Placement Optimization for UAV-Enabled Wireless Power Transfer System

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Abstract. This paper considers an unmanned aerial vehicle (UAV)-enabled wireless power transfer (WPT) system, in which a UAV hovers in a given flying altitude to transfer energy to more than two energy receivers (ERs) on the ground. We consider to maximize the sum energy and weighted sum energy of ERs by UAV placement optimization. As the sum energy and weighted sum energy maximization problem are sum of ratio problems, which are generally NP-hard. It is difficult to give the optimal location of the UAV for those two problems. To tackle those problems, we adopt a novel quadratic transform technique to transfer to an equivalent problem. Based on the equivalent problem, we propose an iterative coordination update algorithm in a closed-form expression, which can converge to the stationary point of the sum energy maximization problem or even the global optimal solution under a sufficient condition of the flight altitude. Simulation results show that the proposed algorithm can achieve nearly the same weighted sum energy for ERs and reduces more than 90% complexity compared to the two-dimensional (2D) exhaustive search method.

Keywords: Unmanned aerial vehicle (UAV) · Wireless power transfer · Placement optimization

1 Introduction

In recent years, with the continuous development of wireless communication, some portable electrical appliances such as laptop computers, mobile devices and music players needing battery charging have had higher and higher constraints on energy consumption. However, the battery life is limited, usually only 3–5 years, which can not meet the needs of people for communication equipment [1]. In addition, in special occasions, such as mining and oil mining, the traditional power transmission has hidden dangers in terms of safety.

This work was partially supported by the National Natural Science Foundation of P.R. China(No.61701064), Basic Research and Frontier Exploration Project of Chongqing(cstc2019jcyj-msxmX0264).

H. Gao et al. (Eds.): ChinaCom 2020, LNICST 352, pp. 64–77, 2021.
https://doi.org/10.1007/978-3-030-67720-6_5

Under the above circumstances, wireless power transmission becomes more and more important and imperative. In the traditional WPT system, energy is usually sent to the energy receiver through a fixed energy beacon (PB) [2]. However, in some practical situations, such energy transfer efficiency is not high. Because the transmission process will be blocked by some obstacles, such as tall buildings and mountains. But against this disadvantage, drones can be effectively circumvented. The high mobility and flexible deployment of UAV can effectively guarantee the line-of-sight link between UAV and ground users, thus achieving reliable and energy-saving energy transmission [3]. Therefore, UAV as a radio frequency energy signal transmitter will be a research hotspot in the future.

We can find that there are many applications of UAV in the WPT system in previous studies. For example, in [4], the authors considered the UAV as an energy source send energy to the ground user, and the ground users exploit the energy received to send information to the UAV. The purpose is to maximize the user's information sum-rate by jointly optimizing the time allocation and position optimization of the UAV. In [5], a parallel wireless information and Power Transmission (SWIPT) technology was proposed to jointly optimize the power allocation and trajectory design of UAV to maximize the minimum energy collected by ground dispersed Internet of Things (loT) devices. However, it is difficult to solve this problem directly, so they focus on finding suboptimal solutions for solving this problem. In addition, WPT was also used in the recently popular UAV mobile edge computing system. [6] considered a kind of edge system, a TDMA working model was proposed, which allows UAV-assisted systems to carry out parallel transmission, downlink energy transfer to charge loT devices, and uplink information transfer. In [7], the authors studied a UAV-supported WPT system, one of which serves two ERs on the ground. The Pareto boundary of the reachable energy region of the two ERs is determined by optimizing the flight path of the UAV under the constraint of maximum velocity. [8] expanded [7] and considered the system model of more than two ERs, where the purpose is to maximize the sum or minimum accepted energy of all ERs by controlling UAV trajectory. Two drones served two ground users was further considered in [9], aim to maximize the uplink shared (minimum) throughput of the two users within a limited UAV mission cycle by jointly optimizing the running trajectories of the two drones and wireless resource allocation. In [10], through the joint optimization of UAV trajectory and wireless resource allocation, the common throughput maximization of wireless powered communication networks (WPCN) supported by UAV was studied.

