



Distributed Opportunistic Channel Access Under Single-Bit CSI Feedback

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Abstract. In this research, the problem of distributed opportunistic channel access is investigated under single-bit channel state information (CSI) feedback in ad hoc networks. In finding an optimal strategy achieving the maximum average system throughput, the optimal stopping approach is used, and an optimal strategy is proposed. In it, each winner source decides when to take transmission opportunity based on single-bit feedback information. Moreover, an optimal quantization vector is also derived for the strategy. Interestingly, it is pure-threshold based, and the optimal quantization vector is fixed. All thresholds can be calculated off-line, and easy implementation is available. Through simulation, the effectiveness and efficiency of the optimal strategy are also verified.

Keywords: Distributed opportunistic access · Single-bit feedback · Optimal stopping approach

1 Introduction

Recent years have witnessed rapid development of wireless network by catering ever-increasing demands. As available spectrum is very limited, emerging innovative techniques improving spectrum utilization receive research attention. In them, the concept of opportunistic channel access is enlightened, designing the link transmission and multiple user channel access in a joint manner. Based on it, centralized opportunistic access has been fully investigated in [1, 2]. In these works, a central controller is deployed in the network, and by scheduling the user of best channel condition to access the channel based on global channel state, the multi-user diversity is harvested, which significantly enhances network performance.

However, centralized scheduling does not work in a distributed network. The use of the shared channel and distributed decisions on channel access rises up new challenges due to coupling between wireless transmission adaptation and multi-user collision. Due to independent design of physical and MAC layer strategy,

the random and time-varying nature of channel access makes network performance not preferable. Thus, distributed opportunistic channel access is not well addressed.

As a seminal work in resolving the difficulty, authors in [3] proposes an efficient channel scheduling strategy based on optimal stopping approach. The strategy is pure-threshold, and can benefit from easy implementation. Extended from it, work in [4] further investigates opportunistic scheduling under interference channels by allowing multiple users simultaneous access. Moreover, for time sensitive service, distributed scheduling under time constraints is studied in [5]. Optimal scheduling strategies are proposed maximizing average system throughput, respectively.

All the above existing works are based on the assumption that in the wireless distributed network, a winner source has accurate channel state information of the channel. However, in practical network environment, channel state information relies on control channel, where limited feedback is only available. Such quantized CSI is included in control packet from the winner destination, and the winner source has to make its decision on channel access. Furthermore, under limited feedback, the optimal quantization vector with respect to observed CSI needs to design, determining the transmission rate for channel access. In finding the optimal quantization vector for all sources and distributed channel access strategy, the average system throughput is to be maximized.

For optimal design of channel access strategies, this research investigates distributed opportunistic scheduling problem under single bit CSI feedback. In terms of network statistics, homogeneous and heterogeneous wireless channels are considered, respectively. In both cases, the quantized CSI thresholds of single bit feedback are derived for all sources, and can be calculated off-line. Interestingly, based on optimal stopping approach, the optimal strategy is in pure-threshold structure, and maximizes the average system throughput.

The rest of the paper is organized as follows. System model under limited feedback mechanism is described in Sect. 2. The optimal opportunistic channel access strategy is proposed and its optimality is proved in Sect. 3. Simulation analysis is then carried out to validate effectiveness in Sect. 4. Finally, the conclusion is presented in Sect. 5.

2 System Model

Consider K source-to-destination pairs in a single-hop ad hoc network. The transmission power of each source is P_s . The wireless channel follows the Rayleigh fading model. The channel gain from the i th source to its destination is denoted by $h_i, i = 1, 2, \dots, K$, which follows a Complex Gaussian distribution with zero mean and variance σ_i^2 .

The channel access is operated in a distributed manner. Each source independently contends for transmission opportunities by sending a request-to-send (RTS) packet at the beginning of a time slot with duration δ . The probability of each source sending RTS is p_0 , and there are three possible results, which

are idle, collision and success. The probability at which the channel is idle is $(1 - p_0)^K$, i.e., there is no source contending the channel in the time slot. The probability of channel collision is $1 - (1 - p_0)^K - Kp_0(1 - p_0)^{K-1}$. It happens when two or more sources contend the channel simultaneously. The probability of successful channel access is $Kp_0(1 - p_0)^{K-1}$ when there is only one source transmitting RTS referred as winner source. When channel is idle or in collision, sources will continue to contend in the next time slot.

