




Berth Allocation Problem in Export Tidal Bulk Ports with Inventory Control

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Abstract. This paper presents the problem of allocating berth positions for vessels in tidal bulk port terminals (BAPTBI), considering the specific export scenario and robust control over goods' stock levels. An integer linear mathematical model is proposed for the discrete case (time and quay). The model controls the minimal and maximum inventory levels. A dataset of 59 instances was generated based on data obtained from a relevant bulk terminal in São Luís, Brazil. Through the experiment using the Gurobi solver, it is noticed that some medium-sized instances take more than 1 h to be solved.

Keywords: Berth allocation problem · Tidal ports · Inventory level constraints · Integer Programming · Mixed Integer Programming

1 Introduction

Bulk cargo is transported unpacked in large quantities of liquid (e.g., petroleum, gasoline, caustic soda, and chemicals) or dry (e.g., coal, grain, iron ore, and bauxite ore). In 2020, about 70% of the seaborne trade volume in tonnes loaded was dry bulk cargo, including iron ore and grains, of which Brazil is a great exporter, reaching in both 23% of world markets [1]. In bulk ports, vessels are loaded using excavators, conveyor belts, or pipelines. Silos or inventory piles for the bulk cargo are often alongside the berth or disposed of in large areas around the port [2].

In Brazil, the total port handling in 2021 was solid bulk 706,635,947 ton, liquid bulk 314,750,969 ton, containerised cargo 132,991,636 ton and general Load 60,004,915 ton, totalling 1,214,383,467 ton that represents 5.09% of increasing when compared to 1,155,608,201 ton in 2020. Ponta da Madeira terminal handled 182.36 million tons, Santos terminal 113.28 million tons, and Tubarão terminal 64.14, from which iron ore (370.57 million tons), oil and results (195.73million tons), and container (132.99million tons) [3].

In Maranhão state, the operational scenario at the maritime industrial port complex of São Luís is shaped by the public terminal of Itaqui port and the private terminals of Ponta da Madeira (Vale Mining Company), Aluminum Consortium of Maranhão (Alumar Alumina refinery), all currently in operation, as well as two new port terminals: the São Luís port terminal, headed by the company WPR, a subsidiary of the WTorre Company; and the Alcântara terminal port headed by GPM (Grão-Pará Multimodal) company [4].

A berth is a quay location equipped with one or more ship loaders. Generally, the more ship loaders, the greater throughput the berth has. The physical space dedicated to the berth can be continuous or discretised. In tidal ports, such as the ones in São Luís, vessels may need to wait for tidal conditions for mooring even when a berth position is available. Thus, the transit from waiting areas to the berth position is done in time windows at regular time intervals, previously known, in general, during high tides [5].

The decision to be taken must consider when (tidal time window) and where to berth a vessel, following the enterprise's berthing politics that considers berth capabilities and throughput, ship features and cargo, etc., and performance measures as overall quay utilisation, turnaround time, and incurred demurrage.

This work is devoted to the heterogeneous berth case of the Berth Allocation Problem in Tidal Bulk ports with Inventory level conditions (briefly named as BAPTBI or BAPTBS [5]), for which is assumed a dynamic BAP on a given discretised planning horizon. The quay is also discretised in berth positions with different load or unload throughput.

The vessels are allocated to time windows with favourable tidal conditions and berth availability regarding the inventory level of bulk cargo transported from/to the mining/refining company yard [5]. It is assumed that each vessel can load or unload multiple bulk cargo types without changing the berth, keeping the same throughput. For loading operations, inventory must be enough to be loaded onto the vessel. Unloading operations prioritise those vessels carrying raw materials at a critical inventory level.

Despite satisfactorily representing a standard bulk port terminal in tidal ports, the proposal in [6,7] differs from the present one mainly because they do not consider operational scenarios related to large mining companies, which work with large volumes of bulk and make decisions based on the levels of inventory in their storage yards. A recent literature review found no studies addressing stock-constrained ports, especially moving bulk in tidal windows [8].