A sub-problem to be solved for the considered system mode in [8,10] is to find the optimal placement of UAV in a given time so as to maximize the total energy of ground users. For two users case, the authors have given the close-form solution of the location for UAV. They have shown that only one location or two locations are optimal, which depends on the height of the UAV and the distance between the two ground users. For more than two users' case, because the objective function is a non-convex problem, it is difficult to look for the optimal location of the UAV. A two-dimensional exhaustive search was conducted for the user area on the ground. In the case of more than two users, using low complexity algorithm to find the optimal or near optimal UAV position is still a problem to be solved.

In this paper, we tend to maximize the sum energy and weighted sum energy of the ERs by optimizing the placement of UAV in more than two user's case. In general, the problem is NP-hard, we first convert it to an equivalent formulation, which is more easier to handle.On this basis, an iterative coordinate updating algorithm is proposed. The simulation results show that the algorithm has lower complexity and almost the same energy compared with the two-dimensional exhaustive search algorithm.

The rest of this article is organized as follows. Section 2 introduces the WPT system model based on UAV. Section 3, the iterative coordinate updating algorithm for solving sum-energy maximization problems is presented. In section 4, numerical results are given to verify the effectiveness of the proposed algorithm. Finally, the fifth part draws the conclusion.

2 System Model and Problem Formulation

We consider a UAV-enabled WPT system with K users, where a UAV is scheduled to charge K ERs on the ground by transferring wireless energy. We set the power transfer time of the UAV as T. The location of ER $k = 1, \cdots, K$ is fixed on the ground. We assume that UAV flies at a fixed altitude H. And under the condition of transmit power P, we need to find an optimal hovering position to maximize the sum of the energy received by all ERs. We consider the channel model of free-space path loss between the UAV and each ER is same as [8,10]. The channel power gain from the UAV to ER $k \in \{1, \cdots, K\}$ is modeled as $h_k = \beta \frac{1}{d_k^2}$, where $d_k = \sqrt{(x - x_i)^2 + (y - y_i)^2 + H^2}$ is their distance and β denotes the channel power gain at a reference distance of $d = 1$ m.

Firstly, we want to find the best placement of the UAV to maximize the sum energy to the K user for the given time T as follows. Assume the UAVs location is (x, y, H), the k-th ground user's location is $(x_k, y_k), k = 1, \cdots, K$, we want to find the optimal (x, y) to the following problem.

$$\underset{x,y}{\text{maximize}} \quad \sum_{i=1}^{K} \frac{PT\beta}{(x - x_i)^2 + (y - y_i)^2 + H^2} \tag{1}$$

(1) is a non-convex optimization problem. Moreover, (1) can be regarded as a multiple-ratio fractional programming which is generally NP-hard problem [11]. Therefore, global optimization methods such as branch-and-bound or 2D exhaustive search method need to be exploited to find the optimal solution [12,13]. However, the complexity of those methods is too much high. In next section, we will give a simple iteration algorithm based on the equivalent problem of (1) to maximize the sum energy of the senors.

3 Iterative Coordinate Update Algorithm

Because the numerators and denominators of object function are constant and convex function respectively, the constraint set is nonempty convex set. (1) is multiple-ratio concave-convex fractional programming. Since the denominator of object function $PT\beta$ is constant for our optimization problem. Thus, we only need to optimize the following problem.

$$\underset{x,y}{\text{maximize}} \quad \sum_{i=1}^{K} \frac{1}{(x-x_i)^2 + (y-y_i)^2 + H^2} \tag{2}$$

As problem (2) can be viewed as sum-of-ratios problem, we can use a novel quadratic transform technique to rewritten it as equivalent optimization problem by the lemma in[14]. So, the equivalent problem is as follow.

$$\begin{aligned} &\underset{x,y,\mathbf{z}}{\text{maximize}} \quad \sum_{i=1}^{K} \left(2z_i - z_i^2 \left((x-x_i)^2 + (y-y_i)^2 + H^2\right)\right) \\ &\text{subject to} \quad x, y, z_1, \cdots, z_K \in R, \end{aligned} \tag{3}$$

where $\mathbf{z}$ is the container for the variables $z_1, \cdots, z_K$. For (3), we can optimize the primal variable x, y and the auxiliary variable $z_1, \cdots, z_K$ iteratively by block coordinate ascent algorithm. For a given $\mathbf{z} = (z_1, \cdots, z_K)$, the object function of (3) is concave function with respective to x and y. Let $w_i = z_i^2 \geq 0 (i = 1, \cdots, K)$, then, (3) is equivalent to the following convex optimization problem for a fixed $(w_1, \cdots, w_K)$.

$$\begin{aligned} &\underset{x,y}{\text{minimize}} \quad \sum_{i=1}^{K} w_i \left((x-x_i)^2 + (y-y_i)^2\right) \\ &\text{subject to} \quad x, y \in R, \end{aligned} \tag{4}$$

Next, using the first order optimality condition for problem (4), we given the closed-form solution to (4) as follows.