In the case of success, upon reception of the RTS, the winner destination, e.g. Destination i obtains instantaneous CSI from Source i . As only a single-bit CSI information is feedback, the destination compares the observed SNR with its local quantization vector, and feedbacks a CTS to the source. In it, a quantized value of instantaneous SNR, denoted as $\omega_{i,j}$ is included. By receiving the CTS, Source i decides to *stop* (i.e. transmit its data for duration τ_d) or to *continue* (i.e. give up its transmission opportunity and re-contend with other sources) (Fig. 1).

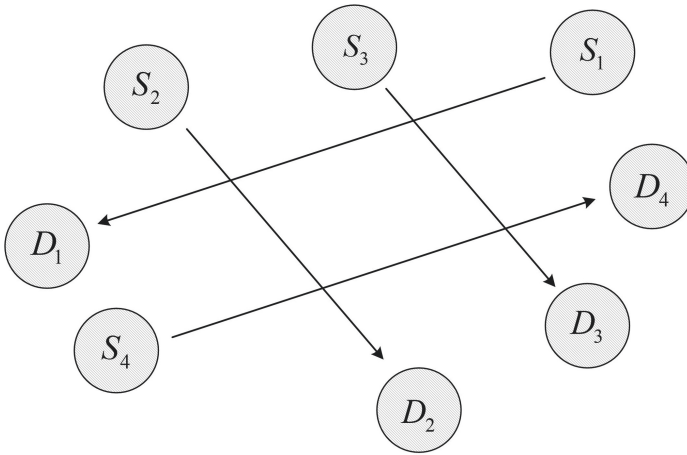


Fig. 1. Network model

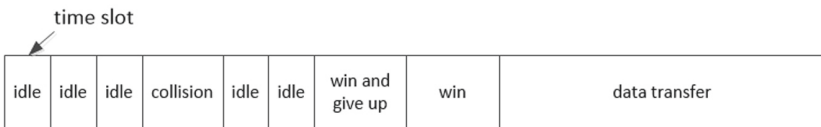


Fig. 2. An example of distributed channel access by multiple sources

An example of distributed channel access in the ad hoc network is shown in Fig. 2. In it, all sources are silent in the first three time slots. Then more than

two sources contend the channel, resulting in a collision. After two idle slots, a winner appears, and obtains CSI. Due to bad channel condition, the winner gives up this opportunity and re-starts channel probing in the next slot. Then another winner appears in the new round. After exchange of RTS and CTS, the winner source transmits data to its destination. By doing it, a successful channel access is finished.

Different from existing works with perfect CSI feedback, influence from single-bit feedback mechanism is investigated, and this mechanism makes distributed opportunistic channel access practical. In particular, each winner destination feedbacks a CTS after channel estimation providing tailored information to its source. In the CTS, channel conditions such as channel state, received power and interference level are included, enabling the source to carry out adaptive transmission.

Furthermore, complete channel gain information is known at each destination, limited information is included in the CTS and sent to the source, based on observed SNR. Limited by bit number $\lceil \log_2 B \rceil$, the information available at each source is the quantized counterpart of instantaneous SNR. For Source $i = 1, 2, \dots, K$, the quantization vector is denoted as

$$\mathbf{w}_i = [\omega_{i,0}, \omega_{i,1}, \dots, \omega_{i,B-1}]. \tag{1}$$

The vector \mathbf{w}_i is shared by source-to-destination pair i . Using the vector, when instantaneous SNR h_i satisfies $\omega_{i,j} < P_s h_i < \omega_{i,j+1}$, $\omega_{i,j}$ represents the maximum achievable SNR by accessing the channel. Without loss of generalization, $\omega_{i,0} = 0$ and an increasing order $\omega_{i,0} < \omega_{i,1} < \dots < \omega_{i,B-1}$ are considered (Fig. 3).

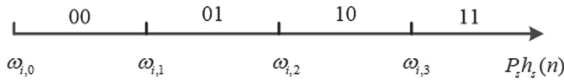


Fig. 3. Example of 2-bits CSI feedback.

3 Optimal Stopping Strategy with Limited Feedback

3.1 Problem Formulation

While quantization vectors $\{\mathbf{w}_i\}_{i=1,2,\dots,K}$ are fixed, we focus on optimal strategy design for the distributed opportunistic channel access, maximizing the average system throughput.

Define an *observation* as the process of channel contention among all sources until a winner source appears. As sources contend the channel independently, the number of contentions in an observation is a random variable of geometrically distribution with parameter $Kp_0(1 - p_0)^{K-1}$. Among all contentions, the last

contention is successful with duration $\tau_{RTS} + \tau_{CTS}$. τ_{RTS} and τ_{CTS} are durations of an RTS and CTS, respectively. An idle slot is with duration δ , and a collision is with duration τ_{RTS} . The mean of the duration of an observation is given as $\tau_0 = \tau_{RTS} + \tau_{CTS} + \frac{(1-p_0)^K}{Kp_0(1-p_0)^{K-1}} \cdot \delta + \frac{1-(1-p_0)^K - Kp_0(1-p_0)^{K-1}}{Kp_0(1-p_0)^{K-1}} \cdot \tau_{RTS}$.

