Reducing Long-Run Average Planned Maintenance Cost Using Markov Decision Modelling Based on Shifting Paradigm and Penalty Model

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Abstract. This paper aims at developing a model for the planned maintenance program and long-run average cost. Markov process decision approach in its discrete version used to model the problem. Penalty cost model due to shifting paradigm as approach applied for textile boiler maintenance program by taking three components of the boiler. Whereas, a three-step policy iteration algorithm, initialization, value determination and policy improvement, for numerical experimentation based on component current age and the schedule maintenance time for the planned maintenance program considered. Relative values (RV), which are not only immediate value from an action taken from planned activity but also are guaranteed values that would be resulted from the accomplishment of the maintenance action specified on the maintenance policy proposed. More importantly, unless a trade-off is made, RVs instead are costs in the long run and this is what the average cost-policy compromise and the policy iteration assures while, as time gets advance the deterioration rate of the components expected to increase, shifting forward will bear an extra cost is the underlying. With this proposed approach, for validation purpose, relative values (RV), based on the two shifting strategies, Forward (FW) and Backward (BW) shifting compared with that of on-schedule (OSC) maintenance, stationary policy based on the long-run average cost obtained to be 2053Birr for the specified case.

Keywords: Maintenance shifting · Markov decision process · Optimal cost · Long run average cost · Deterioration

1 Introduction

Maintenance concept encompasses the area of philosophies, maintenance support levels, work forces, and time required for maintenance. Establishing basis for both maintainability and total maintenance support requirement are the root purpose of the maintenance concept, which could possibly addressed with a proper maintenance analysis. Based on this analysis-anticipated frequency of failures, crew skill levels, spare parts, the tools and the facilities required can be worked out in advance. In general, the objective of maintenance falls to four basic considerations a) ensuring

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system function (availability, efficiency and product quality); b) ensuring system life (asset management), c) ensuring safety and d) ensuring human well-being [1]. Maintenance concept is a prior task for equipment design to meet the design function and the overall maintenance concept. One can then reach in conclusion to understand maintenance concept development is about the requirements, maintainability and facility provision, which in turn show the needs of the system design engineer and the requirement of logistic support planner. The objective behind deploying complex and sophisticated machine (automation) is to achieve higher productivity to have a good return in business (profit). However, this objective is dependent on the functionality and wellbeing of such machines. The measures taken by the industry to keep machine and operating system in trouble free condition are collectively termed maintenance engineering [2]. The reason behind to the out date of "run to failure" is the high cost of the machines than the maintenance cost incurred to make them available. Maintenance is a function to keep the equipment/machine in a working condition by replacing/repairing [3] some of the component of the machine. According to the British Standard (BS 3811-1984) maintenance is: the combination of all technical and associated administrative actions intended to retain an item in or restore it to a state in which it can perform its required function "stated condition". On its, evolution, Fig. 1, maintenance activity comes across different stages starting from run to failure operation until 1950s, proactive maintenance: Condition Based Maintenance (CBM), and Time Based maintenance and to the contemporary Reliability Cantered Maintenance (RCM) and Total Productive Maintenance (TPM). The type of the failure, the time that will failure occur, the condition for failure to happen, the priority to the failure effect and the all-inclusive are the frame work of these evolutions of maintenance. With regard to this maintenance progress and considerable match of the maintenance, type different maintenance policies have been formulated. Preventive maintenance (PM), Predictive maintenance (PdM), Reactive maintenance (RM), proactive maintenance (PrM), condition based maintenance (CBM) and time based maintenance (TBM) policies has found under the two main maintenance categories See [4, pp. 3–3] "planned and Unplanned maintenance programs". Nevertheless, the problem especially in planned maintenance is the accuracy in performing the maintenance activities with the equipment failure condition and time. Since preventive maintenance (PM) which also known as scheduled maintenance is eliminating the future drastic treatment in future [3] (preserving asset, preventing failure and deleting incipient faults preserving asset, preventing failure and deleting incipient faults.

In its usual nature of PM, preventive planned maintenance is the utilization of planned and coordinated routine maintenance activities such as inspection, adjustment, repair and replacement which minimize the interruption of the system with high cost whereas maintenance planning for deteriorating facilities seems to be hard because of their random aspects. For [5] machine in its service of production faces to fates: sudden failure or gradual deterioration which the latter potentially be characterized with frequent repair that lead to higher maintenance cost, however, at decreasing productivity.

Developing model that is insightful in describing aging phenomenon of technical components and that give option for action to cope with anticipated problem is question of recent research interest [1]. While with such distributed random protocol, Markov Decision process (MDP), helps to combine both deterministic and probabilistic nature of the problem domain [6] - an approach that has been mentioned to bridge the gap between analytic model and practice [1].

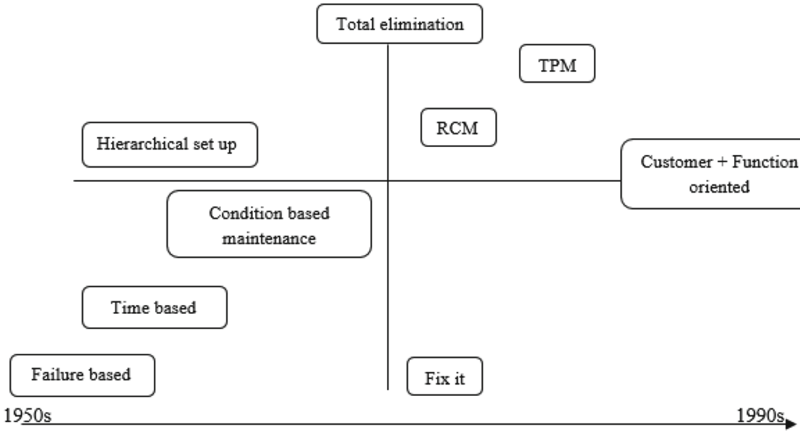

Fig. 1. Evolution of maintenance concept

Then it is the main purpose of this paper to integrate the planned maintenance program with the stochastic nature of the equipment failure and remaining are: Sect. 2 is about recent literature while methods and material presented in Sect. 3 and followed by Case description and it is modelling in Sect. 4. Section 5 provides experimentation and model validation with results and discussion in Sect. 6. Finally, conclusion and forwards presented in Sect. 7.