Lemma 1. *For a given $w_i \geq 0, i \in \{1, \ldots, K\}$, the optimal solution to (4) is given by*

$$x = \frac{\sum_{i=1}^{K} x_i w_i}{\sum_{i=1}^{K} w_i}, y = \frac{\sum_{i=1}^{K} y_i w_i}{\sum_{i=1}^{K} w_i}. \tag{5}$$

Proof. Let $f(x,y) = \sum_{i=1}^{K} w_i \left((x-x_i)^2 + (y-y_i)^2\right)$, we have $\frac{\partial^2 f}{\partial x^2} = 2w_i \geq 0$, $\frac{\partial^2 f}{\partial y^2} = 2w_i \geq 0$, $\frac{\partial^2 f}{\partial x \partial y} = 0$. Therefore, $f(x,y)$ is convex function. Moreover, (4) is a convex optimization problem. The optimal solution $x = \frac{\sum_{i=1}^{K} x_i w_i}{\sum_{i=1}^{K} w_i}, y = \frac{\sum_{i=1}^{K} y_i w_i}{\sum_{i=1}^{K} w_i}$ to (4) can be obtained by the first order condition $\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0$. Thus, lemma 1 is held.

For a fixed x, y, because (3) is concave function with respective to $z_1, \cdots, z_n$. Therefore, the optimal solution $z_1, \cdots, z_n$ to (3) is given by

$$z_i = \frac{1}{\left(x - x_i\right)^2 + \left(y - y_i\right)^2 + H^2}, i = 1, \cdots, K. \tag{6}$$

Using Lemma 1 and (6), we give an iterative coordinate update algorithm as follows.

Because the algorithm proposed in Algorithm 1 can be viewed as the block coordinate descent (BCD) method mentioned in [15] to solve problem (3). Therefore, Algorithm 1 will be convergent due to the object function of (3) obtained by $(x^t, y^t, \mathbf{z}^t)$ is bounded monotonic sequence. Let (x^*, y^*, z^*) be the limit point of the sequence $(x^t, y^t, \mathbf{z}^t), t = 0, 1, \cdots,$. Then, the point (x^*, y^*) will be stationary point of problem (1) by the following theorem.

Algorithm 1. Iterative Coordinate Update Algorithm (ICUA)

Initialization: Let $x^0 = x_{ini}, y^0 = y_{ini}$ such that (x_{ini}, y_{ini}) is located in the convex hull of the region of $(x_i, y_i), i = 1 \cdots, K$;
compute $z_i^0 = \frac{1}{\left(x^0 - x_i\right)^2 + \left(y^0 - y_i\right)^2 + H^2}, i = 1, \cdots, K$;
compute $w_i^0 = \left(z_i^0\right)^2, i = 1, \cdots, K$;
Set $t = 0$;
repeat
t=t+1;
Update $\left(x^t, y^t\right)$:
$x^t = \frac{\sum_{i=1}^{K} x_i w_i^{t-1}}{\sum_{i=1}^{K} w_i^{t-1}}, y^t = \frac{\sum_{i=1}^{K} y_i w_i^{t-1}}{\sum_{i=1}^{K} w_i^{t-1}}$ where $w_i^{t-1} = (z_i^{t-1})^2, i = 1, \cdots, K$,
Update $\mathbf{z^t} = \left(z_1^t, \cdots, z_K^t\right)$:
$z_i^t = \frac{1}{(x^t - x_i)^2 + (y^t - y_i)^2 + H^2}, i = 1, \cdots, K$;
until Some stopping criteria is met
output The coordinate (x, y) of UAV is given by $x = x^t$ and $y = y^t$.

