





# Secrecy Outage Probability Analysis for Indoor Visible Light Communications with Random Terminals

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**Abstract.** This paper focuses on the physical layer security for spatial modulation (SM) based indoor visible light communication (VLC) systems with multi-LED transmitters, a legitimate receiver and multiple eavesdroppers. According to the principle of information theory, a lower bound on the SM-based VLC secrecy outage probability (SOP) is derived by considering the non-negativity, average optical intensity and peak optical intensity constraints. Numerical results show that the lower bound of SOP can be used to evaluate system performance.

**Keywords:** Visible light communications · Secrecy outage probability · Random terminals

## 1 Introduction

As a complementary technology to traditional radio frequency (RF) wireless communication, indoor visible light communication (VLC) is receiving more and more attention and is considered as a promising information transmission technology to meet the growing demand for wireless services. Because it only needs to use the existing lighting infrastructure without additional equipment platform construction costs, spectrum applications and electromagnetic interference. VLC will play an important role in the future fifth generation (5G) wireless Communication.

In indoor VLC, light emitting diode (LED) is used as the light source. The user can receive information from the source when it is illuminated by the LED. The transmission of information on the VLC channel may be eavesdropped by unintended or unauthorized users, which poses a security risk to the transmission of data to legitimate users. Recently, physical layer security has been proposed as an effective method to ensure the security of information theory.

In [1], considering the physical layer security in the VLC system, two cases are mainly discussed, and the closed expression of the upper and lower bounds of the secrecy capacity are derived respectively under the two scenario. [2] described secrecy rate achieved by transmitting beamforming on the multiple-input, single-output (MISO) VLC wiretap channel. For physical layer security in multi-user VLC networks, the secrecy outage probability (SOP) and the ergodic secrecy rate (ESR) are derived in [3]. [4] studied the security performance of a legitimate receiver and a group of eavesdroppers in the VLC system, and derived a closed-form expression of the SOP and the average secrecy capacity without considering the constraints of optical signals. In order to improve physical layer security, a channel determined subcarrier shifting scheme is proposed for orthogonal frequency division multiplexing (OFDM) based VLC in [5].

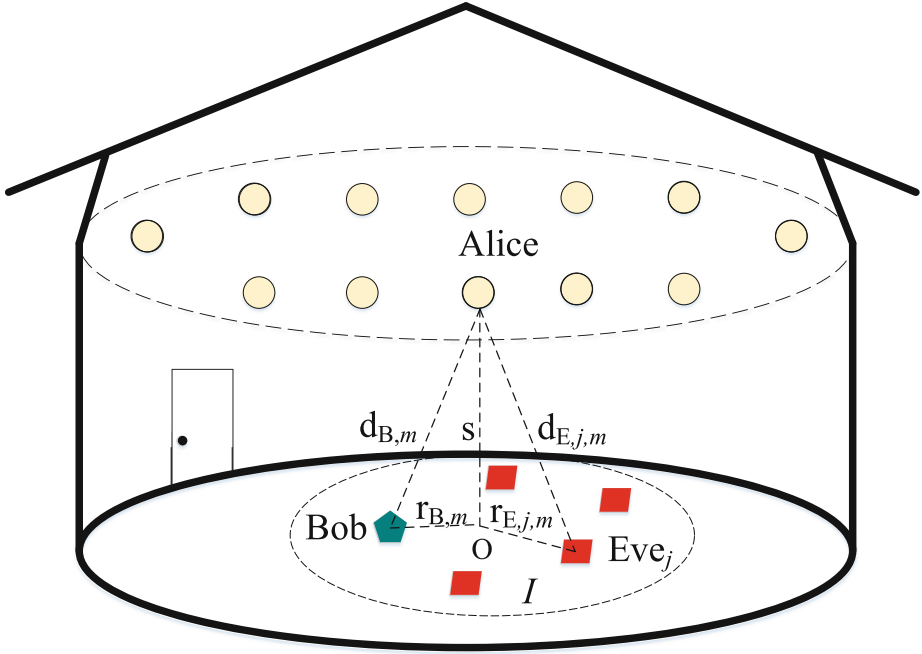
Due to multiple RF chains, the hardware complexity of multi-input multi-output (MIMO) systems is typically high. In order to break this limitation, spatial modulation (SM) using only one RF chain has been proposed as a low complexity solution [6]. Extensive research on SM demonstrates the advantages of SM over MIMO. Recently, SM's investigation has expanded to the field of VLC. The safety performance of the degraded single-input single-output VLC eavesdropping channel is studied in [7]. A new physical layer security transmission strategy, called mapping transform SM, is proposed in [8]. The average symbol error rate of VLC using adaptive SM is studied in [9]. In order to break the limit on the number of transmitters required to be a power of two, a channel adaptive bit mapping scheme is proposed for SM-based VLC in [10]. The above works are based on point-to-point VLC, regardless of the security of information transmission. Recently, physical layer security has attracted great attentions of researchers under various scenarios. However, in the open literature, the physical layer security of SM-based VLC has not been well studied.