2 Recent Literatures

For their discrete time Markov (DTM) model, [3] considered the dynamic nature of system that has to be reviewed at equidistant of time for a possible set of state I that demand action appropriate to it. Since each state $s_i \in I$ and related action $a_i \in A$ are finite, understanding economical consequent through cost at each decision epoch is practical. As an alternative to current state of art of policy for corrective action [7] deployed Markov decision process (MDP) to evaluate corrective maintenance scheduling policies for offshore wind farms using production loss duet distinct policies. On the other hand, [8] proposed a generalized condition based maintenance (CBM) model using Markov decision process for cost effective decision making process. They start by introducing traditional maintenance practice- corrective maintenance (CM) and preventive maintenance (PM) which they specially emphases to one, which is prominent for PM-time, based maintenance (TBM) by arguing with the assumption behind, i.e. two component with similar age will never have similar failure rate which the opposite also holds true. Moreover, if defined well failure rate rather is a function of time than deterministic propagation as observed in time based models [8].

As is evidenced from their review report age of component takes share of failure rate from 15%–20% which the remaining 80%–85% is due to random effect (See also [8]).

Emphasizing to civil infrastructure sector, which unlike mechanical component, deterioration is slow and failure does not show direct economic consequence [1]. Random failure and failure due to deterioration considered by [9] and using policy iteration algorithm an optimum maintenance policy determined for proposed Markove decision model.

Recent work of [10] propose a preventive maintenance schedule for healthcare using Markov chain and were successful both in reducing use of resource and optimizing periodicity of routine maintenance cost. They classified the state of healthcare into nine state of nature (1 = excellent, 9 = unacceptable) and follow a statistical study for determining of probabilistic transition of such identified states. One which is similar to our setup content wise is the work of [11] propose decision model, which help to determine optimal time between periodic inspection. They provide a cost function based on two scenarios-one is for detecting failure without inspection, and the other is for detecting failure only through inspection. A review of probabilistic maintenance model by [12] mad clearly a distinction between deterioration model and decision model with the priori is to estimate the uncertain time to failure while the latter is to optimize the time maintenance activity using result of priori. Consequently, they deployed Markov process to model the uncertain deterioration of an object at concept of Markov Chain (MC). They considered three potential states for number of condition of state set $S = \{0, 1, \ldots .m\}$ an item could attain; state $S = 0$ implies the newness state and $S = m$ indicates state of failure. The three potential state are based on the thresholds for preventive maintenance (r) and corrective maintenance (s) and includes for current state stat of itemcs, (i) functional state $cs < r$ (ii) is marginal state $r \leq cs < s$ and (iii) failed state $s \leq cs$ and for pictorial visualization of these state readers are referred to [11, p. 250]. By marginal state, an item is still functional but ready for an action weather it is preventive or replacement [13].

In general, the work of Kallen and van Noortwijk is motivational but one important contribution of our work is the issue of the paradigm of maintenance shifting (see Sect. 4.3) from principle of joint replacement [14, 15] with the underlying of relative value achieved through shifting the maintenance schedule. Both policy iteration and value iteration of Bellman methods from dynamic programing that are prominent solving method for Markov decision process (MDP) or Semi-Markov decision process (SMDP) [16].

3 Materials and Methods

The main objective of this paper is to formulate a planned maintenance policy by using discrete Markov chain decision process, modelling of the decision process making about the interval of the inspection and modelling the maintenance decision. Figure 2 presents research design that starts with case description of the problem domain and constructing deterioration model for problem to supply state of nature with their probabilistic occurrence to decision model constructed using discrete Markov decision chain (MDC) more preferably Markov Decision Process (MDP) as described in [1].

As contribution to this paper, shifting paradigm is to account relative values and we prefer rather the long-run average cost criteria than the total cost. Cost for various states identified, probabilistic value of each potential state described in [11] determined accordingly by normalizing each statistically. Since in its natural understanding, deterioration model is bas on continuous Markov chain uniformazation is followed approach followed to extract probabilities to each discrete state.