Theorem 1 *Let (x^*, y^*, z^*) be the limit point of the sequence $(x^t, y^t, \mathbf{z}^t), t = 0, 1, \cdots,$ then (x^*, y^*) is the stationary point of problem (1).*

Proof. Let $F(x,y) = \sum_{i=1}^{K} \frac{PT\beta}{(x-x_i)^2+(y-y_i)^2+H^2}$ be the object function of problem (1). The stationary point of problem (1) must satisfied $\frac{\partial F(x,y)}{\partial x} = 0, \frac{\partial F(x,y)}{\partial y} = 0$. Therefore,the stationary point is expressed as follow.

$$\begin{aligned} x &= \frac{\sum_{i=1}^{K} x_i \left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2} \right)^2}{\sum_{i=1}^{K} \left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2} \right)^2}, \\ y &= \frac{\sum_{i=1}^{K} y_i \left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2} \right)^2}{\sum_{i=1}^{K} \left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2} \right)^2}. \end{aligned} \tag{7}$$

From algorithm 1, we have

$$x^{t+1} = \frac{\sum_{i=1}^{n} x_i w_i^t}{\sum_{i=1}^{K} w_i^t}, y^{t+1} = \frac{\sum_{i=1}^{n} y_i w_i^t}{\sum_{i=1}^{K} w_i^t} \tag{8}$$

where $w_i^t = (z_i^t)^2, i = 1, \cdots, K$, and $z_i^t = \frac{1}{(x^t-x_i)^2+(y^t-y_i)^2+H^2}, i = 1, \cdots, K$. Substitute $w_i^t = \left(\frac{1}{(x^t-x_i)^2+(y^t-y_i)^2+H^2} \right)^2$ into (8), we have

$$\begin{aligned} x^{t+1} &= \frac{\sum_{i=1}^{K} x_i \left(\frac{1}{(x^t-x_i)^2+(y^t-y_i)^2+H^2} \right)^2}{\sum_{i=1}^{K} \left(\frac{1}{(x^t-x_i)^2+(y^t-y_i)^2+H^2} \right)^2}, \\ y^{t+1} &= \frac{\sum_{i=1}^{K} y_i \left(\frac{1}{(x^t-x_i)^2+(y^t-y_i)^2+H^2} \right)^2}{\sum_{i=1}^{K} \left(\frac{1}{(x^t-x_i)^2+(y^t-y_i)^2+H^2} \right)^2}. \end{aligned} \tag{9}$$

From (9), we have

$$x^* = \lim_{t \to \infty} x^{t+1} = \frac{\sum_{i=1}^{K} x_i \left(\frac{1}{(x^*-x_i)^2+(y^*-y_i)^2+H^2} \right)^2}{\sum_{i=1}^{K} \left(\frac{1}{(x^*-x_i)^2+(y^*-y_i)^2+H^2} \right)^2}, \tag{10}$$

the last two equalities follow from the continuity of $f_1(x,y) = \frac{\sum_{i=1}^{K} x_i \left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2}\right)^2}{\sum_{i=1}^{K}\left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2}\right)^2}$. Using the same argument, we have

$$y^* = \frac{\sum_{i=1}^{K} y_i \left(\frac{1}{(x^*-x_i)^2+(y^*-y_i)^2+H^2}\right)^2}{\sum_{i=1}^{K}\left(\frac{1}{(x^*-x_i)^2+(y^*-y_i)^2+H^2}\right)^2}, \tag{11}$$

From (10) and (11), we can see that the limit point (x^*, y^*) of the sequence (x^t, y^t) must satisfy equation (7). Therefore, Theorem 1 is held.

In proposed algorithm, we have chosen the initial point in the convex hull of the region of (x_i, y_i), where we can choose randomly from (x_i, y_i) for some $i \in 1, \cdots, K$. Indeed, the initial point can even be choosen outside of the the convex hull of the region of (x_i, y_i). After the algorithm runs only one time, the new location of the UAV will be inside the the convex hull of the region of (x_i, y_i). The initial point can be chosen by $x_{ini} = \frac{\sum_{i=1}^{n} x_i}{n}, y_{ini} = \frac{\sum_{i=1}^{n} y_i}{n}$. However, even for two users' case, the problem has two optimal locations when the hight of the UAV is much less than the distance of the two users. Next, we give a sufficient condition that the problem has a unique optimal solution as follows.

For two ERs case, [8] has proven that there is a unique placement of UAV when the hight of the UAV is larger than a threshold. Next, we give a similar result for more than two users case.

In order to obtain the most essential understanding of the algorithm, we give a sufficient condition among the H and the square of the region of ERs to show that it can converge to a global optimization as follows.