Under the above work, this paper mainly analyzes the SOP of a legitimate receiver and a group of eavesdroppers in a SM-based VLC system. Taking into account the average and peak optical intensity constraints, we derive the expression of a lower bound on the SOP. Numerical results verify the accuracy of derived lower bound.

The rest of this paper is organized as follows. Section 2 shows the system model of an SM-based VLC system that includes a legitimate recipient and multiple eavesdroppers. In Sect. 3, the expression of a lower bound on the SOP of the SM-based VLC is derived and analyzed. Numerical results are given in Sect. 4. Finally, conclusions are drawn in Sect. 5.

## 2 System Model

In this paper, we consider an indoor VLC system consisting of transmitters (Alice) with  $M$  LEDs, a legitimate receiver (Bob) and  $N$  ( $N \geq 1$ ) eavesdroppers (Eve $_j$ ,  $j = 1, 2, \dots, N$ ), as shown in Fig. 1. In this system, Alice is fixed on the ceiling, while Bob and Eve are randomly placed on the receiving plane. Under the SM scheme, only one LED is activated in each symbol period and the activation



**Fig. 1.** An VLC network with  $M$  transmitter, one legitimate receiver and a group of eavesdroppers.

probability of each LED is equal,  $p(h_k = h_{k,m}) = 1/M$ ,  $k = B$  for Bob and  $k = E$  for Eve. Assume that the receiving area is a circular region  $I$  of radius  $D$  and the projection of Alice on the receiving area is point  $O$ . We assume that Bob is uniformly distributed in Region  $I$ , and the position of the eavesdropper is uniformly distributed within the area, while the number of eavesdroppers subjects to the Poisson distribution  $P\{N = k\} = \frac{\mu_s^k}{k!} e^{-\mu_s}$  with  $\mu_s = \pi D^2 \eta$ .

In indoor VLC system, the input signal  $X$  should satisfy  $0 \leq X \leq A$  and  $E(X) = \xi P$ , where  $A$  is the peak optical intensity,  $E(\cdot)$  is the expectation operator,  $\xi$  denotes the dimming target [11], and  $P \in (0, A]$  represents the nominal optical intensity of each LED.

The received signals at Bob and Eve <sub>$j$</sub>  can be respectively expressed as

$$\begin{cases} Y_B = h_{B,m}X + Z_B \\ Y_{E,j} = h_{E,j,m}X + Z_{E,j} \end{cases}, m = 1, 2, \dots, M \quad (1)$$

where  $Z_B \sim N(0, \sigma_B^2)$ ,  $Z_{E,j} \sim N(0, \sigma_{E,j}^2)$ ,  $\sigma_B^2$  and  $\sigma_{E,j}^2$  are the corresponding noise variances.  $h_{k,m}$  represents the channel gain between the  $m$ -th LED and the receiver (Bob and Eve <sub>$j$</sub> ), and can be written as [12]