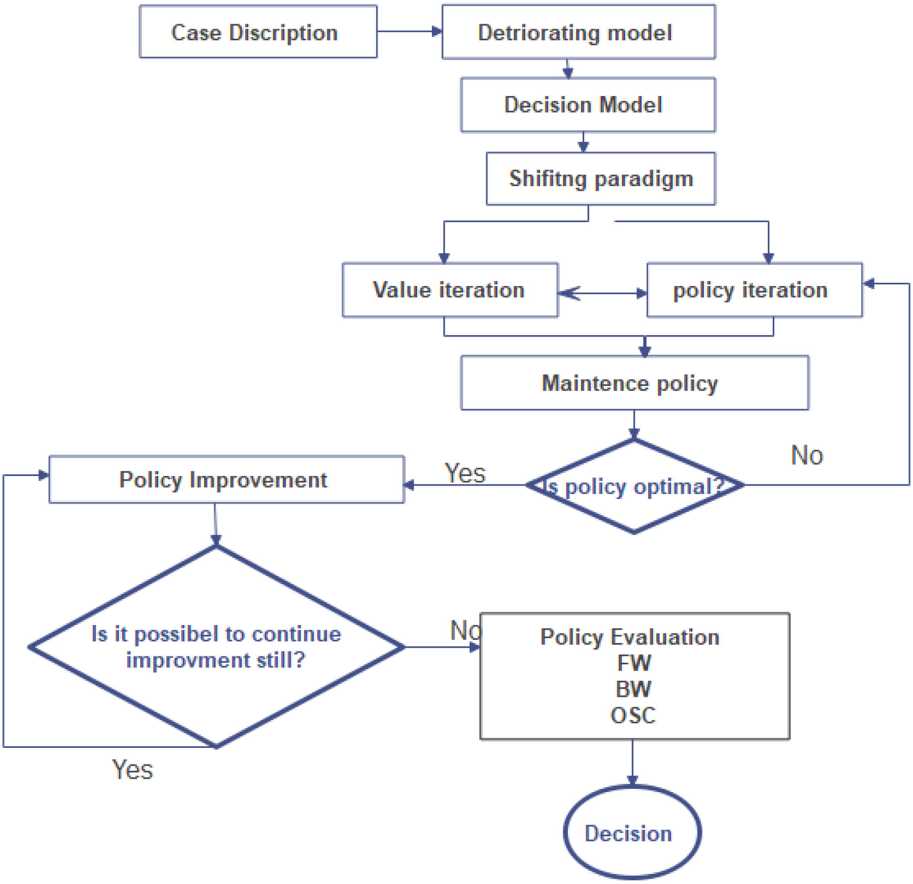

Fig. 2. Research design

4 Cause Description and Modeling

In textile industry, various utilities are utilized for the complete operation of the textile process. A steam or heated water is the most crucial one and can be generated from boiler. A boiler is a system at which water is heated under a pressure. The steam or heated fluid is expected to be circulated out of the boiler for the use of various applications such as sizing, drying, cooling, washing, and so one. It is a closed system and made up of various components. In this paper, the mainly considered components are safety valves, heaters, and temperature sensors. The failure occurs on one of each component causes the failure of the whole system of the boiler. As the function of each of the components and the exposing condition for them is distinct, it enforces the maintenance crew to a distinct time epoch of both preventive and corrective maintenance program. Keeping the routine preventive action as oiling, inspecting and checking as common these different components has different preventive action of time zone and subsequent corrective maintenance epoch. Not only the condition at which each component operate, it is the age of the components which deteriorates with time advance the failure rate of the components in particular and the whole system in general.

Among the various failures modes, overheating, rupture, leaking and cracking or crazing failure modes are the most frequently to occur and are need to be seriously managed. An assumption is made here, though may not it be important, about the failure rate and the planned maintenance activities for each component for the boiler. Each component is assumed to have relativity the same failure rate and repair rate for the same state of conditions. However, the effect related to this failure rate is not to be the same and that is why it is assumed that the planned maintenance activities are made from. The main purpose of this paper is to develop a cost effective maintenance policy via the application of the Discrete Markov chain and considering trade-off between taking separate maintenance action and combined action with respect to the set up cost. Considering only the set up cost is not the deriving factor for the planned boiler maintenance program, it is also to prevent the frequent stoppage of boiler due to each component preventive and corrective action. Each of the considered component are assumed to has similar deterioration model developed in model with the consideration of deterioration and maintenance (Sect. 4.1) which could transient such state of the equipment to the other state the equipment has to follow.

4.1 Discrete Markov Chain as Stochastic Modelling

While modelling a maintenance system using Markov chain defining concept of system and state transition are of basic important. Letting system state as X_n, $n = 0, 1, \ldots$ as time index is to mean that we have knowledge about the system *at* $X_0, X_1 \ldots X_{n-1}, X_n$. However, as far as X_{n+1} is concerned all states before X_n seems redundant, i.e., no memory is needed which is the Markov property. It is claimed that the system state X_n is a stochastic process and the time homogenous discreet Markov chain is assumed here as given in Eq. (1).

$$P(X_{n+1} = j | X_n = i) = P(X_1 = j | X_0 = i) \tag{1}$$

From Eq. (2), we can infer that whatever the value of $X_0, X_1 \ldots X_{n-1}, X_{n-1}$ the left side conditional probability is same. This is to mean that for a given present state of the system (X_n) the future state of the discrete time Markov chain (DTMC) (X_{n+1}) is independent of its past $(X_0, X_1 \ldots X_{n-1})$. Quantity P is called a one-step transition probability of the DTMC at time n. and since, Eq. (2) is a time homogeneous DTMCs, the one-step transition probability depending on *i* and *j* but is the same at all times n.

Therefore, throughout this paper it is to mean that a homogeneous time process when there is an expression of DTMC. Consequently, with the shorthand notation for the one-step transition probability P_{ij}, to convey that given the system is in state i at time$t$, it will be in a state j at time $(t+1)$:

$$P_{ij} = P(X_{n+1} = j|X_n = i)\,for\,i,j = 1,2,\ldots N \tag{2}$$

The transition probabilities are commonly expressed as an $N \times N$ matrix called the transition probability matrix (or transition matrix) P:

$$\boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & p_{22} & \cdots & p_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} \end{bmatrix} \tag{3}$$

$$\sum\nolimits_{j=1}^{n} p_{ij} = 1\,for\,i,j = 1,2,\ldots,N \tag{4}$$

Based on the Chapman-Kolmogorov equation, the probability of the system moving from state i and state j after n periods (n transitions), that is, the *n-step* transition probability matrix, $\boldsymbol{P}^{(n)}$ can be obtained by multiplying the matrix P by itself n times Eq. (5). Thus, Eq. (5.) implies that the application of the Markov chain processes to the development of a deterioration model for a system. Alternatively, an equipment, once a reliable transition probability matrix for the system/equipment is identified, the expected condition of the system/equipment in the future or the expected years that the system will be in a degraded condition can be easily obtained.

$$P^{(n)} = P^n \tag{5}$$

The left side of Eq. (1) of course is general form of Discrete Time Markov Chain, DTMC, and taken as a one-step transition probability P_{ij} of state i and j that must be summed to unity. The model is then tries to integrate the planned maintenance with the characteristics and application of Markov chain by incorporate such three important concerns: Deterioration model, Decision model, cost model and Paradigm of Shifting in each of the respective section next.