Theorem 2. *The proposed algorithm will be convergent to the global optimal solution when H satisfied the following condition:*

$$H > 2\sqrt{2\max\{x_{\max}, y_{\max}\}(x_{\max} + y_{\max})} \tag{12}$$

Proof. The main idea is to use the Banach fixed-point theorem [16]. We only need to prove the jacobian matrix of the proposed iteration is less than 1 under some matrix norm or spectral radius of the jacobian matrix is less than 1. From the proposed algorithm and (7), we can see that the iterative algorithm is fixed point of the following function. Consider a function $\mathbf{v} = \mathbf{f}(x, y)$, such that

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \end{bmatrix} \tag{13}$$

where $f_1(x,y)$ and $f_2(x,y)$ is given by

$$f_1(x,y) = \frac{\sum_{i=1}^{n} x_i \left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2}\right)^2}{\sum_{i=1}^{n}\left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2}\right)^2}, \tag{14}$$

$$f_2(x,y) = \frac{\sum_{i=1}^{n} y_i \left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2}\right)^2}{\sum_{i=1}^{n} \left(\frac{1}{(x-x_i)^2+(y-y_i)^2+H^2}\right)^2}. \tag{15}$$

Then, from (9), the proposed algorithm can be rewritten as the following iteration: Pick an initial point $(x^0, y^0) \in D$, where $D = [x_{min}, x_{max}] \times [y_{min}, y_{max}]$ and defined for $k = 0, 1, 2, \cdots$,

$$\begin{bmatrix} x^{(k+1)} \\ y^{(k+1)} \end{bmatrix} = \begin{bmatrix} f_1\left(x^{(k)}, y^{(k)}\right) \\ f_2\left(x^{(k)}, y^{(k)}\right) \end{bmatrix} \tag{16}$$

Then, the Jacobian matrix of $\mathbf{f}$ is given by

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \tag{17}$$

where

$$\frac{\partial f_1(x,y)}{\partial x} = \frac{-4\sum_{i=1}^{K} x_i(x-x_i)z_i^3(x,y)\sum_{i=1}^{n} z_i^2(x,y) + 4\sum_{i=1}^{n} x_i z_i^2(x,y)\sum_{i=1}^{K} z_i^3(x,y)(x-x_i)}{\left(\sum_{i=1}^{K} z_i^2(x,y)\right)^2}$$

$$\frac{\partial f_1(x,y)}{\partial y} = \frac{-4\sum_{i=1}^{K} x_i(y-y_i)z_i^3(x,y)\sum_{i=1}^{n} z_i^2(x,y) + 4\sum_{i=1}^{n} x_i z_i^2(x,y)\sum_{i=1}^{K} z_i^3(x,y)(y-y_i)}{\left(\sum_{i=1}^{K} z_i^2(x,y)\right)^2}$$

$$\frac{\partial f_2(x,y)}{\partial x} = \frac{-4\sum_{i=1}^{K} y_i(x-x_i)z_i^3(x,y)\sum_{i=1}^{n} z_i^2(x,y) + 4\sum_{i=1}^{n} y_i z_i^2(x,y)\sum_{i=1}^{K} z_i^3(x,y)(x-x_i)}{\left(\sum_{i=1}^{K} z_i^2(x,y)\right)^2}$$

$$\frac{\partial f_2(x,y)}{\partial y} = \frac{-4\sum_{i=1}^{K} y_i(y-y_i)z_i^3(x,y)\sum_{i=1}^{n} z_i^2(x,y) + 4\sum_{i=1}^{n} y_i z_i^2(x,y)\sum_{i=1}^{K} z_i^3(x,y)(y-y_i)}{\left(\sum_{i=1}^{K} z_i^2(x,y)\right)^2}$$

$z_i(x,y) = \frac{1}{(x-x_i)^2+(y-y_i)^2+H^2}$.