$$h_{k,m} = \frac{(l+1)A_r}{2\pi d_{k,m}^2} T_s g \cos^l(\varphi_{k,m}) \cos(\psi_{k,m}) \quad (2)$$

where  $l$  is the lambertian emission sequence;  $A_r$ ,  $T_s$  and  $g$  are the physical areas of the PD, filter gain and concentrator gain.  $d_{k,m}$ ,  $\varphi_{k,m}$  and  $\psi_{k,m}$  are the distance from Alice ( $m$ -th LED) to Bob or Eve $_j$ , the angle of emission and the angle of incidence, respectively.  $\psi_c$  is the field of view of each PD,  $0 \leq \psi_{k,m} \leq \psi_c$ .

Assume that the normal vector of the transceiver plane is perpendicular to the ceiling. Then, we have  $\cos(\varphi_{k,m}) = \cos(\psi_{k,m}) = s/d_{k,m}$ , and the channel gain can be further rewritten as

$$\begin{aligned} h_{k,m} &= \frac{(l+1)A_r T_s g s^{l+1}}{2\pi} (s^2 + r_{k,m}^2)^{-\frac{l+3}{2}} \\ &= \lambda (s^2 + r_{k,m}^2)^{-\frac{l+3}{2}} \end{aligned} \quad (3)$$

where  $r_{k,m}$  is the distance between the projection point of the  $m$ -th LED on the receiving plane and the receiver (Bob or Eve $_j$ ).  $s$  is the vertical height of Alice and the receiving plane, assuming  $\lambda = \frac{(l+1)A_r T_s g s^{l+1}}{2\pi}$ .

### 3 Secrecy Outage Probability

Assume the location of Bob and Eve $_j$  (represented by  $U$ ) are uniformly distributed within the region  $I$ . Then, the probability density function (PDF) for Bob and Eve $_j$  positions (due to the probability distribution of  $M$  LEDs), then  $f_{U,m}(u) = \frac{1}{\pi M D^2}$ .

The cumulative distribution function (CDF) of  $r_{k,m}$  is given by

$$F_{r_{k,m}}(x) = \int_0^{2\pi} \int_0^x \frac{1}{\pi M D^2} r dr d\theta = \frac{x^2}{M D^2} \quad (4)$$

In addition, the PDF of  $r_{k,m}$  can be expressed as

$$f_{r_{k,m}}(x) = \frac{2x}{M D^2}, 0 \leq x \leq D \quad (5)$$

According to (3), (5), the PDF of  $h_{k,m}$  can be further expressed as

$$f_{h_{k,m}}(x) = \frac{2\lambda^{\frac{2}{l+3}}}{(l+3) M D^2} x^{-\frac{l+5}{l+3}} \quad (6)$$

where  $\lambda(D^2 + s^2)^{-\frac{l+3}{2}} \leq x \leq \lambda s^{-l-3}$ .

Therefore, the CDF of  $h_{E,j,m}$  is given by

$$F_{h_{E,j,m}}(x) = -\frac{\lambda^{\frac{2}{l+3}}}{M D^2} x^{-\frac{2}{l+3}} + \frac{1}{M} + \frac{s^2}{M D^2} \quad (7)$$

The largest information gain  $h_{E,\max,m}$  of the eavesdroppers can be expressed as

$$h_{E,\max,m} = \max_{j \in \{1, \dots, N\}} \{h_{E,j,m}\} \quad (8)$$

According to [13], the PDF of  $h_{E,\max,m}$  can be written as

$$f_{h_{E,\max,m}}(x) = \frac{2N(D^2 + s^2)}{M^2 D^4 (l+3)} \lambda^{\frac{2}{l+3}} x^{-\frac{l+5}{l+3}} \left( -\frac{\lambda^{\frac{2}{l+3}}}{D^2 + s^2} x^{-\frac{2}{l+3}} + 1 \right)^{N-1} \quad (9)$$