In the n th observation, let $s(n)$ denote the successful source, $h_s(n)$ and R_n represent the corresponding channel gain and maximal achievable rate, respectively. Due to the block-fading channel feature in wireless environment, R_n is random, and can be calculated as $R_n = \log_2(1 + P_s h_s(n))$. Since the limited feedback, the winner source only obtains the quantified SNR as

$$u(n) = \sum_{i=1}^K I[s(n) = i] \cdot \log_2(1 + \omega_{i,j}),$$

$$\omega_{i,j} = \max_{l=0,1,\dots,B-1} I[\omega_{i,l} \leq P_s h_s(n) < \omega_{i,l+1}] \cdot \omega_{i,l}. \quad (2)$$

It can be observed that based on quantization vector \mathbf{w} , the maximal SNR feedback in the CTS from Destination $s(n)$ to its source will not supersede the instantaneous channel capacity, i.e. $u(n) \leq R_n$.

For opportunistic channel access, after observation n , the winner source obtains the rate $u(n)$ and has to make two choices:

- If the channel condition is good, the winner source will stop the observation and transmit data by rate $u(n)$.
- Otherwise, when the channel condition is poor, the winner source will give up transmission opportunity and start a new observation. At the next slot, all sources re-contend the channel.

Based on the optimal stopping theory, this problem can be transformed into a maximum return problem. For the n th observation, the reward Y_n denotes the total traffic volume to be transferred by the winner source, and the cost T_n is the total waiting time for a successful channel access. In particular, when the winner source stops at N th observation, i.e., stops the observation and transmits data by rate $u(n)$, the instantaneous system throughput is $\frac{Y_N}{T_N}$. The strategy N is also referred as *stopping time*. Following the strategy N for multiple trails of successful data transmission, the average system throughput $\frac{\mathbb{E}[Y_N]}{\mathbb{E}[T_N]}$ is obtained.

In particular, $\frac{\mathbb{E}[Y_N]}{\mathbb{E}[T_N]}$ reflects the trade-off between larger transmitted traffic and the time cost.

In the following, we focus on finding an optimal channel access strategy, including an optimal stopping time and quantization vector, to maximize the average system throughput $\sup_{\mathbf{w} \geq 0} \{ \sup_{N > 0} \frac{\mathbb{E}[Y_N]}{\mathbb{E}[T_N]} \}$. Here $\mathbb{E}[\cdot]$ means expectation.

According to the optimal stopping theory [6], the problem of maximizing the average system throughput can be transformed into an equivalent form with its reward being $\{Y_N - \lambda T_N\}$ and time price λ . To derive optimal strategy N^* given a fixed quantization vector \mathbf{w} , we first need to solve the following problem by maximizing the expected reward

$$V^*(\lambda^*(\mathbf{w})) = \sup_{N > 0} \{ \mathbb{E}[Y_N] - \lambda^*(\mathbf{w}) \mathbb{E}[T_N] \}, \quad (3)$$

where $\lambda^*(\mathbf{w})$ satisfies $V^*(\lambda^*(\mathbf{w})) = 0$. $\lambda^*(\mathbf{w})$ is actually the maximal expected system throughput under a given vector \mathbf{w} .

Specifically, for n th observation, the reward Y_n and the cost time T_n are calculated as $Y_n = \tau_d u(n)$ and $T_n = \sum_{l=1}^n t_l + \tau_d$, respectively. Therefore, the expression (3) is represented as

$$V^*(\lambda^*(\mathbf{w})) = \sup_{N>0} \left\{ \mathbb{E}[\tau_d u(N)] - \lambda^*(\mathbf{w}) \cdot \mathbb{E}\left[\tau_d + \sum_{l=1}^N t_l\right] \right\}, \tag{4}$$

where t_l denotes the duration spent in the l th observation.

For a given quantization vector \mathbf{w} , the following theorem presents the optimal stopping strategy $N^*(\mathbf{w})$ achieving the maximum expected reward $V^*(\lambda^*(\mathbf{w}))$.