We have $z_i(x,y) \in [z_{min}, z_{max}]$ when $(x,y) \in D$, where $z_{min} = \frac{1}{(x_{\max}-x_{\min})^2+(y_{\max}-y_{\min})^2+H^2}$, $z_{max} = \frac{1}{H^2}$. Then, the upper bound of $\left|\frac{\partial f_1(x,y)}{\partial x}\right|$ for $(x,y) \in D$ is given by

$$\begin{aligned} \left|\frac{\partial f_1(x,y)}{\partial x}\right| &\le \frac{4\left|\sum_{i=1}^{K} x_i(x-x_i)z_i^3(x,y)\sum_{i=1}^{n} z_i^2(x,y)\right| + 4\left|\sum_{i=1}^{n} x_i z_i^2(x,y)\sum_{i=1}^{K} z_i^3(x,y)(x-x_i)\right|}{\left(\sum_{i=1}^{K} z_i^2(x,y)\right)^2} \\ &\le \frac{4x_{\max}^2\left|\sum_{i=1}^{K} {z_i}^3(x,y)\sum_{i=1}^{n} {z_i}^2(x,y)\right| + 4x_{\max}^2\left|\sum_{i=1}^{n} {z_i}^2(x,y)\sum_{i=1}^{K} {z_i}^3(x,y)\right|}{\left(\sum_{i=1}^{K} {z_i}^2(x,y)\right)^2} \\ &= \frac{8x_{\max}^2\left|\sum_{i=1}^{K} {z_i}^3(x,y)\sum_{i=1}^{n} {z_i}^2(x,y)\right|}{\left(\sum_{i=1}^{K} {z_i}^2(x,y)\right)^2} \le 8x_{\max}^2 z_{\max} = \frac{8x_{\max}^2}{H^2} \end{aligned} \tag{18}$$

Using the same argument, we have

$$\left|\frac{\partial f_1(x,y)}{\partial y}\right| \le \frac{8x_{\max}y_{\max}}{H^2} \tag{19}$$

$$\left|\frac{\partial f_2(x,y)}{\partial x}\right| \le \frac{8x_{\max}y_{\max}}{H^2} \tag{20}$$

$$\left|\frac{\partial f_2(x,y)}{\partial y}\right| \le \frac{8y_{\max}^2}{H^2} \tag{21}$$

Using (18)-(21), we can find an upper bound for $||J||_\infty$.

$$\begin{aligned} \|J\|_\infty &= \max\left(\left|\frac{\partial f_1}{\partial x}\right| + \left|\frac{\partial f_1}{\partial y}\right|, \left|\frac{\partial f_2}{\partial x}\right| + \left|\frac{\partial f_2}{\partial y}\right|\right) \\ &\le \max\left(\frac{8x_{\max}^2}{H^2} + \frac{8x_{\max}y_{\max}}{H^2}, \frac{8x_{\max}y_{\max}}{H^2} + \frac{8y_{\max}^2}{H^2}\right) \\ &\le \frac{8\max\{x_{\max}, y_{\max}\}(x_{\max}+y_{\max})}{H^2} \end{aligned} \tag{22}$$

Then, we have $\|J\|_\infty < 1$ because (12) is held. Therefore, f is a contraction map in D. Therefore, the proposed algorithm will converge to the unique fixed point, which is unique stationary point of the optimization problem (2). As the stationary point is unique, this stationary point is the global optimal for (2) because the minimum value point of (2) can't be inside the region D. This is can be see from that the infimum of (2) is zero as x and y approaches infinity. Thus, Theorem 2 is held.

In the proof of Theorem 2, we have assumed all the coordinate of ERs is located in the first quadrant which means that $x_i \ge 0$ and $y_i \ge 0$ is held. Without loss of generality, this condition can be assumed to be hold because we can always change the coordinate of each ERs by coordinate transformation to satisfy it. Moreover, the sufficient condition given by (12) for H is general not tight. We may choose different matrix norm to find some other lower bound for H.

Next, we extend the proposed algorithm to the weight sum energy maximization problem. The weigh sum energy maximization problem is given as follows.

$$\max_{x,y} \quad \sum_{i=1}^{K} \frac{a_i PT\beta}{(x-x_i)^2 + (y-y_i)^2 + H^2} \tag{23}$$

where $a_i > 0$ is weight factor for user i to emphasizes the different priority for harvesting energy at sensor node i. In real communication system, the weight factor for user i can be choose to balance the harvesting energy between the SNs. For example, for the sensors who has less remain energy, they can choose the higher weight. A simpler weight for each user can be proportion to the percentage of energy it has consumed compare with its battery storage capacity.