In VLC system with non-negative, peak, and average constraints, when the primary channel is worse than the eavesdropping channel ( $h_{B,m}/\sigma_B < h_{E,\max,m}/\sigma_{E,\max}$ ), the secrecy rate is 0. When the primary channel is superior to the eavesdropping channel ( $h_{B,m}/\sigma_B > h_{E,\max,m}/\sigma_{E,\max}$ ), according to [1], the lower bound of instantaneous secrecy rate is known as

$$R_s = \begin{cases} \frac{1}{2M} \sum_{m=1}^M \ln \left[ \frac{3\sigma_E^2 (A^2 h_{B,m}^2 + 2\pi e \sigma_B^2)}{2\pi e \sigma_B^2 (h_{E,\max,m}^2 \xi^2 P^2 + 3\sigma_{E,\max}^2)} \right], & \text{if } \frac{h_{B,m}}{\sigma_B} > \frac{h_{E,\max,m}}{\sigma_{E,\max}} \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

To further simplify the derivation, the instantaneous secrecy rate can be further expressed as

$$R_s = \begin{cases} \frac{1}{2M} \sum_{m=1}^M \ln \left[ \frac{J_B + 1}{J_{E,\max} + 1} \right], & \text{if } \frac{h_{B,m}}{\sigma_B} > \frac{h_{E,\max,m}}{\sigma_{E,\max}} \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

where  $J_B$  and  $J_{E,\max}$  are

$$\begin{cases} J_B = \frac{A^2 h_{B,m}^2}{2\pi e \sigma_B^2} \\ J_{E,\max} = \frac{h_{E,\max,m}^2 \xi^2 P^2}{3\sigma_{E,\max}^2} \end{cases} \quad (12)$$

According to (6), (9) and (12), the PDF of  $J_B$  and  $J_{E,\max}$  can be further expressed as

$$\begin{cases} f_{J_B}(y) = \alpha y^{-\frac{l+4}{l+3}}, u_{\min} \leq y \leq u_{\max} \\ f_{J_{E,\max}}(z) = \beta z^{-\frac{l+4}{l+3}} \left( -\frac{\lambda^{\frac{2}{l+3}}}{D^2 + s^2} \left( \frac{3\sigma_{E,\max}^2 z}{\xi^2 P^2} \right)^{-\frac{1}{l+3}} + 1 \right)^{N-1}, v_{\min} \leq z \leq v_{\max} \end{cases} \quad (13)$$

where  $\alpha$  and  $\beta$  can be expressed as

$$\begin{cases} \alpha = \frac{\lambda^{\frac{2}{l+3}}}{(l+3)MD^2} \left( \frac{2\pi e \sigma_B^2}{A^2} \right)^{-\frac{1}{l+3}} \\ \beta = N \frac{D^2 + s^2}{M^2 D^4} \frac{\lambda^{\frac{2}{l+3}}}{l+3} \left( \frac{3\sigma_{E,\max}^2}{\xi^2 P^2} \right)^{-\frac{1}{l+3}} \end{cases} \quad (14)$$

Moreover,  $v_{\min}$ ,  $v_{\max}$ ,  $u_{\min}$ ,  $u_{\max}$  can be expressed by

$$\begin{cases} u_{\min} = \frac{\lambda^2 A^2}{2\pi e \sigma_B^2} (D^2 + s^2)^{-l-3} \\ u_{\max} = \frac{\lambda^2 A^2}{2\pi e \sigma_B^2} s^{-2l-6} \\ v_{\min} = \frac{\lambda^2 \xi^2 P^2}{3\sigma_{E,\max}^2} (D^2 + s^2)^{-l-3} \\ v_{\max} = \frac{\lambda^2 \xi^2 P^2}{3\sigma_{E,\max}^2} s^{-2l-6} \end{cases} \quad (15)$$

In this paper, SOP is defined as

$$\begin{aligned} SOP(C_{\text{th}}) &= \Pr\{C_s \leq C_{\text{th}}\} \\ &= \Pr\left\{\frac{1}{2} \ln\left(\frac{J_B+1}{J_E+1}\right) \leq \frac{1}{2} \ln(\gamma_{\text{th}})\right\} \end{aligned} \quad (16)$$

where  $\Pr\{\cdot\}$  represents the PDF of an event.  $C_{\text{th}} = 0.5 \ln(\gamma_{\text{th}})$  represents the threshold of the secrecy capacity, and  $\gamma_{\text{th}} \geq 1$  is the equivalent threshold of signal-to-noise ratio (SNR).