4.2 Deterioration Model

Equipment pertains to a units' position in a state space with greater probability of failure than a former position. It can be modelled using general path model, random process model Markovian stochastic process model and time series model. Individual randomness and dynamic environment always cases for temporal uncertainty and is difficult to apply both general path and random process rather the Markovian stochastic process as it is flexible to such practical deterioration indicators. For the given deterioration, X_n in discrete time n and all possible of it contained in state space S can shift to each other according to the transient matrix with special state X_f called absorbing state representing the failure state of the asset. Equipment deterioration multi factor dependent and of these the way they are treated, preventive maintenance and inspection, takes the higher share. In Fig. 3 (b) N is nominal operation state and D_i is deterioration state or marginal before being state malfunctioning F. Since, Fig. 3(a) and Fig. 3(b) that respectively portrayed model of three state and general equipment models of deterioration are based on continuous Markov chain (CMC), uniformization based on Eq. (6) for uniform transition rate γ deployed. Dividing the original transitions by γ therefore gives probability value for each discrete state of Markov chain. By assuming the instant state after the maintenance is performed as a new state, the transition probability can be constructed using Eq. (7).

$$\gamma = \boldsymbol{\lambda}_1 + \boldsymbol{\lambda}_2 + \boldsymbol{\lambda}_3 + \ldots + \mu_1\mu_2\mu_3 \tag{6}$$

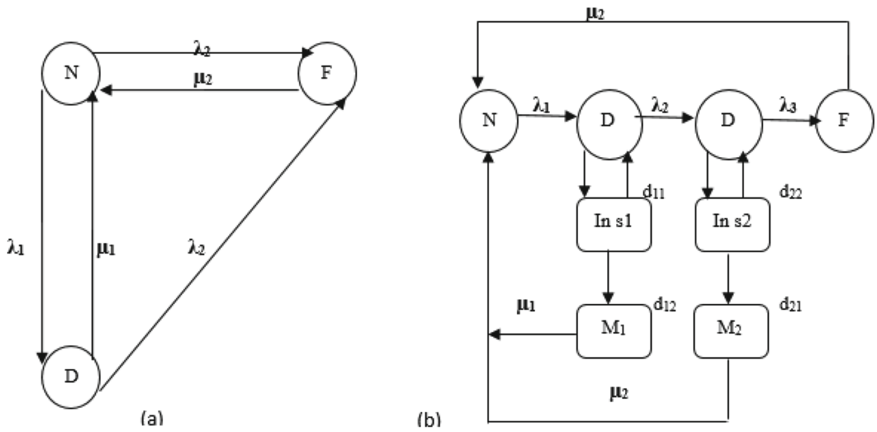

Fig. 3. Three state and general equipment model

$$P = \begin{bmatrix} NN & ND_1 & ND_2 & NF \\ D_1N & D_1D_1 & D_1D_2 & D_1F \\ D_2N & D_2D_1 & D_2D_2 & D_2F \\ FN & FD_1 & FD_2 & FF \end{bmatrix} = \begin{bmatrix} 1-\boldsymbol{\lambda}_1/\gamma & \boldsymbol{\lambda}_1/\gamma & 0 & 0 \\ \boldsymbol{\mu}_1/\gamma & 1-\boldsymbol{\lambda}_2/\gamma & \boldsymbol{\lambda}_2/\gamma & 0 \\ \boldsymbol{\mu}_2/\gamma & 0 & 1-\boldsymbol{\lambda}_3/\gamma & \boldsymbol{\lambda}_3/\gamma \\ \boldsymbol{\mu}_3/\gamma & 0 & 0 & \boldsymbol{\mu}_3/\gamma \end{bmatrix} \tag{7}$$

4.3 Decision Model

The equipment at the beginning of each day found to be in one of the working condition $S_i = 1, 2, \ldots, N$ and is obvious that working condition i is better than working condition $S_j = S_{i+1}$. Given the current condition i with no repair, probability of equipment to be on working condition j is $q_{s_i s_j}$ and is zero for $S_i > S_j$. An enforced repair action elapsed with m days has to take for $S_i = N$ while if the equipment is found between $1 < S_i < N$ there is a possibility to take or not to take a maintenance action. Hence, for the maintenance action Eq. (7) gives an alternative action. A repaired system has a working condition $S_i = 1$ and the cost of enforced repair is C_f and Cp_{s_i} is a pre-emptive repair cost. This problem can be put in the framework of a discrete-time Markov decision model and cost criterion from the maintenance point of view has been articulated as long-run average cost, total cost, marginal cost, or discount cost. Time horizon in the caused problem for maintenance action is assumed infinite hence; the long-run average cost is preferred rather than total cost criterion. Since an enforced repair takes m, days and the state of the system has to be defined at the beginning of each day, there should be an auxiliary state for the situation in which an enforced repair is in progress already for y days. Thus, the set of possible states of the system is chosen as $S = \{1, 2...N, N+y\}$. State S_i with $1 \leq S_i \leq N$ corresponds to the situation in which an inspection reveals working condition i, while state $N+1$ corresponds to the situation in which an enforced repair is in progress already for y day. Hence, if, a_i action it can be stated as Eq. (8).

$$a_i = \begin{cases} 0, \textit{if no repair is done} & 1 \leq j < N \\ 1 \textit{ if preventive action is done} & 1 < j < N \\ \quad 2 \textit{ enforced repair done} & \end{cases} \tag{8}$$

With repair time of $y < m$, the one-step transition probabilities $p_{ij}(a)$ are given:

- $p_{s_i, s_j}(0) = q_{s_i s_j} \textit{for } 1 \leq s_i < N$
- $p_{s_{i,} 1}(1) = 1 \textit{ for } 1 < s_i < N$
- $p_{N, N+y}(2) = p_{N+y, 1}(2) = 1$

And it is institutive that for $p_{ij}(a) = 0$, $C_{si}(0) = 0, C_{si}(1) = Cp_{si}$, $C_N(2) = C_f$ and $C_{N+y}(2) = 0$.