This paper is organized as follows: in Sect. 2, a flat bibliographic review is presented; the mathematical model is proposed in Sect. 3; the instance generator, the instance dataset, and the main computational results are depicted in Sect. 4; the main findings are highlighted in Sect. 5.

2 Related Works

The problem of planning and scheduling integrated guaranteeing that products are stored and shipped within the established schedule has been approached

in [9]. A mathematical model was formulated as a Product Flow Planning and Scheduling Problem, solved by a column generation procedure and a branch-and-price algorithm.

The results have shown that the proposed method can reach optimal solutions in small and medium instances, producing upper and lower bounds for medium and large sizes instances related to operational scenarios where optimisation packages are not effective [9].

A mathematical model that deals with tidal constraints was proposed by [7], specifying aspects such as allocation of sections of quays and arrival-departure of vessels. The quay boundary is discretised into sections with the same length, each equipped with only one fixed quay crane. Depending on the vessel's length, one or two quay cranes can load or unload the vessel. CPLEX and a Genetic Algorithm were able to solve the model. The latter was proposed to deal with the above 20 vessels [7].

[10] proposes a model dealing with berth locations on an opposite or adjacent side in a continuous quay. Hence the model manages spatial constraints limiting the mooring and departure of vessels. Such real-world restrictions are not taken into account in the literature. The authors have proposed a mixed-integer linear programming formulation and a heuristic-based solver algorithm to obtain optimal or near-optimal solutions using instances inspired by an actual tank terminal in Belgium.

A stochastic version of a previous deterministic model [11] has been proposed based on a network berth-flow, dealing with delays on the planning horizon. The model considers different arrival times for each vessel, and a network is constructed for each type of berth. The results have shown that the stochastic model surpasses the manual allocation.

In [12], a mathematical model is presented, assuming a dynamic arrival of vessels, discrete berths, and operations without interruption before service completion. The work presented the results of a mathematical model validated through numerical experiments over six instances inspired by data of the bulk port of *Sfax*, Tunisia. The model considers specific features like length and draft of vessel, length and draft of berth, and security control time between vessels. The authors compared the results obtained by CPLEX solver with those obtained by the Sidi Youssef Port's planning process (SYPP), reaching a total service time saved from 6% to 17% [12].

Based on the port of Jorf Lasfar [13], the largest in Africa, a model was proposed to deal with the restrictions of routes made between the storage hangars and the berths with different water depths and heterogeneous berth speeds. The model's objective is to maximise the difference between dispatch and demurrage for all berthed vessels. Even being inspired by a specific port, the model is flexible to be easily adapted to any bulk port.

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3 Mathematical Modelling

The Berth Allocation Problem in Tidal Bulk ports with Inventory levels constraints (BAPTBI) is a lineup decision problem on a set of vessels with an Estimated Time of Arrival (ETA) within a given planning horizon. The lineup is a set of decisions that leads to the vessels' Estimated Time of Berthing (ETB). Once the berth and the docking TTW are assigned, it is possible to define the Estimated Time of Sailing (ETS).

The Berth Allocation Problem in Tidal Bulk ports with Inventory levels constraints (BAPTBI) has been modelled with time, and berth discretised [5]. A Tidal Time Window (TTW) happens in a regular and known time frequency, contemplating high tides or even low tides but under particular current conditions in which it is possible to tow a large ship through the narrow sea channel safely. The planning horizon is divided by M TTW and all the time scale (vessel's arrival times, days on hand, etc.) is discretised and expressed as a multiple of TTW.

For convenience, TTWs happen in a regular and known time-frequency TF ($TF \cong 12$ h, if contemplating only high tides). The planning horizon is divided by M TTWs and the time scale (vessel's arrival times, days on hand, etc.) is discretised and expressed as a multiple of TTW . Such assumption is sufficiently flexible to represent fine or coarse-grained discrete times. Even eventual not regular TTWs, like those with low tidal currents, can be represented by reducing the time-frequency (fine-grained discrete-time) and including constraints that disable specific low tide windows.