Theorem 1. *The optimal stopping rule $N^*(\mathbf{w})$ maximizing system throughput $\sup_{N>0} \frac{\mathbb{E}[Y_N]}{\mathbb{E}[T_N]}$ exists and is given by $N^*(\mathbf{w}) = \min \{n \geq 1 : u(n) \geq \lambda^*(\mathbf{w})\}$. The threshold $\lambda^*(\mathbf{w})$ is the unique solution such that $\mathbb{E}[\max\{u(n) - \lambda, 0\}] = \frac{\lambda \tau_0}{\tau_d}$.*

Proof. The proof is similar to that in [3].

While the maximal average system throughput λ^* satisfies the optimal equation in Theorem 1, by the monotonicity of both sides, the problem of maximizing the system throughput transfers to finding an optimal vector \mathbf{w}^* to achieve system throughput $\sup_{\mathbf{w}>0} \lambda^*(\mathbf{w})$.

In the following, we consider the problem where only 1-bit CSI feedback is available, and the problem under multiple bits feedback can be analyzed similarly. Moreover, for wireless channel environments, cases with homogeneous links and heterogeneous channels are investigated, respectively.

3.2 The Case Under Homogeneous Channels

We consider the homogeneous case where all channels between source-to-destination pairs have the same channel statistics. For all pairs, the quantization vector is the same with $\mathbf{w} = [0, \omega_1]$. It thus suffices to find the optimal value ω_1 to maximize throughput λ^* . The optimal strategy is derived in the following theorem.

Theorem 2. *Under homogeneous channels, the optimal channel access strategy with 1-bit feedback is $N^* = \min\{n \geq 1 : P_s h_s(n) \geq \omega_1^*\}$. The threshold ω_1^* satisfies the equation*

$$\frac{\log_2 e}{(1 + \omega_1^*)} \left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_1^*}{P_s \sigma^2}}\right) - \frac{\tau_0}{\tau_d P_s \sigma^2} \log_2 \left(1 + \omega_1^* e^{\frac{\omega_1^*}{P_s \sigma^2}}\right) = 0,$$

and can be calculated off-line. The optimal average throughput is $\lambda^* = \frac{\log_2(1 + \omega_1^*)}{1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_1^*}{P_s \sigma^2}}}$.

Proof. For distributed channel access, in the n th observation, Destination $s(n)$ has two decisions:

- when the instantaneous SNR is less than threshold ω_1^* , i.e., $P_s h_s(n) < \omega_1^*$, the feedback bit symbol is 0, meaning that the practical rate is 0.
- when $P_s h_s(n) \geq \omega_1^*$, the feedback bit symbol is 1.

After receiving the symbol, Source $s(n)$ will compare the practical rate $\log_2(1 + \omega_1^*)$ with λ^* , and decide whether to stop or not.

From the result of Theorem 1, threshold λ^* is the unique solution of equation $\mathbb{E}[\max\{u(n) - \lambda, 0\}] = \frac{\lambda\tau_0}{\tau_d}$. Moreover, the problem of finding ω_1^* such that $\sup_{\omega_1 > 0} \lambda^*(\omega_1)$ is equivalent to solve the solution such that

$$\mathbb{E}[\max\{u_n(\omega_1) - \lambda, 0\}] = \frac{\lambda\tau_0}{\tau_d}. \quad (5)$$

By observing that the left side (LHS) of Eq. (5) is a monotonically increasing function, we focus on finding optimal value ω_1 .

The LHS of Eq. (5) can be calculated as follows:

$$\mathbb{E}\left[\sum_{i=1}^K I[s(n) = i] \cdot \max\{u_i(n) - \lambda, 0\}\right] = \frac{1}{K} \sum_{i=1}^K \mathbb{E}[\max\{u_i(n) - \lambda, 0\}]. \quad (6)$$

As link $i = 1, 2, \dots, K$ has channel gains $P_s h_i(n)$ following exponential distribution with probability density function $\frac{1}{P_s \sigma^2} e^{-\frac{x}{P_s \sigma^2}}$, expression (6) can be further calculated as

$$\begin{aligned} \text{LHS} &= (\log_2(1 + \omega_1) - \lambda) \int_{\omega_1}^{+\infty} \frac{1}{P_s \sigma^2} e^{-\frac{x}{P_s \sigma^2}} dx \\ &= (\log_2(1 + \omega_1) - \lambda) e^{-\frac{\omega_1}{P_s \sigma^2}}. \end{aligned} \quad (7)$$

Based on it, Eq. (5) is simplified into

$$(\log_2(1 + \omega_1) - \lambda) \cdot e^{-\frac{\omega_1}{P_s \sigma^2}} = \frac{\lambda\tau_0}{\tau_d}. \quad (8)$$