4.4 The Paradigm of Shifting

A machine as a system, if m_i is the maximum life time of the component i, the possible state of the component are $m_i + 1$ and the state space is a result of the sum product of the number of product n and $m_i + 1$ equivalent to the $\prod_{i=1}^{n} mi + 1$. Each of the component limited to the control limit h_i^*(age) for replacement analogize to age of preventive action for the planned maintenance activity especially for time based preventive maintenance. In planned maintenance activity, different component may have different lifetime or have being exposed for different operational condition with different deterioration (limitation of planned maintenance). For the control limit age h* of the component, the expected age for the preventive maintenance, plays an important role to draw an argument about the long run average cost of preventive action and breakdown maintenance. The long-term average cost $g_i(h)$ in the control limit h_i^* of component i will have a relative value of V_i^j

that cane linearly formulated as given in Eq. (9) and a normalization equation $v_s = 0$ as a unique solution.

$$V^j{}_i = \min_a\{C^j{}_i(a) + \sum_{k\in S} P_i^{jk}(a)v_s{}^k(h^*)\} - g_i^* \tag{9}$$

Where a and $C_i^j(\alpha)$Cʲᵢ(a) are action and cost due to this action for component i and $P_i^j(\alpha)$Cʲᵢ(a)Pᵢʲᵏ(a) is the probability of transition of component i from state j to state k when action a is chosen. It is clear that different relative value can be inferred at each transition state as given by Eq. (10). It is stated that earlier that the objective in this paper is to make a trade-off between separate and combined maintenance activity under shifting paradigm.

$$V_i^j = \begin{cases} P_i^j V_i^j + q_i^j V_i^j - g_i \quad for\ j = 0..h^* - 1 \\ Cp_i + V_i^0 \quad for \quad j = h^* m_i \\ b_i + Cp_i + V_i^0 \quad for\ j = m_i + 1 \end{cases} \tag{10}$$

Where Cp_i and b_i are cost of preventive and break down action for component respectively. While $V^0{}_i$ is value paid to bring component i to state zero. If we denote the average, set up cost by Δc the possibility to have cost effective action via taking combined preventive maintenance activities is given as $\Delta c + V^n{}_i > Cp_i + V^0{}_i$ and V_i^n indicates the value that would be paid if the preventive action on component $i$ii were taken. Our trade-off to the combined action, however, has another challenge: penalty cost due to shifting, which can be forward (FW) or backward (BW) as given in Eq. (11). If the pre-planned preventative scheduling time, called epoch time is t_i, the three scenarios expected are: (1) if the component fail at its due date but prevention action is shifted forward (positive shifting), (2) if shift is backward (negative shifting) (3) if no shift.

$$shifting = \begin{cases} Postive, for\ t > t_i, \Delta t = t - t_i, \Delta t > 0 \\ none \quad for\ t = t_i, \Delta t = 0 \\ negative, \quad for\ t < t_i, \Delta t < 0 \end{cases} \tag{11}$$

Equation (11) confirms that for no shifting of prevention action, component i gets immediate response as per the schedule and only the cost for planned activity incurs. On the other hand, there exist an expected cost of penalty costs due to postponing ($\Delta t > 0$) or due to early action ($\Delta t < 0$) and can be valued relatively. For control limit h^* and some constant number n of (Δt ($\Delta t = n$), if the component is not failed before $t_i + 1$ the next prevention action then is at epoch $t_i + n$ but at probability of $p_i^{h^*}$, otherwise malfunction with probability $q_i^{h^*}$, i.e., $1 - p_i^{h^*} = q_i^{h^*}$. A state space $j = j + 1$ and $j = m_i + 1$ are respectively operating and failed state and the average cost g_i^* for one period transition can be saved probabilistically. For a preventive action with probability $p_i^h.p_i^{h+1}$, the relative value is $V_i^{h+2} - g_i^*$, otherwise a failure tends to occur with probability $q_i^h.q_i^{h+1}$ and relative value is $V_i^{m+1} - g_i^*$ if response of corrective maintenance action taken. However, if these all not happen, state $j = n$ is valued as V_i^h

and a penalty cost $p_i(n)$ entertained and generalized using Eq. (12). Arranging along with the stationary policy for the Markov chain to Eq. (13) and cost for shifting preventive action is given Eq. (14) for shifting time Δt.

$$P_i(n) = p_i^h\left(V_i^{h+1} - V_i^{h+1} - g_i^*\right) + q_i^h\left(V_i^{m+1} V_i^h - g_i^*\right) + p_i^h q_i^{h+1}\left(V_i^{m+1} V_i^{h+1} g_i^*\right) + p_i^h q_i^{h+1}\left(V_i^{m+1} - V_i^{h+1} - g_i^*\right) \tag{12}$$

$$P_i(n) = \left(q_i^h b_i - g_i^*\right) + + p_i^h\left(q_i^h b_i - g_i^*\right) \tag{13}$$

$$p_i(\Delta t) = \begin{cases} \sum_{j=h_i}^{h_i+\Delta t-1} \left(q_i^j b_i - g^*\right) \sum_{l=h_i}^{j-l} p_i^l & \text{if } \Delta t > 0 \\ \sum_{j=h_i+\Delta t}^{h_i-1} \left(g^* - q_i^j b_i\right) \sum_{l=h_{i+\Delta t}}^{j-l} p_i^l & \text{if } \Delta t < 0 \end{cases} \tag{14}$$