In indoor VLC system, it provides a large SNR to meet the illumination requirements. Therefore, we can assume  $J_B \gg 1$ ,  $J_{E,\text{max}} \gg 1$ ,  $J_B > J_{E,\text{max}}$ , and thus  $(J_B + 1) / (J_{E,\text{max}} + 1) < J_B / J_{E,\text{max}}$ .

Therefore, the upper bound of the instantaneous secrecy capacity can be expressed as

$$C_s = \frac{1}{2} \ln\left(\frac{J_B + 1}{J_{E,\text{max}} + 1}\right) < \frac{1}{2} \ln\left(\frac{J_B}{J_{E,\text{max}}}\right) \quad (17)$$

According to (16) and inequality (17), the lower bound of SOP can be expressed as

$$\begin{aligned} SOP(C_{\text{th}}) &\geq \Pr\left\{\frac{1}{2} \ln\left(\frac{J_B}{J_E}\right) \leq \frac{1}{2} \ln(\gamma_{\text{th}})\right\} \\ &= \Pr\{J_B \leq \gamma_{\text{th}} J_E\} \triangleq SOP_L \end{aligned} \quad (18)$$

The lower bound of the SOP can be discussed separately in four cases. Case 1: when  $\gamma_{\text{th}} v_{\text{max}} \leq u_{\text{min}}$ , we have

$$SOP_L = \int_{v_{\text{min}}}^{v_{\text{max}}} \int_0^{\frac{u_{\text{min}}}{\gamma_{\text{th}}}} f_{J_B}(y) f_{J_{E,\text{max}}}(z) dy dz = 0 \quad (19)$$

Case 2: when  $\gamma_{\text{th}} v_{\text{min}} \leq u_{\text{min}}$  and  $u_{\text{min}} \leq \gamma_{\text{th}} v_{\text{max}} \leq u_{\text{max}}$ , we have

$$\begin{aligned} SOP_L &= \int_{\frac{u_{\text{min}}}{\gamma_{\text{th}}}}^{v_{\text{max}}} \int_{u_{\text{min}}}^{\gamma_{\text{th}} z} f_{J_B}(y) f_{J_{E,\text{max}}}(z) dy dz \\ &= -(l+3) \alpha \beta \frac{v_{\text{max}}}{\gamma_{\text{th}}} \left[ (\gamma_{\text{th}} z)^{-\frac{1}{l+3}} - (u_{\text{min}})^{-\frac{1}{l+3}} \right] z^{-\frac{l+4}{l+3}} \left( -\frac{\lambda}{D^2+s^2} \left( \frac{3\sigma_E^2 z}{\xi^2 P^2} \right)^{-\frac{1}{l+3}} + 1 \right)^{N-1} dz \\ &= A_1 \end{aligned} \quad (20)$$

Case 3: when  $u_{\text{min}} \leq \gamma_{\text{th}} v_{\text{min}} \leq u_{\text{max}}$  and  $u_{\text{max}} \leq \gamma_{\text{th}} v_{\text{max}}$ , we have

$$SOP_L = \underbrace{\int_{v_{\text{min}}}^{\frac{u_{\text{max}}}{\gamma_{\text{th}}}} \int_{u_{\text{min}}}^{\gamma_{\text{th}} z} f_{J_B}(y) f_{J_{E,\text{max}}}(z) dy dz}_{T_1} + \underbrace{\int_{\frac{u_{\text{max}}}{\gamma_{\text{th}}}}^{v_{\text{max}}} \int_{u_{\text{min}}}^{u_{\text{max}}} f_{J_B}(y) f_{J_{E,\text{max}}}(z) dy dz}_{T_2} \quad (21)$$

