Daoud, A., Alpcan, T., Agarwal, S., Alanyali, M.: A Stackelberg game for pricing uplink power in wide-band cognitive radio networks. In: Proceedings of 47th IEEE Conference on Decision and Control (CDC) (2008)
2. Altman, E., Avrachenkov, K., Garnaev, A.: Jamming in wireless networks under uncertainty. Mob. Netw. Appl. **16**, 246–254 (2011). https://doi.org/10.1007/s11036-010-0272-4
3. Aziz, F., Shamma, J., Stuber, G.L.: Jammer type estimation in LTE with a smart jammer repeated game. IEEE Trans. Veh. Technol. **66**, 7422–7431 (2017)

4. Feng, Z., Ren, G., Chen, J., Zhang, X., Luo, Y., Wang, M., Xu, Y.: Power control in relay-assisted anti-jamming systems: a Bayesian three-layer Stackelberg game approach. IEEE Access **7**, 14623–14636 (2019)
5. Garnaev, A., Petropulu, A., Trappe, W., Poor, H.V.: A power control game with uncertainty on the type of the jammer. In: Proceedings of IEEE Global Conference on Signal and Information Processing (GlobalSIP) (2019)
6. Garnaev, A., Petropulu, A., Trappe, W., Poor, H.V.: A jamming game with rival-type uncertainty. IEEE Trans. Wireless Commun. **19**, 5359–5372 (2020)
7. Garnaev, A., Petropulu, A., Trappe, W., Poor, H.V.: A multi-jammer game with latency as the user's communication utility. IEEE Commun. Lett. **24**, 1899–1903 (2020)
8. Garnaev, A., Petropulu, A., Trappe, W., Poor, H.V.: A switching transmission game with latency as the user's communication utility. In: Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP) (2020)
9. Garnaev, A., Trappe, W.: The rival might be not smart: revising a CDMA jamming game. In: Proceeding of IEEE Wireless Communications and Networking Conference (WCNC) (2018)
10. Garnaev, A., Trappe, W., Petropulu, A.: Combating jamming in wireless networks: a Bayesian game with jammer's channel uncertainty. In: Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pp. 2447–2451 (2019)
11. Han, Z., Niyato, D., Saad, W., Basar, T., Hjrungnes, A.: Game Theory in Wireless and Communication Networks: Theory, Models, and Applications. Cambridge University Press, New York (2012)
12. He, G., Debbah, M., Altman, E.: k-player Bayesian waterfilling game for fading multiple access channels. In: 3rd IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP), pp. 17–20 (2009)
13. Jia, L., Yao, F., Sun, Y., Niu, Y., Zhu, Y.: Bayesian Stackelberg game for antijamming transmission with incomplete information. IEEE Commun. Lett. **20**, 1991–1994 (2016)
14. Li, L., Zhang, H., Yang, H., Wan, X.: Security estimation of stochastic complex networks under Stackelberg game framework. In: Proceedings of 37th Chinese Control Conference (CCC) (2018)
15. Sagduyu, Y.E., Berry, R.A., Ephremides, A.: Jamming games in wireless networks with incomplete information. IEEE Commun. Mag. **49**, 112–118 (2011)
16. Scalabrin, M., Vadori, V., Guglielmi, A.V., Badia, L.: A zero-sum jamming game with incomplete position information in wireless scenarios. In: Proceedings of 21th European Wireless Conference, pp. 1–6 (2015)
17. Xiao, L., Chen, T., Liu, J., Dai, H.: Anti-jamming transmission Stackelberg game with observation errors. IEEE Commun. Lett. **19**, 949–952 (2015)
18. Yang, D., Xue, G., Zhang, J., Richa, A., Fang, X.: Coping with a smart jammer in wireless networks: a Stackelberg game approach. IEEE Trans. Wireless Commun. **12**, 4038–4047 (2013)
19. Yang, D., Zhang, J., Fang, X., Richa, A., Xue, G.: Optimal transmission power control in the presence of a smart jammer. In: Proceedings of IEEE Global Communications Conference (GLOBECOM), pp. 5506–5511 (2012)
20. Yuan, L., Wang, K., Miyazaki, T., Guo, S., Wu, M.: Optimal transmission strategy for sensors to defend against eavesdropping and jamming attacks. In: Proceedings of IEEE International Conference on Communications (ICC) (2017)