4.5 Maintenance Policy

An optimal maintenance activity and a stationary policy is a convenient one for average-cost Markov decision model with finite state space and finite action set. A stationary policy R is a policy that assigns a fixed action $a = R_j$ to each state j and always used whenever the system is in state$j$. By acknowledging the effort made, this section is based on the work of [15] with given state $j = n$ for $n = 0, 1, ..$ to define the state of the system X_n at the n^{th} decision epoch, the markov chain is defined as $P\{X_{n+1} = k | X_n = j\} = p_{jk}(R_j)$ regardless of the past history of the system up to time n. The stochastic process $\{X_n\}$ is a discrete-time markov chain with one-step transition probability $p_{jk}(R_j)$ with incurred cost $c_i(R_j)$ each time the system visits state j and the long –run average cost per time unit under a given stationary policy could be invoked. Finding the optimal policy for average cost is NP hard for each stationary in separate and needs an effective algorithm to have an optimal average cost policy. Policy iteration and value iteration are the most widely used algorithms to compute an average cost optimal policy. Policy iteration arises from the unchain assumptions of Markov chain and works on a policy space and generate a sequence of improved policies whereas value iteration approximates the minimal average cost through a sequence value function. The relative values associated with a given policy R provide a tool for constructing a new policy $\overline{R}$ whose average cost is no more than that of the current policy R. In order to improve a given policy R whose average cost $g(R)$) and relative values $V_j(R), j \in S$ have been computed from the terms $g = g(R)$ and $V_j = V_j(R)$.

$$C_j\left(\overline{R}_j\right) - g(R) + \sum_{k \in S} p_{jk}\left(\overline{R}_j\right) V_k \leq V_j \tag{15}$$

By constructing a new policy $\overline{R}$ for each state $j \in S$, the cost based on the improved policy is as given by Eq. (14) for $g\left(\overline{R}\right) \leq g(R)$ and for $a_j = R_j, a_j \in A$, the three step in policy iteration algorithm is as follow:

- Step 0 (initialization)

$$R^1 = R_j = a_j = \begin{cases} 0\, if\ no\ repair\ is\ done\ 1 < j < N \\ 1\, if\ preventive\ action\ is\ done,\ 1 < j < N \\ 2\, enforced\ repair\ is\ done\ j = N \end{cases} \tag{16}$$

Where, $N \in S$

- Step 1 (value determination):

The current unique solution $g(R), V_j(R)$ for the given stationary point is computed for an arbitrary chosen state s with value of $v_s = 0$ and Eq. (17) used to determine the value V_j for $j = 1, 2, \ldots..N$ and for $j = N + 1$, $V_j = v_s$ and known as normalizing value.

$$V_j(a, R) = C_j(a = R) - g(R) + \sum_{k \in S} p_{jk} V_j(R) \tag{17}$$

- Step 2 (Policy improvement):

At this step the possible minimum action a_j is to be taken for a minimum and guaranteed using testing Eq. (18) and current policy Eq. (19) while Eq. (20) for selecting an action with minimum $T_j(a, R)$.

$$T_j(a_j, R) = C_j(a) - g(R) + \sum_{k \in S} p_{jk} V_j(R) \tag{18}$$

$$T_j(a_j, R) = V_j(R) \tag{19}$$

$$\min_{a \in A(j)} C_j(a) - g(R) + \sum_{k \in S} p_{jk}(a_j) V_k(R) \tag{20}$$

These all then give an optimal to Eq. (21) of the relative value for component i and the new policy $\overline{R}_j = a_j$ for $j = 1, 2, \ldots N$.

$$V_i^j = \min_a \{C_i^j(a) + \sum_k p_i^{jk}(a) V_i^k - g_i^* \tag{21}$$

- Step 3 (convergence test):

Now it is possible to compare the action for the optimum cost with old policy R and if $\overline{R} = R$, the algorithm is stopped otherwise go to step1 with new policy $\overline{R}$.

5 Experimentation and Verification

5.1 Input Data

Based on these arguments and the data synthesized, the following failure rate and maintenance rate both in months are: λ1 = 4, λ2 = 3, λ3 = 2 and μ1 = 0.133, μ2 = 0.1233, μ3 = 0.33. By using equation, 3.1 and the deterioration model based on equation could be as given in Table 1. The time table for the planned maintenance to the boiler is seated as per the time horizon to take either the preventive or the corrective action. However, the corrective action depends on the decision taken upon the inspection time, which is different for the different components. As it is mentioned earlier for the components, i = 1, 2, 3…n. Table 2 is then the time schedules for the preventive maintenance actions. Again, it is understood that the age also is the other factor for the deterioration should be taken in to account and each components will have different life span and deterioration rate then when we assume the preventive action at time t_i the age is hi.

Table 1. The deterioration of the component ($\boldsymbol{q_{ij}}$)

State (i)	Overheating	Rupturing	Leaking	Cracking
Overheating	0.6	0.4	0	0
Rupturing	0.01	0.69	0.3	0
Leaking	0.01	0	0.79	0.2
Cracking	0.03	0.08	0.1	0.79

Table 2. Control limits of the components for maintenance action (hi)

Components	Current age	Ti	t = 5, 6, 8		
Safety valve	3	6	−1	0	2
Heaters	5	8	−3	−1	0
Temperature sensors	2	5	0	−1	−3

5.2 Model Verification

Using Eq. (13)–(21) the possible action that would be taken is iterated to come with the average cost to have a stationary policy R. The states that the component will found are four N = 4 and the possible action would be taken in response of the component conditions are given in Eq. (8). In the stationary policy rule, $R_i = a_i$ while the following simultaneous equation demonstrates the first iteration of the solution.