where  $T_1$  and  $T_2$  are given by

$$\begin{aligned} T_1 &= \int_{v_{\text{min}}}^{\frac{u_{\text{max}}}{\gamma_{\text{th}}}} \int_{u_{\text{min}}}^{\gamma_{\text{th}} z} f_{J_B}(y) f_{J_{E,\text{max}}}(z) dy dz \\ &= -(l+3) \alpha \beta \int_{v_{\text{min}}}^{\frac{u_{\text{max}}}{\gamma_{\text{th}}}} \left[ (\gamma_{\text{th}} z)^{-\frac{1}{l+3}} - (u_{\text{min}})^{-\frac{1}{l+3}} \right] z^{-\frac{l+4}{l+3}} \left( -\frac{\lambda}{D^2+s^2} \left( \frac{3\sigma_E^2 z}{\xi^2 P^2} \right)^{-\frac{1}{l+3}} + 1 \right)^{N-1} dz \end{aligned} \quad (22)$$

and

$$\begin{aligned}
 T_2 &= \int_{\frac{u_{\min}}{\gamma_{\text{th}}}}^{u_{\max}} \int_{u_{\min}}^{u_{\max}} f_{J_B}(y) f_{J_{E,\max}}(z) dy dz \\
 &= -(l+3) \alpha \beta \int_{\frac{u_{\max}}{\gamma_{\text{th}}}}^{u_{\max}} \left( u_{\max}^{-\frac{1}{l+3}} - u_{\min}^{-\frac{1}{l+3}} \right) z^{-\frac{l+4}{l+3}} \left( -\frac{\lambda}{D^2+s^2} \left( \frac{3\sigma_B^2 z}{\xi^2 P^2} \right)^{-\frac{1}{l+3}} + 1 \right)^{N-1} dz
 \end{aligned} \tag{23}$$

Therefore, (21) can be expressed as

$$SOP_L = T_1 + T_2 = A_2 \tag{24}$$

Case 4: when  $u_{\max} \leq \gamma_{\text{th}} u_{\min}$ , we have

$$SOP_L = \int_{v_{\min}}^{v_{\max}} \int_{u_{\min}}^{u_{\max}} f_{J_B}(y) f_{J_{E,\max}}(z) dy dz = 1 \tag{25}$$

Based on the analysis of the above four cases, the lower bound of the SOP can be finally expressed as

$$SOP_L = \begin{cases} 0, & \gamma_{\text{th}} u_{\max} \leq u_{\min} \\ A_1, & \gamma_{\text{th}} u_{\min} \leq u_{\min}, u_{\min} \leq \gamma_{\text{th}} u_{\max} \leq u_{\max} \\ A_2, & u_{\min} \leq \gamma_{\text{th}} u_{\min} \leq u_{\max}, u_{\max} \leq \gamma_{\text{th}} u_{\max} \\ 1, & u_{\max} \leq \gamma_{\text{th}} u_{\min} \end{cases} \tag{26}$$

where  $A_1$  is given by (20),  $A_2$  is given by (24).