The data for BAPTBI is given by [5]:

- N : set of ships, $n = |N|$;
- M : set of TTWs, $m = |M|$;
- L : set of berth positions, $l = |L|$
- a_i : expected arrival time (ETA) of the ship i (TTW);
- h_{il} : handling time of the ship i if berthed at l (TTW);
- d_i : demurrage for ship i ;
- e_k : the initial stock level for the raw material k ;
- w_k : the amount of consumption or production for raw material k ;
- q_{ik} : the cargo capacity of the ship i with respect to raw material k .

The bulk k may be imported or exported (input or output cargo). Hence, the operation type (unload or load) is defined by the signal of w_k and q_{ik} .

- importation: $w_k, q_{ik} > 0$;
- exportation: $w_k, q_{ik} < 0$;

Handling time, h_{il} is given by:

$$h_{il} = \frac{\sum_{k \in K} q_{ik}}{v_l} \quad (1)$$

The BAPTBI mathematical model is decided by the variable y_{ijl} that represents the relationship between vessels, TTWs and berths (Eq. 2).

$$y_{ijl} = \begin{cases} 1 & \text{if ship } i \text{ is allocated to TTW } j \text{ and allocated to berth } l \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Equations 3 and 4 ensure that each vessel may be moored in a berth, at a TTW, after the ETA. Equation 5 avoids a vessel to be moored before the previous vessel is out of the berth. The last service TTW can be obtained by $j + h_{il} - 1$.

$$\sum_{j=1}^{a_i-1} \sum_{l=1}^{|L|} y_{ijl} = 0, \quad \forall i \in N \quad (3)$$

$$\sum_{j=a_i}^{|M|} \sum_{l=1}^{|L|} y_{ijl} = 1, \quad \forall i \in N \quad (4)$$

$$\sum_{\substack{n=1 \\ n \neq i}}^{|N|} \sum_{\substack{m=j \\ m \leq |M|}}^{j+h_{il}-1} y_{nml} \leq (1 - y_{ijl})|N||M|, \quad \forall i \in N, j \in M, l \in L \quad (5)$$

$$\sum_{i=1}^{|N|} \sum_{l=1}^{|L|} \sum_{z=a_i}^j \frac{\min(j - a_i + 1, h_{il})}{h_{il}} \times q_{ik} \times y_{izl} \leq j \times c_k + e_k, \quad \forall j \in M, k \in K \quad (6)$$

$$\sum_{i=1}^{|N|} \sum_{l=1}^{|L|} \sum_{z=a_i}^j \frac{\min(j - a_i + 1, h_{il})}{h_{il}} \times q_{ik} \times y_{izl} \geq j \times c_k + e_k - Q_k, \quad \forall j \in M, k \in K \quad (7)$$

Equation 6 refers to inventory control in an importation port, i.e., incoming vessels arrive loaded with the raw material necessary for the manufacturing activities that take place in the industry that manages the port (usually large transnational enterprises). On the other hand, Equation represents an exportation port inventory constraint in which the incoming vessels only may berth if there is a sufficient amount of bulk cargo to be loaded in.

The minimisation of the service time (Eq. 8) is defined as the sum of the waiting time, $j - a_i$, added to the handling time, h_{il} , for each ship. Note that TTW j meets the expected berth time (ETB).

$$\min \sum_{i=1}^{|N|} \sum_{j=1}^{|M|} \sum_{l=1}^{|L|} (j - a_i + h_{il}) \times y_{ijl} \quad (8)$$

4 Computational Experiments

In this section, the results of computational experiments are presented to enable the assessment of the proposed mathematical model. It is important to mention that the new proposition maintains the same realistic elements modelled previously, however featuring a better running time for resolution.