- Step-1: value determination

1. Policy initiation: the first policy $R_i^{(1)}$ initiated as to have the policy of $R_i^{(1)} = (0, 0, 1, 2, 2)$.)
2. Relative value of the policy: Based on Eq. (17) the following linear equations give both the relative value and the average cost for the given policy.

$$V_1 = 0 - g + 0.6V_1 + 0.4V_2$$

$$V_2 = 0 - g + 0.01V_1 + 0.69V_2 + 0.3V_3$$

$$V_3 = 1500 - g + 0.01V_1 + 0.79V_3 + 0.2V_4$$

$$V_4 = 2500 - g + 0.03V_1 + 0.97V_4$$

$$V_5 = 0 - g + 1V_1$$

Using $V_j = V_5$ for $j = N + 1 = 5$ known as normalization equation with the value of $V_5 = 0$ and gives $\boldsymbol{V_1 = g}$.

Consequently,$V_1R_i^{(1)} = 2038.1, V_2R_i^{(1)} = 7,133.5, \quad V_3R_i^{(1)} = 14,0833, \quad V_4R_i^{(1)} = 17,446.3$ and hence, $gR_i^{(1)} = 2038$.

- Step 2: Policy improvement

Using Eq. (18) and Eq. (19) respectively for testing and current policy i.e.: $T_j(a, R) = C_j(a) - g(R) + \sum_{k \in s} p_{jk} V_j(R)$ And $T_j(a, R) = V_j(R)$ for $a = R$ and take $j = 2$ for instance,

$$T_2(a_i, R) = C_2(a = 0) - g\left(R^{(1)}\right) + p_{21}v_1 + p_{22}v_2 + p_{23}v_3 + p_{24}v_4$$

$T_2(a_i, R) = 600 - 2038.1 + 0.01 * 2038.1 + 0.69 * 7,133.5 + 0.3 * 14,083.3 =$ **7,729.4** and continuing as if, consequently gives:

$$T_2(a_i, R) = C_2(a = 0) = 7,729.4, T_2(a_i, R) = C_2(a = 1) = 600$$

$$T_3(a_i, R) = C_2(a = 0) = 12.587.3, T_3(a_i, R) = C_2(a = 1) = 1,500$$

$$T_4(a_i, R) = C_2(a = 0) = 14,946, T_4(a_i, R) = C_2(a = 1) = 2500$$

Then, new improve solution based on Eq. (20) i.e. $\min(T_2(a = 0, a = 1), R_i^{(1)}))$ gives $R_i^{(2)} = (0, 1, 1, 1, 2)$. Using $R_i^{(2)}$ and continuing iteration II in similar fashion and the result gives then the policy of R(^{4}) = (1, 1, 1, 2) Which is equivalent to the R (^{3r}) = (1, 1, 1, 2) But the average cost g^*(R) for R(^{3}) is 2085 whereas for R^4 g^*(R) = 2053 and different relative values. The reason and the best policy will be discussed in the result and discussions part.

6 Results and Discussion

The calculation result obtained in the determination of the long run average cost of the maintenance action indicates that the cost is a function of different circumstance.

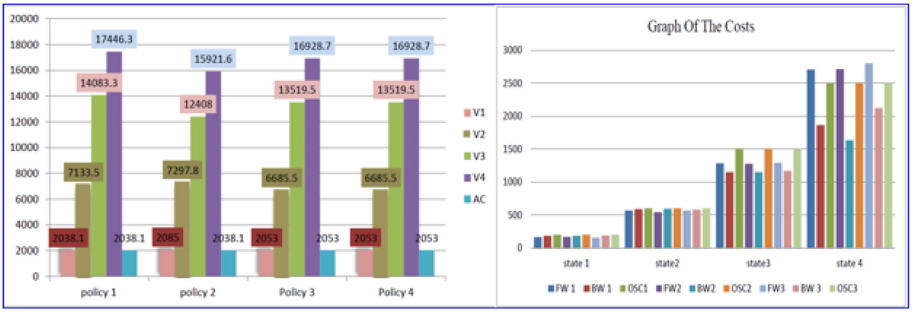

Fig. 4. Graphs of the relative values (left) and average costs for the different state of components (right)

Especially it is dependent on the period at which the age of the component taken, as the point of the action and it is the type of the response for the system or system's components current condition. Based on the formulated model for the long-run average cost of the maintenance action the discrete Markov chin results enables as to take trade-off about taking of the various maintenance action response towards the inspected and observed condition of the system or the component. As it is clearly, shown different relative values and an average cost could be obtaining through the iterative application of the policy iterations. Graphs in Fig. 4 depict the relative values of the maintenance action at the different state of the system. From the graph one an inferred that by applying the various maintenance policies which are relatively sufficient enough, it is possible to increase the relative values that could be obtained in return of performing of such policy. The relative value designated by the v1 is the immediate value that is obtained with respect to the current condition of the components. On the other hand, the other relative values (v2, v3 v4) are not the immediate values rather they are the guaranteed values resulted from the accomplishment of the maintenance action specified on the maintenance policy. Table 3 shows that penalty vs. scheduled cost at different state of the components. It is not the immediate result that to be taken for the maintenance action rather the long run values to be considered. Policy 1 $R_i^{(1)}$ = (0, 0, 1, 2, 2) state the application of no action at the first and second state of the components **(a = 0)** and preventive action when the system is found at state three **(a = 1)** and a corrective action at stat four and five if it will exist still. Whereas, with regard to the policy 2 of the model R^2 = (0, 1, 1, 1, 2) the preventive action is expected at the second, third and fourth state and the system is to be kept as it's in its first state whatever its condition is and a corrective action is at the fifth state. According to the policy improvement iterations and the test results the stationary policy R^2 = (0, 1, 1, 1, 2) and R^3 = (0, 1, 1, 1, 2) are the same. However, if policy R^3 is improved to $R^{3'}$ = (1, 1, 1, 1, 2) which is equivalent to R^4 and the average cost will become 2,053.