Because maximizing the sum of the weighted energies is also a sum-of-ratios problem. We can use the same argument as the sum energy maximization problem to design the iterative coordinate update algorithm. Use Lemma 1 and omit the constant value $PT\beta$ in the numerator in problem (23), then the weight sum energy maximization problem is equivalent to the following problem.

$$\begin{aligned} \underset{x,y,\mathbf{z}}{\text{maximize}} \quad & \sum_{i=1}^{K} \left(2z_i\sqrt{a_i} - z_i^2 \left((x-x_i)^2 + (y-y_i)^2 + H^2 \right) \right) \\ \text{subject to} \quad & x, y, z_1, \cdots, z_K \in R, \end{aligned} \tag{24}$$

where $\mathbf{z}$ refers to a collection of variables $z_1, \cdots, z_K$. Using the same method as the sum energy maximization problem, we can extend the ICUA to handle weight sum energy maximization problem by algorithm 2 as follows.

Algorithm 2. ICUA for weight sum energy maximization

Initialization: Let $x^0 = x_{ini}, y^0 = y_{ini}$ such that (x_{ini}, y_{ini}) is located in the convex hull of the region of $(x_i, y_i), i = 1 \cdots, K$;
compute $z_i^0 = \frac{\sqrt{a_i}}{(x^0-x_i)^2+(y^0-y_i)^2+H^2}, i = 1, \cdots, K$;
compute $w_i^0 = \left(z_i^0\right)^2, i = 1, \cdots, K$;
Set $t = 0$;
repeat
t=t+1;
Update $\left(x^t, y^t\right)$:
$x^t = \frac{\sum_{i=1}^{K} x_i w_i^{t-1}}{\sum_{i=1}^{K} w_i^{t-1}}, y^t = \frac{\sum_{i=1}^{K} y_i w_i^{t-1}}{\sum_{i=1}^{K} w_i^{t-1}}$ where $w_i^{t-1} = (z_i^{t-1})^2, i = 1, \cdots, K$,
Update $\mathbf{z^t} = \left(z_1^t, \cdots, z_K^t\right)$:
$z_i^t = \frac{\sqrt{a_i}}{(x^t-x_i)^2+(y^t-y_i)^2+H^2}, i = 1, \cdots, K$;
until Some stopping criteria is met
output The coordinate (x, y) of UAV is given by $x = x^t$ and $y = y^t$.

4 Simulation Results

This section presents the simulation results of the proposed algorithm and compares it with the two heuristic algorithm. The main idea of the first heuristic algorithm 2D grid search method is to find an optimal UAV position by searching for M^2 points in an $M \times M$ grid [8,10]. The second heuristic algorithm is searching with four directions under a given step size from the same initial point with proposed algorithm $x_{ini} = \frac{\sum_{i=1}^{n} x_i}{n}, y_{ini} = \frac{\sum_{i=1}^{n} y_i}{n}$ until it reaches the maximum number of iterations. For the sake of brevity, we name the first and second heuristic algorithm as heuristic I and heuristic II.

4.1 Simulation Results Versus the Number of Users

The number of SNs is set from 4 to 60. The weight factor for users is randomly generated from $[0, 1]$. β is set to be 10^{-2}, the height of the UAV H ,the transmission power of UAV P and energy transfer time T are respectively 5 m,40 dBm and 1000 s.The convergence threshold of the proposed algorithm and heuristic II algorithm is set to be 10^{-1}.The random distribution region of the sensor is 30×30 m^2.The simulation results are obtained randomly according to the sensor position. The heuristic I search method have divided the region of the SNs into 30×30 grid-points to find the best location for UAV. The step size for heuristic II is 0.5 m. The maximum iterations for proposed algorithm and heuristic II algorithm is set to be 900, which is the same with number of the searching point as the heuristic I algorithm.

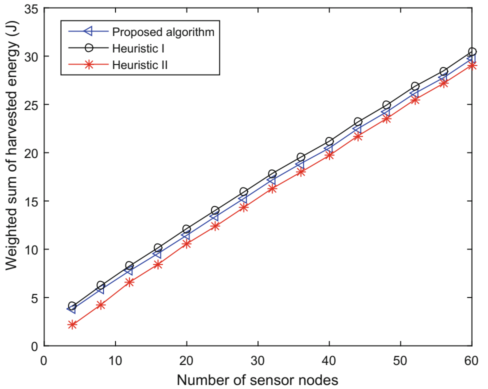

Fig. 1. Weighted sum of energy versus number of SNs

As can be seen from Fig. 1, with the increase in the number of SNs, the energy weighted sum of the three algorithms is increasing. The proposed algorithm has little performance loss compare with the Heuristic I algorithm. In addition, for all SNs quantities, this algorithm is superior to heuristic I algorithm. When the number of SNs is 28, the proposed algorithm can obtained 95.79% and 106.42% weighted sum energy compared with the Heuristic I and II algorithm, respectively.