In order to define a more challenging dataset for a commercial solver and eventually for approximate algorithms, an instance generator is proposed for the BAPTBI.

4.1 Instance Generator

We have run an instance generator for building problem instances based on the operational scenarios in the private bulk port terminals in São Luís: Ponta da Madeira port (Vale Mining Company), Alumar port (Alumina refinery). The former mainly exports large volumes of *sinter-feed* iron ore through six ship-loaders that feature four berths. The latter imports raw materials, such as caustic soda and bauxite, through all two-berth positions. Nevertheless, occasionally Alumar port employs a single berth that is the only one equipped for load operations, and exports refined alumina destined to big manufacturers worldwide.

4.2 Instance Dataset

We have provided a total of 59 export instances¹, ranging combinations from $N \in [10, 50]$, $L \in [4, 6]$, and $K \in [4, 6]$. Such ranges of values cover parameters that well represent port terminals that are slightly smaller or larger than those found in São Luís' port terminal. All vessels handle only one bulk cargo at a time.

Allocating berths to ships affects handling times and consequently can expose the inventory to critical levels (close to zero or above Q_{max}), especially in scenarios where each ship only transports one raw material per trip. In these cases, it is more likely situations where it is required to prioritise ships with cargo whose inventory level is reaching a critical level. However, sometimes there is no possible decision that saves the instance from unfeasibility.

The problem instances are named in the form `nnN.bb.kk` to specify nn vessels, b berths and k different cargo.

4.3 Commercial Solver

Exact methods play an essential role in validating heuristic algorithms, as they serve as a baseline for comparing results. In this work, we use Gurobi's solver, version 9.5.1 with academic license².

¹ dataset available at <http://LACMOR>.

² <https://www.gurobi.com/academia/academic-program-and-licenses/>.

The solver uses simplex or barrier methods for continuous models and branch-and-cut for MILP³. The user can modify a significant number of parameters allowing different configurations.

This particular solver was chosen by the resources available, the detailed documentation, and the library provided in many programming languages besides performance.

4.4 Computational Experiment

In the second computational experiment, we run the solver GUROBI on the dataset considering the turnaround time objective function (Eq. 8) in the time limit of 4,800s.

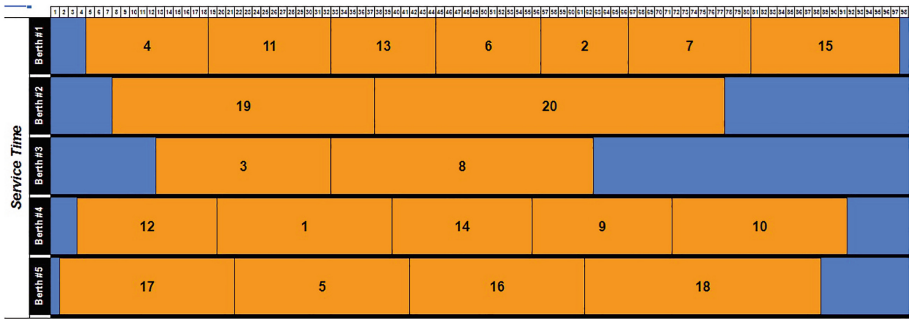


Fig. 1. Solution Service Time for instance 20N.5B.5P.20D

Table 1. Computational Experiment Results Using GUROBI

Instance	Solution (TTW)	Time (s)	GAP (%)
10N.4B.5K	236	3.66	0.00
10N.4B.6K	285	7.90	0.00
10N.5B.4K	167	2.63	0.00
10N.5B.5K	190	2.12	0.00
10N.5B.6K	210	2.73	0.00
10N.6B.4K	107	0.94	0.00
10N.6B.5K	136	1.45	0.00
10N.6B.6K	151	1.91	0.00

(continued)

³ https://www.gurobi.com/documentation/9.5.1/refman/cpp-grbmodel_optimize.html.