A closed form expression of solution λ^* is derived with respect to ω_1 below:

$$\lambda^*(\omega_1) = \frac{\log_2(1 + \omega_1)}{1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_1}{P_s \sigma^2}}}. \quad (9)$$

We then analyze the relationship between throughput function $\lambda(\omega_1)$ and threshold ω_1 . The first derivative of the throughput $\lambda(\omega_1)$ can be expressed as

$$\frac{d\lambda^*(\omega_1)}{d\omega_1} = \frac{\frac{\log_2 e}{(1 + \omega_1)} (1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_1}{P_s \sigma^2}}) - \frac{\tau_0}{\tau_d P_s \sigma^2} \log_2(1 + \omega_1) e^{\frac{\omega_1}{P_s \sigma^2}}}{(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_1}{P_s \sigma^2}})^2}. \quad (10)$$

Valuing $\frac{d\lambda^*(\omega_1)}{d\omega_1} = 0$, the optimal ω_1^* satisfies the following equation

$$\frac{\log_2 e}{(1 + \omega_1^*)} \left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_1^*}{P_s \sigma^2}}\right) - \frac{\tau_0}{\tau_d P_s \sigma^2} \log_2(1 + \omega_1^*) e^{\frac{\omega_1^*}{P_s \sigma^2}} = 0. \tag{11}$$

And the second derivative of $\lambda(\omega_1)$ at $\omega_1 = \omega_1^*$ satisfies the inequality that

$$\left. \frac{d^2 \lambda^*(\omega_1)}{d\omega_1^2} \right|_{\omega_1 = \omega_1^*} = \frac{-\frac{\log_2 e}{(1 + \omega_1^*)^2} \left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_1^*}{P_s \sigma^2}}\right) - \frac{\tau_0}{\tau_d P_s^2 \sigma^4} \log_2(1 + \omega_1^*) e^{\frac{\omega_1^*}{P_s \sigma^2}}}{\left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_1^*}{P_s \sigma^2}}\right)^2} < 0. \tag{12}$$

Based on inequality (12), we conclude $\lambda^*(\omega_1)$ is concave and ω_1^* is optimal value satisfying Eq. (12).

Moreover, we prove the uniqueness of ω_1^* . By dividing $e^{\frac{\omega_1}{P_s \sigma^2}}$, Eq. (11) can be transformed into a form that

$$\frac{\log_2 e}{(1 + \omega_1)} \left(e^{-\frac{\omega_1}{P_s \sigma^2}} + \frac{\tau_0}{\tau_d}\right) = \frac{\tau_0}{\tau_d P_s \sigma^2} \log_2(1 + \omega_1). \tag{13}$$

As the left side is monotonically decreasing and the right side is monotonically increasing, the point ω_1^* is unique and maximizes $\lambda^*(\omega_1)$ at $\omega_1 = \omega_1^*$.

Based on the result of Theorem 2, threshold ω_1^* can be calculated off-line. By following the optimal channel access strategy N^* , the transmission rate $\log_2(1 + \omega_1^*)$ is always greater than threshold λ^* , which makes opportunistic channel access efficient. In particular, after n th channel contention, when the winner Source $s(n)$ has channel condition $P_s h_s(n)$ better than threshold ω_1^* , symbol 1 is notified from the destination to allow the channel access. Then, the winner source transmits data by rate $\log_2(1 + \omega_1^*)$. Otherwise, when channel gain is worse than the threshold, symbol 0 is notified, and the winner source actively drops the transmission opportunity and re-contentends with other sources in the next slot. In this case, it is found that different from perfect CSI feedback, the optimal decision on when to transmit depends on threshold λ^* .

3.3 The Case Under Heterogeneous Channels

In most wireless networking environment, different links may have different channel characteristics. We consider the heterogeneous case where all channels between source-to-destination pairs have different channel statistics. For pair $i = 1, 2, \dots, K$, channel gain h_i follows exponential distribution with expectation σ_i^2 .

In this case, each pair has its quantization vector $\mathbf{w}_i = [0, \omega_{i,1}]$, $i = 1, 2, \dots, K$. Based on the result in Theorem 1, for given vectors \mathbf{w}_i , $i = 1, 2, \dots, K$, the system throughput $\lambda^*((\mathbf{w}_1, \dots, \mathbf{w}_K))$ is calculated as

$$\lambda^*((\mathbf{w}_1, \dots, \mathbf{w}_K)) = \frac{\frac{1}{K} \sum_{i=1}^K \log_2(1 + \omega_{i,1}) e^{-\frac{\omega_{i,1}}{P_s \sigma_i^2}}}{\frac{1}{K} \sum_{i=1}^K e^{-\frac{\omega_{i,1}}{P_s \sigma_i^2}} + \frac{\tau_0}{\tau_d}}. \tag{14}$$

We then focus on maximizing average throughput $\lambda^*((\mathbf{w}_1, \dots, \mathbf{w}_K))$ in expression (14).