Table 3. Penalty vs scheduled costs at different state of the components

	State 1				State 2				State 3				State 4			
	RV	FW	BW	OSC	RV	FW	BW	OSC	RV	FW	BW	OSC	RV	FW	BW	OSC
1	2053	1642	1843	200	66855	5683	5883	600	13519.5	12844	11492	1500	16928.7	2708	1862.2	2500
2	2053	1683	1852	200	66855	5405	59952	600	13519	12756	11511	1500	16928.7	2711	1634.7	2500
3	2053	1548	1861	200	66855	5702	5802	600	13519	12902	1170.7	1500	16928.7	2800	21213	2500
Total	6159	4873	5561	600	200565	1679	71637	1800	40557.5	38502	3471	4500	50786.1	8219	5618.2	7500

Note: RV = Relative value, FW = Forward shifting, BW = Backward shifting, OSC = on schedule

One may get confusion about the conclusion of giving that the optimum policy is policy relay on either policy three or four due to the small value of the average cost (2038) that would incur or the long run relative value of this policy (7,133.5, 14,083.3 and 1746.3) would be secured at each state. It is this mysterious which makes the long run average cost criterion policy is the best for the infinite time horizon of the maintenance action. What to mean here is that it is not to men that this relatives values are simply to be obtained they also are costs unless the trade-off is made. This is what the average cost-policy compromise and the policy iteration assures. The lose that would resulted in failing apply policy1 is then higher than to the other polices (immediate value = 0, and a maximum of the long run lose is 15407). Then policy 1 is not a feasible solution and as the average cost of operation is relatively higher for policy 2 than of policy 3 and the long run values that would be fail-safe is comparatively less and is not viable so. The last and not the least consideration that could be made, if needed, is to see the difference between policy 3 and policy 4. These policies only has a difference of whether to take the preventive action at the first inspection result of the components and this depends on the components exposure and the maintenance cost and effect on the production. However, as much as average cost is concerned policy 3 is an optimum maintenance policy. As it is depicted in the graph, observed from Table 3, both the forward and backward shifting penalty costs are smaller than the scheduled cost especially in state one, and state two of the component conditions. However, as the time gets advance the deterioration rate of the components expected to increase, shifting forward will bear an extra cost, and this is shown clearly on the graph above. For instance the penalty cost due to forward shifting for components 1, 2, 3, respectively in state 1 is 164.2, 168.3 and 154.8 whereas the backward shifting penalty costs are 184.8, 185.2 and 186.1. These figures convinced us about the importance of the forward shifting up to the extent of the component's condition, which is assumed to be not severs. However, it is the backward shifting that is to be used for the components found at the condition of beyond the moderately sever. Based on our model the state of conditions of the components are taken four on which the first state is assumed to have less effects and possibly to or not to take any maintenance response equipment will let to continue even with the knowledge of some sort of failure symptoms. The second and the third state conditions are the most and the valuable states depending to the response for them. It is here at which the critical decision plays an important role, it is the knowledge, and understanding as well as the experience of the maintenance manager and the crews under it to imitate and interpret the conditions occurred. Nevertheless, as the graph depicts it is recommended to take the backward shifting at such state of components. Hence the on-scheduled response action is then preferable than that of the forward shifting it is not only it is cost advantage but also to prevent the components from further deterioration and stoppage of the production for long time. With regard to the domain dimension of the system especially for machines having different components with different design function and exposure condition, the alignment, the preventive actions, the operative performance speed, and the loading factors have an essential influence. These and other circumstance then makes the planned maintenance program a decision under uncertainty and regret will develop in each of the programs schedule.

7 Conclusion

This paper aims to articulates the basics of the maintenance engineering and rooted from the basics and philosophy of the maintenance action as well as tried to explore the evolution of them maintenance as an introduction. It is the long-run average cost criterion, which is used in this paper as the criterion to develop the decision model about the actions. On the other hand, the decision about the response action for the various equipment condition needs other rules. Stationary policy is then the other considered idea in this paper to set the actions for the different state of the equipment or the systems at various conditions. Therefore, the decision model developed based on the long run average cost criterion and guided by the stationary policy. In the model for the average cost determination, the policy iteration algorithm is used to improve the policies to get an optimum policy that could be used as a maintenance program under the consideration of both the aging and deterioration factors. In this paper, not only developing of model for the maintenance considered, but also the paper tried to compromise the inefficiency of planned maintenance by the strategy of the application of the shifting of the maintenance schedule. Both the Discrete Markov Chain model for the long run average cost and the penalty cost model for the shifting strategy are test with a numerical case study for the textile boiler maintenance program by considering three components of the boiler with their current age and the schedule maintenance time for the planned maintenance program. Based on the analysis made and the result discussed the maintenance long run average cost will be attained if the stationary policy is based on the decision policy of R (i) = (0, 1, 1, 1, 2) with the long run average cost of 2053birr.

In general, in this paper the application of the Discrete Markov Chain is used to develop a maintenance model, which gives a long run average cost of maintenance cost under the consideration of the infinite time horizon of the maintenance action. Again, the paper introduces and proofs the importance of the shifting strategy of the maintenance action scheduled in the planned maintenance program.

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