The normalized CPU time versus number of SNs is given in Fig. 2. And you can see that the number of SNs reaches 32, the proposed algorithm can save about 94% and 34% CPU time compared with the Heuristic I and II algorithm, respectively.

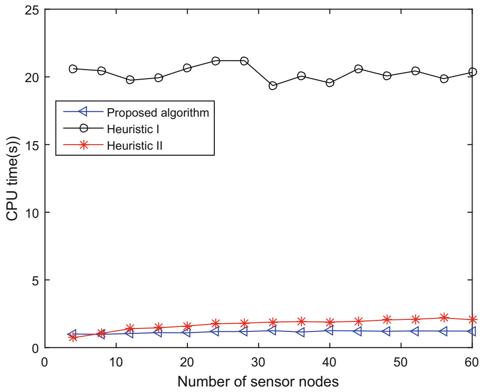

Fig. 2. Normalized CPU time versus number of SNs

4.2 Simulation Results Versus Number of Grids (step Size)

When the grid number (step size) tends to infinity, the heuristic I algorithm can obtain the optimal solution. However, this results in huge CPU computing time. Simulation parameters are as follows:

The number of SNs is set to 30. Other parameters are the same as subsection A. The sensors are randomly distributed in a two-dimensional area of 10×10 m^2,and We can see that in section B, the performance of our proposed algorithm is slightly different from that of heuristic I algorithm. Therefore, the proposed algorithm may not find the optimal solution in some cases. However, we found that as the size of SNs region decreases, our proposed algorithm will find the optimal solution just like the heuristic I algorithm. In brief, as the size of the region for SNs decreases, the sufficient condition for Theorem 2 has more probability to be satisfied. Then, the optimal solution can be sought out by the

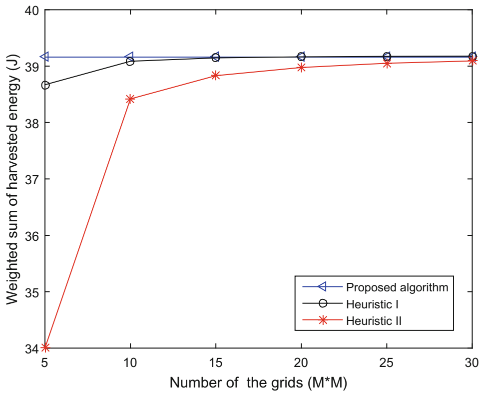

Fig. 3. Weighted sum of energy versus the number of grids

proposed algorithm. The simulation result of 10^3 is obtained randomly according to sensor's position and weighted factor. Next,we will show the influence of grid number on three algorithms.

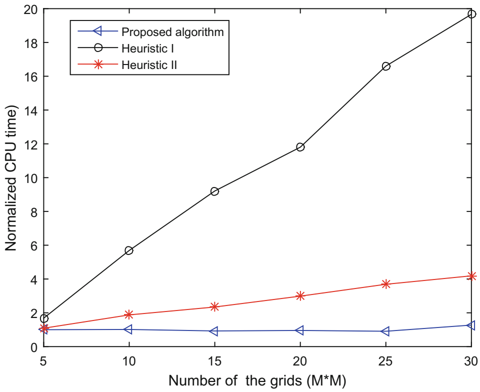

Fig. 4. Normalized CPU time versus number of grids

From above discussion, it can seen that the proposed algorithm can obtain the optimal solution as the heuristic algorithm with much less computation time.

5 Conclusion

In this paper, we studied the placement problem of the UAV that maximizes the sum energy and weighted sum energy of the ERs.Using a novel quadratic transform technique to transfer the non-convex optimization problem to an equivalent problem, we proposed a low-complexity iterative coordinate update algorithm to solve the placement problem. Simulations have shown that the proposed algorithm performs very close to the 2D-exhaustive search algorithm with a significant reduction in run time.

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