We define the optimal quantization thresholds as $\{\omega_{i,1}^*\}_{i=1,2,\dots,K}$. The first-order derivative function can be calculated as

$$\left[\frac{\partial \lambda^*}{\partial \omega_{1,1}}, \frac{\partial \lambda^*}{\partial \omega_{2,1}}, \dots, \frac{\partial \lambda^*}{\partial \omega_{K,1}} \right] = \left\{ \frac{\frac{\log_2 e}{1+\omega_{i,1}} \left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_{i,1}}{P_s \sigma_i^2}}\right) - \frac{\tau_0}{\tau_d P_s \sigma_i^2} \log_2(1 + \omega_{i,1}) e^{\frac{\omega_{i,1}}{P_s \sigma_i^2}}}{\left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_{i,1}}{P_s \sigma_i^2}}\right)^2} \right\}_{i=1,2,\dots,K}. \quad (15)$$

By valuing the derivative as 0, we obtain the stationary point as $(\omega_{1,1}^*, \dots, \omega_{K,1}^*)$ where $\omega_{i,1}^*$ satisfies that

$$\frac{\log_2 e}{1 + \omega_{i,1}} \left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_{i,1}}{P_s \sigma_i^2}}\right) - \frac{\tau_0}{\tau_d P_s \sigma_i^2} \log_2(1 + \omega_{i,1}) e^{\frac{\omega_{i,1}}{P_s \sigma_i^2}} = 0. \quad (16)$$

The point $(\omega_{1,1}^*, \dots, \omega_{K,1}^*)$ exists uniquely.

Moreover, we derive second-order derivative matrix $\mathbf{J} = \left[\frac{\partial^2 \lambda}{\partial \omega_{i,1} \partial \omega_{j,1}} \right]_{i,j}$ as

$$\mathbf{J} = \begin{bmatrix} \frac{\partial^2 \lambda^*}{\partial \omega_{1,1}^2} & \frac{\partial^2 \lambda^*}{\partial \omega_{1,1} \partial \omega_{2,1}} & \dots & \frac{\partial^2 \lambda^*}{\partial \omega_{1,1} \partial \omega_{K,1}} \\ \dots & \dots & \dots & \dots \\ \frac{\partial^2 \lambda^*}{\partial \omega_{K,1} \partial \omega_{1,1}} & \frac{\partial^2 \lambda^*}{\partial \omega_{K,2} \partial \omega_{2,1}} & \dots & \frac{\partial^2 \lambda^*}{\partial \omega_{K,K}^2} \end{bmatrix}. \quad (17)$$

By analysis, since each first-order partial derivative depends on variable $\omega_{i,1}$ only, the second-order derivative elements have the following characteristics.

When $i = j$, the matrix element has the form that

$$\frac{\partial^2 \lambda^*}{\partial \omega_{i,1}^2} \Big|_{(\omega_{1,1}^*, \dots, \omega_{K,1}^*)} = \frac{-\frac{\log_2 e}{(1+\omega_{i,1}^*)^2} \left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_{i,1}^*}{P_s \sigma_i^2}}\right) - \frac{\tau_0}{\tau_d P_s^2 \sigma_i^4} \log_2(1 + \omega_{i,1}^*) e^{\frac{\omega_{i,1}^*}{P_s \sigma_i^2}}}{\left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_{i,1}^*}{P_s \sigma_i^2}}\right)^2} < 0. \quad (18)$$

When $i \neq j$, $\frac{\partial^2 \lambda^*}{\partial \omega_{i,1} \partial \omega_{j,1}} \Big|_{(\omega_{1,1}^*, \dots, \omega_{K,1}^*)} = 0$.

Combining the results together, the second derivative matrix \mathbf{J} is diagonal matrix as follows

$$\mathbf{J} \Big|_{(\omega_{1,1}^*, \dots, \omega_{K,1}^*)} = \begin{bmatrix} \frac{\partial^2 \lambda^*}{\partial \omega_{1,1}^2} \Big|_{(\omega_{1,1}^*, \dots, \omega_{K,1}^*)} & 0 & \dots & 0 \\ 0 & \frac{\partial^2 \lambda^*}{\partial \omega_{2,1}^2} \Big|_{(\omega_{1,1}^*, \dots, \omega_{K,1}^*)} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{\partial^2 \lambda^*}{\partial \omega_{K,1}^2} \Big|_{(\omega_{1,1}^*, \dots, \omega_{K,1}^*)} \end{bmatrix}. \quad (19)$$

Since all diagonal elements are negative, the second derivative matrix is negative definite. The function $\lambda^*(\mathbf{w})$ is thus concave, and points $(\omega_{1,1}^*, \dots, \omega_{K,1}^*)$ are optimal. Based on the result, the following theorem is obtained, presenting the optimal channel access strategy.