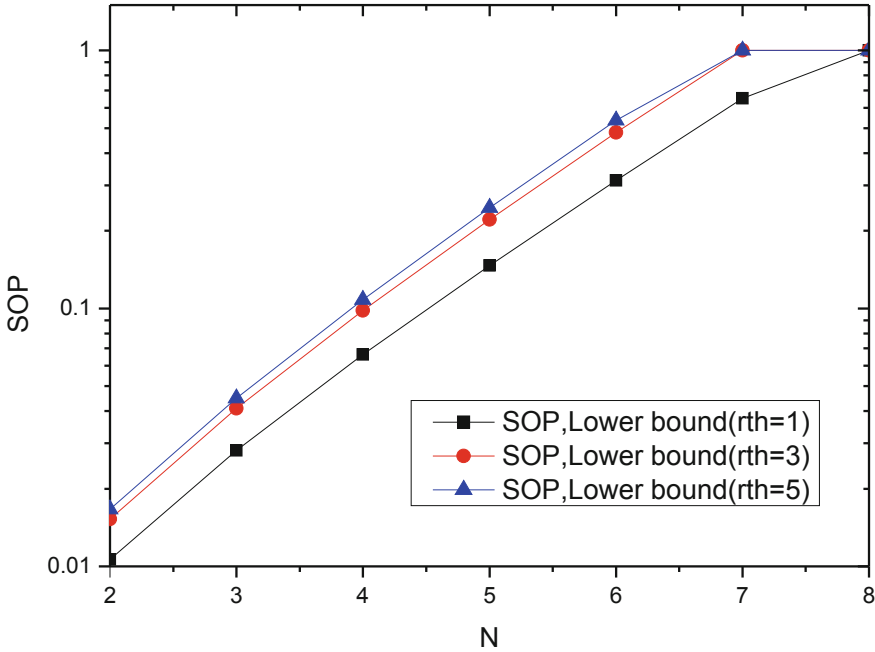


Fig. 2. SOP versus  $N$  with different  $\gamma_{\text{th}}$  when  $P = 30$  dB and  $D = 2$  m.

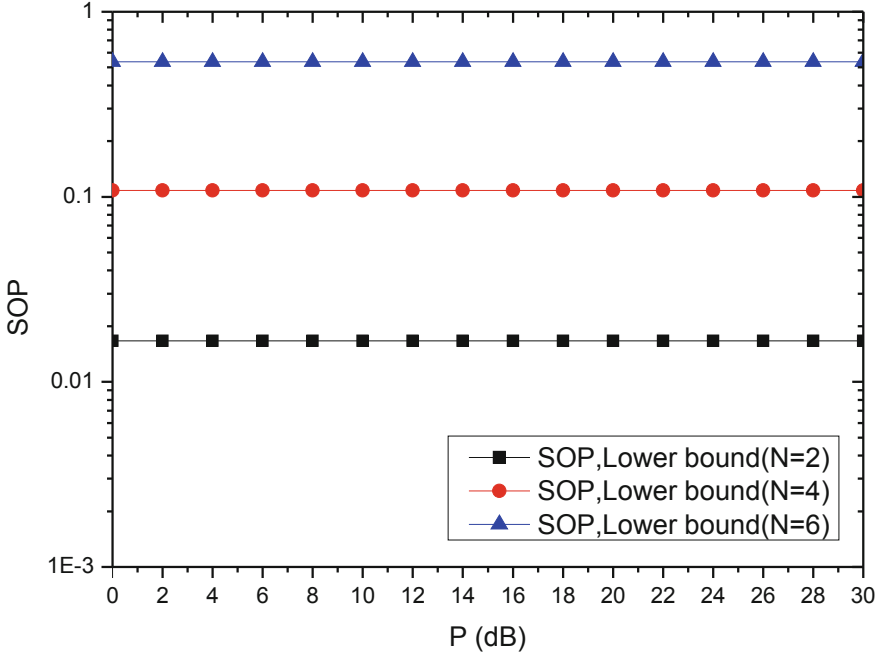


Fig. 3. SOP versus  $P$  with different  $N$  when  $\gamma_{\text{th}} = 5$  and  $D = 2$  m.

## 4 Numerical Results

In this section, we consider an indoor VLC system, where area  $I$  includes a legal recipient (Bob) and multiple eavesdroppers (Eve $_j$ ,  $j = 1, 2, \dots, N$ ). Assume there are  $M = 8$  LEDs fixed on the ceiling at a height of 3 m, and the receiving planes of the receiver and the eavesdropper are located in the same height of 0.5 m. LED dimming coefficient is set to be  $\xi = 0.5$ . The noise variances are set as  $\sigma_B^2 = \sigma_E^2 = -104$  dBm.