Table 1. (*continued*)

Instance	Solution (TTW)	Time (s)	GAP (%)
15N.4B.4K	501	82.31	0.00
15N.4B.5K	644	77.90	0.00
15N.4B.6K	845	125.49	0.00
15N.5B.5K	620	117.80	0.00
15N.5B.6K	765	90.77	0.00
15N.6B.4K	217	12.47	0.00
15N.6B.5K	317	20.94	0.00
15N.6B.6K	396	37.90	0.00
20N.4B.4K	1016	236.32	0.00
20N.4B.5K	1122	4800.00	2.41
20N.4B.6K	1311	4800.00	0.23
20N.5B.4K	523	54.30	0.00
20N.5B.5K	671	4800.00	36.07
20N.6B.4K	465	41.89	0.00
20N.6B.5K	605	4800.00	39.34
20N.6B.6K	807	147.59	0.00
25N.4B.4K	1251	395.35	0.00
25N.4B.5K	1273	4800.00	3.30
25N.5B.6K	1258	4800.00	1.74
25N.6B.5K	991	4800.00	29.87
30N.5B.4K	922	4800.00	51.19
30N.5B.6K	1119	4800.00	0.45
30N.6B.5K	1413	930.68	0.00
30N.6B.6K	1192	4800.00	0.76
35N.4B.4K	922	284.07	0.00
35N.4B.5K	1172	622.62	0.00
35N.4B.6K	1435	4800.00	0.84
35N.5B.4K	1242	4800.00	30.19
35N.5B.5K	1452	659.00	0.00
35N.5B.6K	1637	4800.00	0.49
35N.6B.6K	2061	4800.00	0.44
40N.4B.4K	1742	4800.00	28.76
40N.4B.5K	2294	4800.00	0.22
40N.4B.6K	2114	4800.00	0.99
40N.5B.4K	1995	4800.00	0.40
40N.5B.5K	2290	4800.00	3.49

(continued)

Table 1. (*continued*)

Instance	Solution (TTW)	Time (s)	GAP (%)
40N.5B.6K	2470	4800.00	1.70
40N.6B.5K	2062	4800.00	0.63
45N.4B.4K	1854	4800.00	54.64
45N.4B.5K	1640	4800.00	40.24
45N.4B.6K	2180	4800.00	1.01
45N.5B.4K	1957	994.93	0.00
45N.5B.5K	2128	4800.00	0.61
45N.5B.6K	2460	4800.00	0.53
50N.4B.4K	1871	4800.00	0.11
50N.4B.5K	2403	4800.00	58.43
50N.4B.6K	2804	4800.00	69.72
50N.5B.5K	2929	4800.00	58.01
50N.5B.6K	3474	4800.00	37.25

5 Conclusion

This work shows a slightly better response of GUROBI for the new maximum inventory level constraint tested for 81 instances. It happens due to the reduction of the search space with the inclusion of this new constraint, so the solver can spend its total time for new areas, reducing the final gap. From 81 instances tested 21 were considered infeasible by the solver, or 4800 was not enough time to find a solution.

Table 1 above shows the results obtained for the instances, the total time used by GUROBI to reach the best solution and the final gap of the search space.

Already in Fig. 1 we have a graphic example of solution obtained by the solver where we can see the proper distribution of vessels along berths and tidal windows. To reach the minimum service time, the proposed solution agglutinate the vessels without free TTW between them to find the lower value for the objective function. For the 20N.5B.5P instance, the solution prioritise the faster berths (#1, #4 and #5) to attend the vessels, reducing the time usage for the slower berths (#2 and #3)

Future researches can improve the instance generator quality, working with higher number of vessels and comparing larger datasets with controlled differences between them. Other point is to apply heuristics and metaheuristics that can compete against the commercial solvers or work with them inside hybridization strategies to reach better results.

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