Theorem 3. Under heterogeneous case, the optimal strategy achieving the maximal throughput λ^* is $N^* = \min\{n \geq 1 : P_s h_s(n) \geq \omega_{s(n),1}^*$, and $u(n) \geq \lambda^*\}$, and for pair $i = 1, 2, \dots, K$ thresholds $\omega_{i,1}^*$ are fixed such that

$$\frac{\log_2 e}{1 + \omega_{i,1}^*} \left(1 + \frac{\tau_0}{\tau_d} e^{\frac{\omega_{i,1}^*}{P_s \sigma_i^2}}\right) - \frac{\tau_0}{\tau_d P_s \sigma_i^2} \log_2(1 + \omega_{i,1}^*) e^{\frac{\omega_{i,1}^*}{P_s \sigma_i^2}} = 0.$$

Moreover, the optimal throughput λ^* satisfies

$$\lambda^* = \frac{\frac{1}{K} \sum_{i=1}^K \log_2(1 + \omega_{i,1}^*) e^{-\frac{\omega_{i,1}^*}{P_s \sigma_i^2}}}{\frac{1}{K} \sum_{i=1}^K e^{-\frac{\omega_{i,1}^*}{P_s \sigma_i^2}} + \frac{\tau_0}{\tau_d}}.$$

In accordance with Theorem 3, the proposed optimal stopping strategy N^* is in a bi-thresholds based structure. For each pair, thresholds $\omega_{i,1}^*$ and λ^* can be calculated off-line. Threshold $\omega_{i,1}^*$ is local value for each source-to-destination pair, λ^* is global. All these thresholds can be calculated off-line, which much benefits the strategy implementation.

Following the optimal strategy, channel access by multiple source-to-destination pairs are operated as follows:

After n th successful channel contention, a winner source $s(n)$ appears, and sends a RTS to its destination. Then, the destination estimates the CSI and compares it with its local threshold $\omega_{s(n),1}^*$. Then, if $P_s h_s(n) \geq \omega_{s(n),1}^*$, it inserts symbol 1 in the CTS; otherwise, it inserts symbol 0. Subsequently, the CTS is feedback to the winner source. After receiving CTS, the winner source will make a decision.

- If the feedback in CTS is 1, a winner Destination $s(n)$ compares the practical rate $u(n) = \log_2(1 + \omega_{s(n),1}^*)$ with the throughput threshold λ^* . If $u(n) \geq \lambda^*$, it lets Source $s(n)$ access the channel by rate $u(n)$.
- otherwise, it will give up this transmission opportunity, and let all sources re-contend the channel.

Upon a successful transmission, one transmission round is finished and all sources contend the channel in the next slot.

Under the heterogeneous case, all links share the same threshold λ^* . Different from the homogeneous case, the quantization rate is different for each pair. Moreover, due to statistical variety of the wireless channels, probability of channel access by each pair is different. In particular, if a pair has a poor channel condition, it may never access the channel, while others in better channel conditions could obtain more opportunities for channel access.

4 Performance Evaluation

This section carries out numerical simulation to validate theoretic results. Consider 5 source-to-destination pairs in the wireless network, and channels from

sources to destinations experience i.i.d. Rayleigh fading. Channel contention parameters of sources are set as $p_0 = 0.3$, $\delta = 25 \mu\text{s}$, $\tau_{RTS} = \tau_{CTS} = 50 \mu\text{s}$.