Figure 2 shows the relationship between SOP and the number  $N$  of eavesdroppers and changes with different equivalent thresholds when  $P = 30$  dB and  $D = 2$  m. It can be intuitively seen that the SOP performance deteriorates with the increase of  $N$ , because the increase in the number of Eve will increase the diversity gain of information eavesdropping, and the secrecy performance of the system will definitely decrease. In addition, as the  $\gamma_{\text{th}}$  increases, the value of the SOP also becomes larger, which means that the system's safe transmission performance is degraded.

Figure 3 shows the relationship between SOP and  $P$  with a different number of eavesdrops  $N$  when  $\gamma_{\text{th}} = 5$  and  $D = 2$  m. It can be seen that when the number  $N$  of eavesdroppers is determined, the SOP performance hardly changes with an increase in  $P$ . The results show that in this case, increasing the optical intensity does not improve the SOP performance of the system. This is because

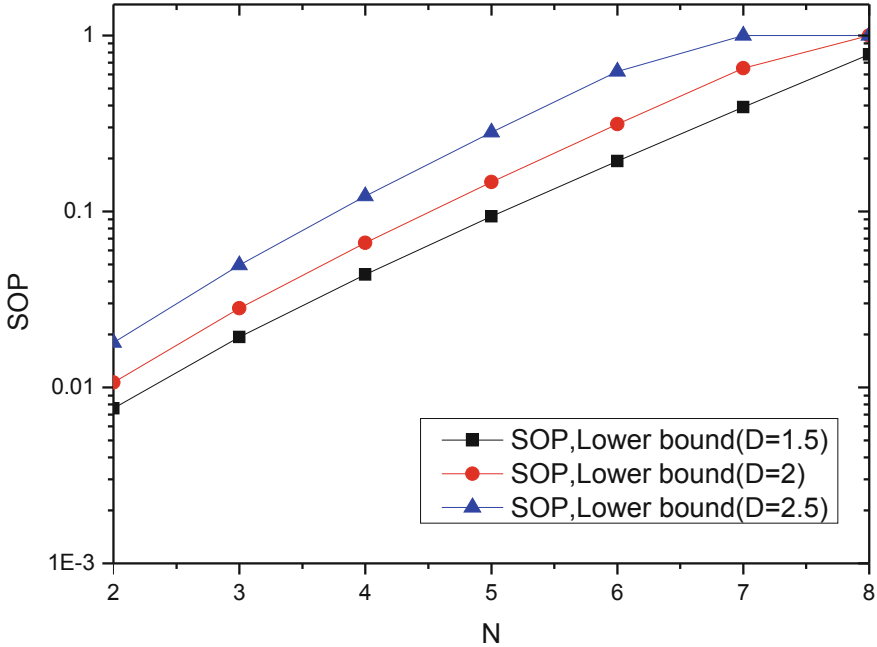


Fig. 4. SOP versus  $N$  with different  $D$  when  $\gamma_{th} = 1$  and  $P = 30$  dB.

Bob and Eve follow the same distribution in the circular area, and increasing the optical intensity will increase Bob’s SNR and Eve’s SNR. Similar to Fig. 2, the SOP in the figure also increases as the number  $N$  of eavesdroppers increases.

Figure 4 shows the relationship between the SOP and the number  $N$  of eavesdroppers with different radius sizes  $D$  when  $P = 30$  dB and  $\gamma_{th} = 1$ . When  $N$  is determined, it can be observed that the SOP performance improves as  $D$  decreases. This is because as the radius of the area increases, Eve has a greater chance of approaching Alice’s projection on the floor, resulting in better channel gain than Bob. In addition, the SOP also increases with  $N$ , which is similar to Fig. 2.

## 5 Conclusion

In this paper, we have studied the SOP in indoor VLC system. A closed-form expression of the lower bound of the SOP has been derived. Numerical results show that the derived lower bound of the SOP can be used to evaluate system performance. Besides, the number of eavesdroppers has a large impact on SOP, and increasing optical intensity does not improve SOP performance. In contrast, the size of the radius of the area has a negative impact on the SOP.

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