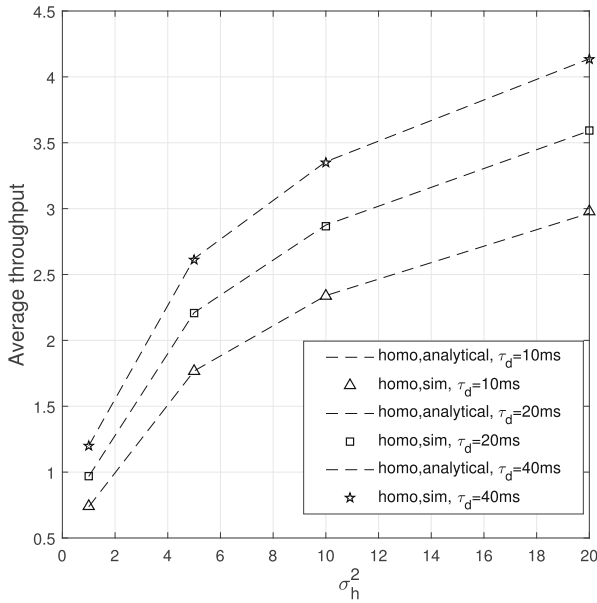


Fig. 4. Performance analysis under homogeneous channels

First, we consider a homogeneous wireless network where average SNR of all channels is σ_h^2 . When data transmission time τ_d varies from 10 ms to 40 ms, the average system throughput is analyzed. In Fig. 4, for a fixed τ_d , the statistical average result shown as ‘analytical’ and the time average result shown as ‘sim’ are compared. For each value of τ_d , throughput curves match well, verifying the theoretic analysis. Moreover, the relation between average system throughput and quantized threshold is analyzed, as shown in Fig. 5. The two curves represent system throughput when σ_h^2 is 10 and 20, respectively.

Secondly, we consider a heterogeneous wireless network where average SNR of all channels may be different. The channel average SNR parameter of all source-to-destinations are $(0.5\sigma_h^2, 0.8\sigma_h^2, \sigma_h^2, 2\sigma_h^2, 2.5\sigma_h^2)$, respectively. As transmission time τ_d are 10 ms, 20 ms and 40 ms, the average system throughput curves are simulated when σ_h^2 varies from 1 to 10. In Fig. 6, the statistical average result shown as ‘analytical’ and the time average result shown as ‘sim’ match well with each other.

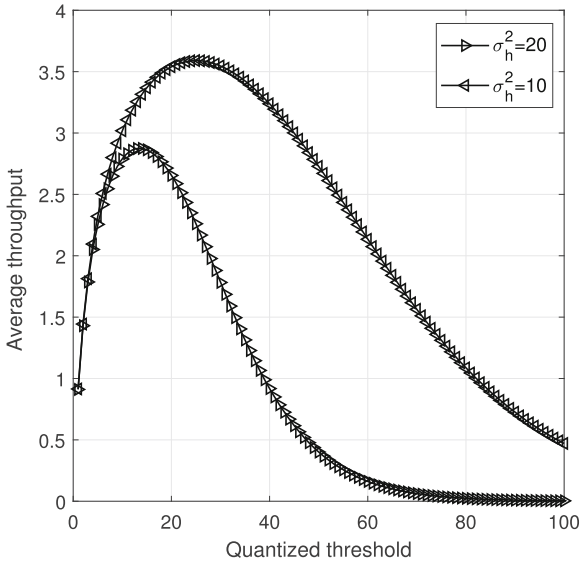


Fig. 5. Influence from quantized threshold

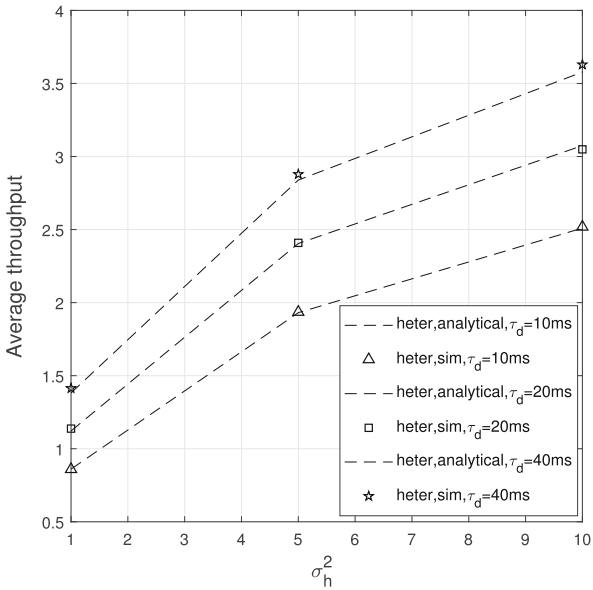


Fig. 6. Performance analysis of heterogeneous channels

5 Conclusion

In practical wireless networks, the channel state information can only be obtained through limited feedback. This research investigates the optimal distributed opportunistic channel access under single-bit CSI feedback. The channel homogeneity and heterogeneity are also studied, and optimal opportunistic channel access strategies are proposed with the optimality rigorously proved. For network implementation, the strategies' structure are analyzed and easy network operation is available. This research should open the potential direction for distributed networking approach under limited CSI.

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